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THE DYNAMO

ITS THEORY, DESIGN, AND MANUFACTURE

C. C. HAWKINS, M.A., M.I.E.E.
Assoc.Amer.I.E.E.

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VOLUME III
(ALTERNATORS)

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PREFACE TO VOLUME III

So large a number of books and papers have been published of recent years on alternating-current machinery that many of the detailed investigations here touched upon have elsewhere been carried to greater depths. Especially is this the case in connexion with eddy-current losses and the phenomena accompanying sudden short-circuits. In all such matters of detail, important and interesting though they are, considerations of space have forbidden more than an attempt to provide for the student a basis for their further consideration with reference to other sources of information, and so far as this has been done with any measure of success, the book will not have failed in its purpose.

In Chapter XXIV attention has been paid to the subject of opened wave windings with simple examples of each kind, since within the experience of the writer the object and the method of determining the openings present great difficulties to the student who has not had much factory experience. In Chapter XXVI the theoretical validity of the treatment of tooth-ripples by calculation of quarter stages will perhaps not be questioned ; its practical usefulness must depend on our ability to map out the flux-wave accurately for the four stages. The process is admittedly tedious, and at each stage it really demands a consideration of the *whole* magnetic circuit, so that it is difficult to secure a sufficiently high degree of accuracy; yet it is just this difficulty which in the author's opinion prevents the immediate solution of the question as to the likelihood of tooth-ripples in any given case or as to their magnitude by any direct mathematical formulæ. All that is wanted is an accurate physical knowledge of the flux of one tooth-pitch for all positions within a pole-pitch.

For the treatment of the effect of armature reaction in the salient-pole machine, the two-reaction method of Prof. A. Blondel is adopted and strongly advocated. For additional evidence as to its accuracy may be cited the determination of the characteristics of two synchronous machines as given by Prof. V. Karapetoff in a paper[1] describing his ingenious instrument, the " Blondelion," which is based on Prof. Blondel's theory. The supposed complexity in the practical application of the method has been much exaggerated, and I would hazard the opinion that its application to the semi-salient-pole rotor of the modern turbo-alternator would be justified by predeterminations more accurate than are given by the methods commonly in vogue.

The maximum instantaneous stator current upon sudden short-circuit is treated under two aspects, (*a*) by means of coefficients of inductance, self and mutual, (*b*) by means of ampere-turns of reaction, direct and alternating ; the two points of view are not inconsistent, but while the first lends itself to a preliminary understanding of the physical actions, the second is the better adapted for approximate calculation of magnitudes. It is hoped that the expression " permanent short-circuit current " will be pardoned ; it is introduced to allow of the use of the symbol, I_{psc}, in clear distinction from the " sudden short-circuit current," I_{ssc}, the suffixes for which are liable to be misinterpreted and are indeed sometimes used for " steady " or " sustained short-circuit."

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[1] *Trans. Amer. I.E.E.* Vol. 42, p. 144.

For many points in connexion with the open-circuit and full-load curves of flux in the turbo-alternator, I would refer the reader to my article on the subject in *Papers on Alternating Current Machinery* (Pitman & Sons), since space does not permit of their repetition here. I have to thank my collaborators in that volume, Prof. S. P. Smith and Mr. S. Neville, for many helpful suggestions and kindly assistance. I am also greatly indebted to Dr. M. G. Say, who was good enough to undertake much of the work of proof-reading for me. For such errors of fact or theory as may still remain, the responsibility is mine, and I must crave the reader's leniency towards them in view of the large amount of detail involved.

For permission to reproduce photographs and other details, my thanks are due especially to Messrs. The Allis-Chalmers Manufacturing Co., Milwaukee, Wis., U.S.A. ; The British Thomson-Houston Co., Ltd. ; The Canadian General Electric Co. ; The General Electric Co., Ltd., Witton ; The General Electric Co., Schenectady, N.Y., U.S.A. ; The Metropolitan-Vickers Electrical Co., Ltd. ; Oerlikon, Ltd. ; La Société Alsacienne de Constructions Mécaniques, Belfort.

C. C. HAWKINS.

1925.

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LIST OF CHIEF SYMBOLS ADDITIONAL TO THOSE OF VOL. I

A . . . = constant amplitude of joint fundamental M.M.F. of polyphase alternator in AT (Chap. xxvii, §§ 8, 15).

AT_a . . = $k_{\mathrm{D}} JZ/4p$ = maximum possible direct-magnetizing armature AT per pole (Chap. xxvii, § 44).

AT_{CM} . . = cross-magnetizing armature AT from all phases of a polyphase alternator as acting on half the cross circuit (eq. 244).

AT_{DM} . . = direct-magnetizing armature AT from all phases of a polyphase alternator as acting on a half magnetic circuit, *i.e.* per pole (eq. 242).

$\overline{AT}_{\mathrm{DM}}$ . . = *average* value of direct-magnetizing armature AT per pole in a salient-pole single-phase alternator (Chap. xxvii, § 29).

AT_{E} . . = field AT expended over single air-gap in non-salient-pole machine including effect of armature AT (eq. 241).

AT_{fl} . . = field excitation per pole under load.

AT_{fo} . . = field excitation per pole on open circuit.

AT_{fsc} . . = field excitation per pole on short circuit, giving I_{sc}.

AT_{gl} . . = air-gap excitation per pole under load (Chap. xxvii, § 41).

AT_{go} . . air-gap excitation per pole on open circuit (Chap. xxvii, § 41).

AT_s . . . = ampere-conductors within a stator slot.

AT_r . . . = resultant of field and armature AT.

$\mathbf{a}$ . . . = maximum amplitude of fundamental M.M.F. of single-phase alternator (eq. 236*a*).

ac . . . = ampere-conductors of stator per unit length of periphery of air-gap.

ac' . . . = ampere-conductors of rotor per unit length of periphery of air-gap.

a_a . . . = attenuation factor of armature (Chap. xxviii, §§ 9, 16).

a_f . . . = attenuation factor of field (Chap. xxviii, §§ 10, 16).

c_{A} . . . = factor which when multiplied by the turns of a coil of one phase on a non-salient-pole machine gives the amplitude of their joint fundamental

M.M.F. in AT for a constant current of one ampere (Chap. xxvii, § 7).

c_{C} . . . = factor which when multiplied by the turns of a coil of one phase on a salient-pole machine gives the amplitude of the joint fundamental of their cross-magnetizing AT for a constant current of one ampere (Chap. xxvii, § 27).

c_{D} . . . = the same as above but for direct-magnetizing AT.

d . . . = deflection of particle with mass-eccentricity e (Chap. xxx, § 47).

δ_e . . . = angle of displacement (electrical) of rotor or E.M.F. vector from its position for steady running at constant speed (Chap. xxxi, §§ 12, 24).

δ_o . . . = initial displacement (electrical) due to cyclic speed-irregularity of engine without synchronizing torque (eq. 316).

E . . . = coefficient of fluctuation of energy (Chap. xxxi, § 28).

E_a . . . = E.M.F. induced in armature phase (if flux undistorted in salient-pole machine, Chap. xxvii, § 4).

E_c . . . = cross E.M.F. of salient-pole armature per phase (Chap. xxvii, § 33).

E_i . . . = internal E.M.F. induced in armature phase.

E_o . . . = open-circuit or no-load virtual E.M.F. per phase.

e_{sa} . . . = E.M.F. self-induced in end-connexions and slots of armature per phase.

e_{sb} . . . = $I\,x_b$ = apparent cross E.M.F. of armature per phase divided by $\cos\phi_o$ (Chap. xxvii, § 35).

ε . . . = cyclic speed-irregularity of engine (Chap. xxxi, § 27).

F_{AR} . . . = fundamental of M.M.F. wave (*i.e.* of armature reaction) in polyphase alternator (Chap. xxvii, § 15, eq. 238).

F_{R} . . . = amplitude of fundamental of excitation in AT on non-salient-pole rotor for air-gap including effect of F_{AR} (eq. 240).

F_g . . . = amplitude of resultant fundamental AT acting on the air-gap in a non-salient-pole alternator (Chap. xxvii, § 17).

f . . . = frequency.

= H.C.F. of S and p.

f_c . . . = centrifugal force per inch run of copper in a rotor slot (eq. 280).

f_i . . . . = centrifugal force per inch run of insulation (eq. 281).
f_w . . . . = centrifugal force per inch run of wedge (eq. 279).
$\hat{f}_t$ . . . . = centrifugal force per inch run of tooth (eq. 282).
f_e . . . . = frequency per second of forced oscillation of rotor (Chap. xxxi, § 29).
f_o . . . . = frequency per second of free oscillation of rotor (eq. 311–314).

G . . . . = specific torque coefficient.
= weight in metric tons.
g . . . . = slots per pole (Chap. xxv, § 6).
g' . . . . = slots per pole per phase.
g_o . . . . = wound slots per pole.
g_o' . . . . = wound slots per pole per phase.
γ . . . . = phase-angle (electrical) between induced E.M.F., E_i, and terminal voltage, V_e (Chap. xxxi).

h_1 . . . . = height of straight side of slot above h_w (Fig. 543*a*).
h_2 . . . . = height of sloping side of half-closed slot.
h_3 . . . . = height of lip of slot-opening.
h_w . . . . = height of winding in slot.

I_c . . . . = corrective component of current, at right angles to V_e, per phase in alternator working in parallel (Chap. xxxi, § 5).
I_c' . . . . = interchange current per phase, flowing through bus bars or between stations (Chap. xxxi, §§ 9, 10).
I_f . . . . = field exciting current.
I_f' . . . . = value to which field current rises at any moment after sudden short-circuit, $= I_f + i_f$ (Chap. xxviii, § 12).
I_{fr} . . . . = *rise* of the field current under sudden short-circuit of 3-phase armature (Chap. xxviii, § 13*b*).
I_{fdr} . . . . = the direct-current component of I_{fr} (Chap. xxviii, § 13*b*).
I_n . . . . = normal full-load current per phase.
$\hat{I}_o$ . . . . = half of inferred maximum crest value of short-circuit current (Chap. xxviii, § 11).
$\hat{I}_{psc}$ . . . . = crest value of permanent short-circuit current (Chap. xxviii, § 11).
$\hat{I}_{ssc}$ . . . . = crest value of sudden short-circuit current (Chap. xxviii, § 11).
$\hat{I}_t$ . . . . = crest value of alternating component of s. c. current, less $\hat{I}_{psc}$.
$\mathbf{I}_{ssc}$ . . . = initial amplitude of armature current in alternator suddenly short-circuited (Chap. xxviii, § 12).
$\mathbf{I}_{ssc\ max}$ . . . = ditto when short-circuit occurs at zero voltage.

I_{sc} . . . = virtual value of permanent short-circuit current under any particular excitation, when it is not necessary to distinguish it as I_{psc} from I_{ssc}.

I_{scf} . . . = the above under full-load excitation.

I_s . . . = virtual value of synchronizing current per phase.

IS . . . = interpolar line of symmetry.

i_f . . . = additional alternating current in field-winding after short-circuit (Chap. xxviii, § 12).

$\hat{i}_f$ . . . = amplitude of i_f.

i_{ssc} . . . = instantaneous value of armature current per phase after sudden short-circuit (Chap. xxviii, § 12).

K . . . = $k_t\ Q/t_i$ (Chap. xxx, § 24), with suffixes *a* for embedded portion, *b* for end-connexions.

K_c . . . = $k_{fc} \times k_{dc}$, *i.e.* product of form factor and differential factor in relation to cross E.M.F. (Chap. xxvii, § 33).

K_r . . . = regulation factor (Chap. xxix, § 4) = $AT_{fl}/(JZ/4p)$.

k_A . . . = factor for calculating fundamental of armature AT reaction on non-salient-pole machine (eq. 239).

k_Q . . . = factor for calculation of cross-magnetizing armature AT on salient-pole machine (eq. 245).

k_D . . . = factor for calculation of direct-magnetizing armature AT on salient-pole machine (eq. 243).

k_c . . . = thermal conductivity of copper in watts per 1 in. cube per 1° C.

k_e . . . = eddy-current coefficient (Chap. xxx, § 23).

k_{ea} . . . = average eddy-current coefficient for slot copper.

k_o . . . = ratio of the field AT on short-circuit to the armature AT from the short-circuit current, = AT_{fsc}/AT_a (Chap. xxvii, § 44).

k_{s1}, k_{s3}, k_{s5} . . . . k_{sn}, differential factor for fundamental, 3rd, ... *n*th harmonic.

k'_{s1}, k'_{s3}, k'_{s5} . . . . k'_{sn}, belt differential factor for fundamental, 3rd, ... *n*th harmonic.

k''_{s1}, k''_{s3} k''_{s5} . . . . k''_{sn}, pitch differential factor for fundamental, 3rd, ... *n*th harmonic.

k'''_{s1}, k'''_{s3}, k'''_{s5} , ... k'''_{sn}, group differential factor for fundamental, 3rd, ... *n*th harmonic.

k_{r3}, k_{r5}, ... k_{rn}, reduction factor for 3rd, 5th, ... *n*th harmonic.

k_t . . . = thermal conductivity of insulation in watts per 1 in. cube per 1° C.

kW_{su} . . = synchronizing kilowatts per radian of electrical displacement, = $W_{su}/1000$.

$\mathscr{L}_a$ . . . = inductance of armature phase (or of one branch of phase when $q > 1$) in henrys.

$\mathscr{L}_{ao}$ . . . = inductance of armature phase under sudden short-circuit, corresponding to equivalent reactance, x_o (Chap. xxviii, § 16).

$\mathscr{L}_c$ . . . = apparent cross inductance of armature phase (or of one branch of phase when $q > 1$) in henrys (Chap. xxvii, § 35).

$\mathscr{L}_d$ . . . = apparent direct self-inductance of armature phase (or of one branch of phase when $q > 1$) in henrys (Chap. xxvii, § 34).

$\mathscr{L}_f$ . . . = inductance of field circuit, due to leakage flux (Chap. xxviii, § 14).

$\mathscr{L}_{fdr}$ . . . = inductance of field circuit due to transient flux (Chap. xxviii, § 16).

L . . . = gross length of armature core between end-flanges in cm.

l . . . . = net iron length of armature core between end-flanges in cm.

l_1 . . . = total length of straight side of coil (Fig. 543*b*) in cm.

l_2 . . . = direct distance between centres of coil-sides in cm.

l_3 . . = length of circumferential end-connexion in cm.

λ_c . . . = $4\pi \times$ equivalent permeance of bore-surface, per cm. of axial length (Chap. xxvii, § 48, eq. 255).

λ_{ca} . . . = ditto of bore surface due to flux linked with part only of belt of conductors.

λ_{cb} . . = ditto of bore surface due to flux linked with whole of belt of a phase.

λ_e . . . = ditto of circumferential end-connexions of coil, per cm. of length (eq. 257).

λ_p . . . = ditto of straight projections, per cm. of length (eq. 256).

λ_s . . . = ditto of slots, per cm. of axial length (eq. 254).

$\mathscr{M}$ . . . = mutual inductance between rotor and stator (Chap. xxviii, § 12).

m . . . = factor for converting total flux of pole-pitch into virtual volts of phase (Chap. xxvii, § 33).
= number of groups of vectors (Chap. xxv, § 2).
= special eddy-current factor (eq. 264).
= special ohmic loss factor (Chap. xxx, § 24).

n . . . . = order of harmonic.

n_e . . . = number of engine impulses per revolution.

NL . . . = neutral line where flux changes direction from entering to leaving stator, or *vice versa*.

$\mathfrak{P}_l$ . . . = leakage permeance.
$\mathfrak{P}_l'$ . . . = leakage permeance for flux linked with all turns of field (Chap. xxviii, § 14).

Q . . . = ratio of short circuit current under full-load excitation to normal full-load current, = I_{scf}/I_n (Chap. xxvii, § 46).
= mean area of heat path through insulation in sq. ins. per unit length axially, with suffixes *a* for embedded portion, *b* for end-connexions (Chap. xxx, § 24).

R_e . . . = resistance of external network.
$\mathfrak{R}_{cc}$ . . . reluctance of half cross circuit = reluctance of air-gap approx.
$\mathfrak{R}_d$ . . . = sum of the direct reluctances, $\mathfrak{R}_c + \mathfrak{R}_t + \mathfrak{R}_g + \nu\,(\mathfrak{R}_m + \mathfrak{R}_y)$.
$\mathfrak{R}_{do}$ . . . = ditto on open circuit for low saturations or on short-circuit.
r_a' . . . = resistance of armature phase in ohms, increased to allow for eddy-currents.
r_i . . . = resistance of interconnector between stations per phase.

s . . . = slip.
σ . . . = ratio of final to initial displacement of alternator in parallel (eq. 321, Chap. xxxi, § 29).

$T_c°$ . . . = temperature of copper in ° C.
T_d . . . = temperature difference between copper and outer surface of insulation in ° C.
T_e . . . = periodic time of forced oscillation from prime mover in seconds (Chap. xxxi, § 26).
T_{iu} . . . = synchronizing torque per radian of electrical displacement, = T_u/p (Chap. xxxi, § 19).
$T_i°$ . . . = temperature of iron in ° C.
= absolute temperature of ventilating air at inlet in ° C.
T_m . . . = conductors per pole on rotor.
$\mathbf{T}_{mf}$ . . . = final amplitude of effective variable torque (Chap. xxxi, § 29).
$\mathbf{T}_0$ . . . = amplitude of variable part of external driving torque (Chap. xxxi, § 20).
$T_o°$ . . . = absolute temperature of ventilating air at outlet in ° C.
T_{pd} . . . = periodic time of oscillation of rotor with damping (Chap. xxxi, § 21).

T_p . . . = free undamped periodic time of oscillation of rotor (Chap. xxxi, § 22).
T_r° . . . = temperature rise of air in ° C.
T_s . . . = synchronizing torque.
$\mathbf{T}_{sf}$ . . . = final synchronizing torque.
$\mathbf{T}_{so}$ . . . = initial synchronizing torque due solely to the variable driving torque (Chap. xxxi, § 29).
T_u . . . = synchronizing torque per radian of mechanical displacement (Chap. xxxi, § 19).
t_i . . . = thickness of insulation in inches.
t_v . . . = instantaneous value of variable part of driving torque (Chap. xxxi, § 26).
θ . . . = mechanical angle of displacement in radians (Chap. xxxi. § 19), corresponding to δ_e (electrical), *i.e.* $= \delta_e/p$.
θ_0 . . . = initial amplitude of mechanical displacement due to variable driving torque without effect of synchronizing torque (Chap. xxxi, § 26).
θ_{mf} . . . = final maximum angle of mechanical displacement (eq. 320).

v . . . = magnetic potential in AT on stator bore due to armature currents.
v_0 . . . = $t\sqrt{2}\,I/q$ = max. AT of belt of a stator phase (Chap. xxviii, § 7).

W_{eu} . . . = synchronizing watts per radian of electrical displacement, $= W_s/\delta_e$.
W_s . . . = synchronizing watts (Chap. xxxi, § 14).
w_c . . . = number of periods of any variable component of driving torque per revolution (Chap. xxxi, § 29).

x_a . . . = true reactance of a complete armature phase.
x_a' . . . = synchronous reactance of armature per phase.
x_b . . . = apparent cross reactance on salient-pole machine (eq. 248) per armature phase.
x_d . . . = apparent direct reactance of armature per phase (Chap. xxvii, § 42).
X_e . . . = reactance of external network.
x_f . . . = reactance of field circuit due to leakage lines.
x_f' . . . = the above referred to an armature phase (Chap. xxviii, § 15).
x_i . . . = reactance per phase of interconnector between stations.
x_o . . . = equivalent reactance of armature phase, $= x_a + x_f'$ (Chap. xxviii, § 15).

y . . . = effective width of field.
y' . . . = mean pitch between sides of a coil.
Y . . . = pole-pitch.

Z_e . . . = impedance of network, expressed in ohms.
Z_t . . . = total impedance of network and alternators.
z_a . . . = impedance of armature, expressed in ohms.
z_a' . . . = synchronous impedance of armature phase (Chap. xxvii, § 42), expressed in ohms.
z_t . . . = impedance per phase of interconnector between stations.
ζ . . . = $k_o \, k_D$ (Chap. xxvii, § 45).
Φ_c . . . = flux of cross field of armature (Chap. xxvii, § 32).
ϕ_a . . . = angle of lag of armature current behind E.M.F. consumed by true armature impedance.
ϕ_a' . . . = angle of lag of armature current behind E.M.F. consumed by synchronous impedance,
which also = ϕ_o on permanent short-circuit.
ϕ_e . . . = angle of lag or lead of external current in relation to terminal voltage, V_e.
ϕ_i . . . = angle of lag or lead of current per phase in relation to induced voltage, E_i, per phase.
ϕ_{lo} . . . = leakage flux of field-magnet on no-load.
ϕ_o . . . = angle of lag or lead of current-vector in relation to interpolar line of symmetry, IS.
Ω . . . = mean angular velocity of alternator.
ω . . . = angular velocity in radians per sec.
ω' . . . = angular velocity of shaft whirling round centre-line between bearings (Chap. xxx, § 49).
ω_e . . . = angular velocity in electrical radians per sec. $= 2\pi f$.
ω_k . . . = special value of ω' yielding a stable deflection for a perfectly balanced shaft, and closely = critical speed for whirling of shaft (Chap. xxx, § 49).
ω_o . . . = maximum value of variable part of velocity of alternator.
ω_v . . . = instantaneous value of variable part of velocity of alternator (Chap. xxxi, § 26).
ω_{crit} . . = critical angular velocity for whirling of unbalanced concentrated mass (Chap. xxx, § 49).

Note—Clarendon (that is, heavy) letters indicate the maximum value of quantities varying in time.

THE DYNAMO

CHAPTER XXIV

ARMATURE WINDING OF ALTERNATORS

§ 1. Suitability of the alternator for high pressures.—While the continuous-current closed-circuit dynamo is limited to pressures not exceeding 4000 volts[1] owing to the difficulty of commutation at the brushes, and partly also of insulating the conductors of its rotating armature, the alternator is by contrast well adapted on every score for the production of high voltages. Its armature coils may be stationary and heavily insulated; there is no commutator with its liability to trouble from sparking; lastly, the alternating-current transformer with its stationary windings affords a simple, economical, and thoroughly reliable means for transforming electrical energy from a high to a low pressure or *vice versa* just as the circumstances demand. In the early days of electric lighting, single-phase alternators were largely employed in central stations for the smaller cities and suburbs which could not be served by the low-tension continuous-current system; at pressures of 1000 or 2000 volts step-down transformers dotted over large scattered areas could be fed without an excessive amount of copper in the transmitting lines. As the demand for lighting increased and also became more concentrated, the spread of the three-wire system and the raising of the pressure at the consumer's terminals to 240 volts enabled the continuous-current system to reach a wider area of dense supply with reasonable economy of copper in the mains; at the same time energy could not be supplied for motive purposes on the alternating-current system, owing to the want of a satisfactory single-phase motor. On both counts, therefore, the continuous-current system found most favour. With the introduction, however, of the induction motor and polyphase alternator, the last-named objection was removed, and the alternating current once again recommended itself from the ease with which the energy could be transmitted at high pressures up to as much as 60,000 or 100,000 volts, and from the flexibility with which it could be adapted to meet every kind of demand. The generating station could now be placed on some convenient and economical site, perhaps near to a waterfall or to a coal-field, at a considerable distance from the area to be supplied; the alternator could itself be built directly for pressures of 6600, or in large sizes for as much as 11,000 to 15,000 or even 20,000 volts, while the energy could be supplied at high pressure[1] and transformed down at a substation, or again converted into direct-current form for traction purposes by means of the motor-generator or rotary converter. The latter serves as the connecting link between the alternating and continuous systems, and is largely employed for traction or railroad work, for which the continuous-current series-wound motor is well suited on account of its accelerating and other characteristics.

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[1] A special dynamo with stationary armature giving 25,000 volts was built in 1902 to the design of M. Thury for testing purposes in connexion with the St. Maurice-Lausanne transmission. For 12,000-volt dynamos for wireless work, see S. R. Bergman, *Trans. A.I.E.E.*, Vol. 42, p. 910.

As standard voltages for alternating-current generators, 2200, 3300, 6600, and 11,000 volts are in general use for apparatus taking 2000, 3000, 6000, and 10,000 volts respectively.

§ 2. Single-phase drum armature winding.—Owing to the absence of the commutator, the winding of the alternator armature is somewhat simpler than that of the analogous continuous-current machine. The open-ended wave principle of winding, which is the type most commonly employed, has already been described in its simplest forms in Chapter XI, § 7; it only remains to deal with more highly developed cases, and in the first place with it in its single-phase form, as being the basis from which polyphase forms can afterwards easily be derived.

Using the same method of representation as in Figs. 95–97, in which the front end-connexions of the coils in the left-hand portion of the diagram appear on the inner periphery of the radial or disc diagram on the right-hand side, Fig. 443 shows a winding concentrated in a single slot per pole. This in general yields an E.M.F. wave of somewhat unfavourable shape, and the armature reaction is high; further, if there is only a single large undivided[2] coil per pair of poles (Fig. 443), the inductance of the conductors reaches a maximum (although this is not always a disadvantage), and the cooling conditions both for the portions of the coils embedded in the slots and for the end connexions are poor. On the other hand with this arrangement the maximum difference of potential between any wires situated in the same slot can never exceed that of one coil, and the beginning and end of the entire series of coils between which exists the full voltage of the machine are kept well apart; further, with an even number of pole-pairs, the top half of the stator can be removed after undoing two connexions between the coils above and below the horizontal plane, as shown by the dividing line on the horizontal diameter of the stator. With an uneven number of pole-pairs, the division must be made above the horizontal plane to avoid having to undo one complete coil. An inside end of a coil is joined to the outside of its next neighbour, and it should be noticed that if only the connexions of the coils are considered, the scheme becomes that of the simple wave-winding of Fig. 95.

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[1] Transmission at 220,000 volts is now within the field of practice in dry climates.

[2] For "divided" and "undivided" (or "hemitropic") coils, see Chapter IX, § 5.

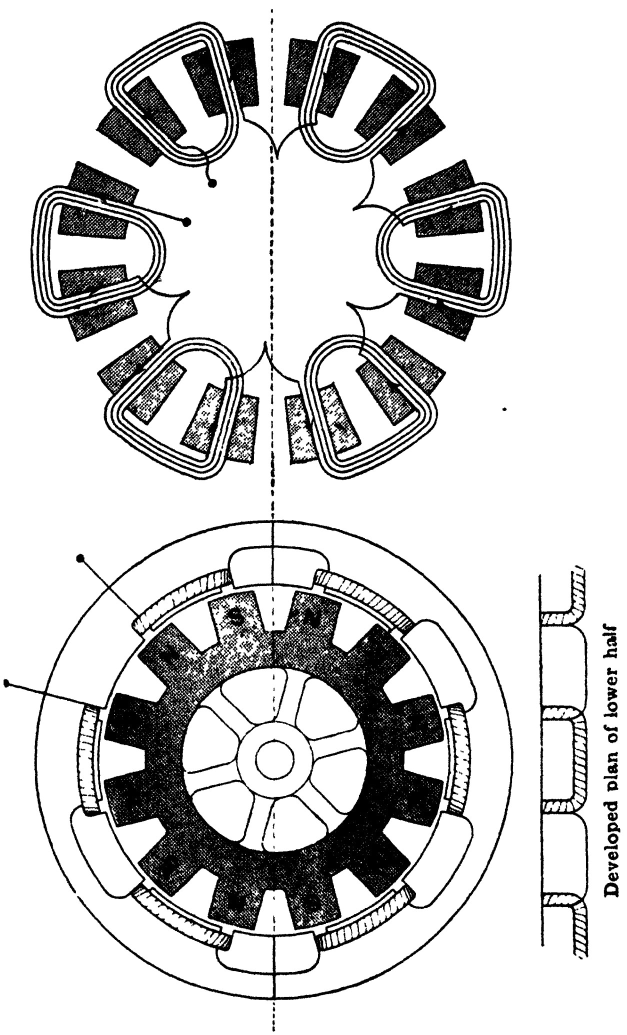

FIG. 443.—Single-phase winding, one coil per pole-pair, *i.e.* undivided and concentrated in one slot per pole. The stator can be split.

Although the simplest type, and here introduced first as forming the basis of polyphase windings in common use, yet for single-phase alternators the undivided coil owing to its unsymmetrical effect on the distribution of magnetic potentials in the magnetic circuit is not so suitable as divided coils.

When the coils are divided so that there are as many as there are poles (Fig. 444), each coil has its outside end connected to the outside end of the adjacent coil on one side, and its inside end connected to the inside end of the adjacent coil on the other side. With such divided coils, as already explained in Chapter IX, § 5, the overhanging projection of the end-connexions is less, there is less copper, the inductance is less, and the cooling effect is better. On the other hand, if the winding is connected in series right round the armature, in one slot containing the adjacent halves of the first and last coils the strain on the insulation of contiguous wires

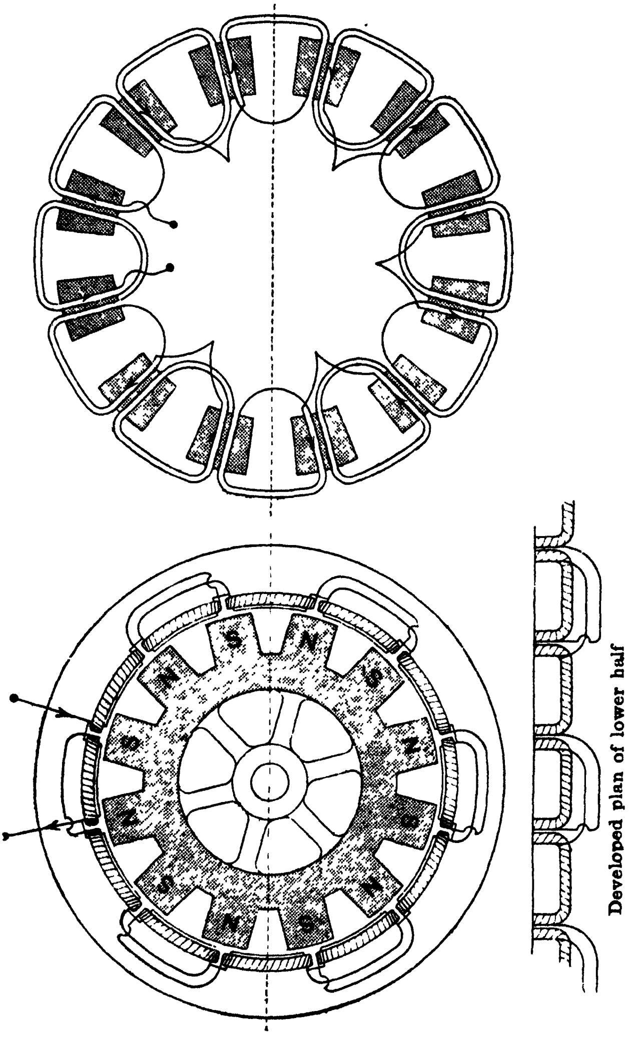

Fig. 444.—Single-phase winding, one coil per pole (*i.e.* divided), concentrated in one slot per pole. Two coils must be unwound or removed to divide stator on a horizontal diameter.

amounts very nearly to the total voltage of the machine. This objection may be removed by dividing the winding into two halves in parallel; the coils which are in series are then wound in the

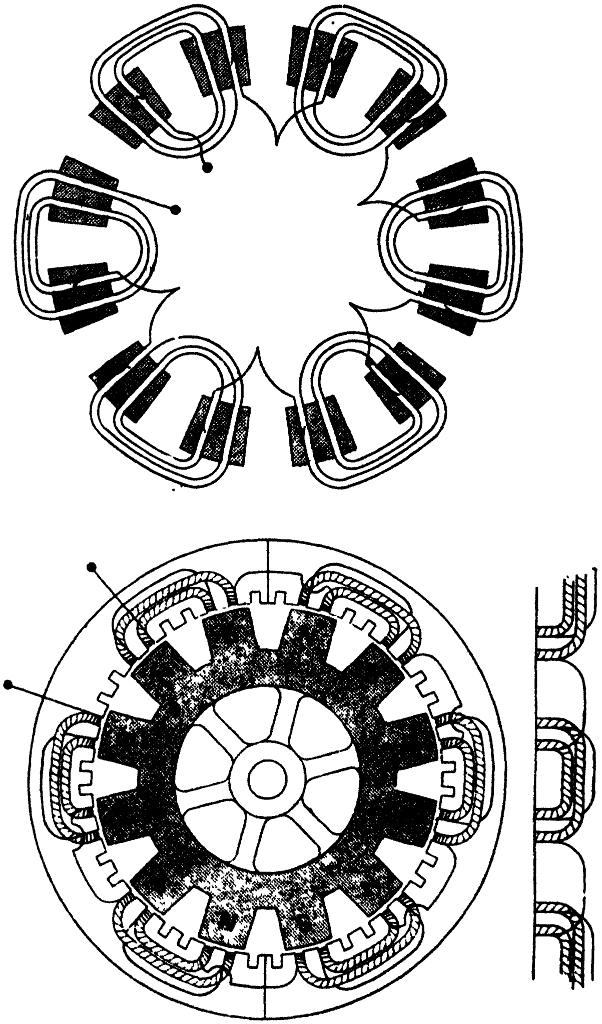

FIG. 445.—The same as FIG. 443, but with each coil-side grouped in two slots. The stator can be split.

same direction, and this direction is reversed at the points where the two halves are united in parallel. When the number of pairs of coils is even, the junctions are made by connecting together contiguous inside ends of one pair of coils and the distant inside ends of the opposite coils. With an uneven number of pairs of coils, contiguous inside and contiguous outside ends are joined together. The maximum difference of potential between any neighbouring wires is then equal to that generated by a pair of coils, and so in a machine giving, *e.g.* 2200 volts of virtual E.M.F. with 12 poles does not amount to more than about 1050 volts at the moment of maximum induced E.M.F.

The method of parallel division is, however, accompanied by the disadvantage that there are then twice as many active wires, each of half the area, and consequently there is more space wasted in insulation. It is therefore only suited to comparatively low voltages or to high speeds with large currents when the wires or bars become too massive to bend; in such cases it can be carried to the extreme of dividing the whole into p identical circuits in parallel with undivided coils or $2p$ identical circuits with divided coils.

It is evident that in either Fig. 443 or Fig. 444 each coil-side may be divided between two or more slots, the winding becoming thus grouped instead of completely concentrated within a single slot per pole. Thus Fig. 445 (*cp.* Fig. 68) shows the winding of Fig. 443 when the side of each coil is divided into two groups situated near together; the advantages of Fig. 443 are retained, and the maximum difference of potential between wires in the same slot is reduced to one-half or with 3 slots to one-third, and so on; the slot inductance is also reduced, and the cooling conditions are improved, since a greater iron area is available to conduct the heat away from the coils; lastly the wave shape is bettered, so that the arrangement is often employed for high voltages. In the single-phase case the wound slots are usually fairly close together to avoid undue reduction of E.M.F.; hence, as mentioned in Chapter IX, § 7, in order to give a more constant reluctance to the magnetic circuits, and to make use of a standard slotting which is also suitable for polyphase machines, the periphery of the stator may have a number of uniformly distributed slots, some of which are unwound. When the same grouped distribution of the winding is applied to Fig. 444, the advantage is gained that the stator becomes divisible on a plane *between* two wound slots by merely undoing two connexions (Fig. 446). In the front views of Fig. 445 and 446 the coils are shown bent back to expose the empty slots.

So far the coils even when divided between 2 or more slots per coil-side have been shown wound concentrically as it were, one inside another (*cp.* Fig. 68), and disposed in a single layer round the stator periphery.

Where transmission over long distances necessitates the use of high voltages, step-up transformers are employed. The generating machines can then be built for low voltage with fewer conductors, and little loss of space in insulation. Thus, instead of the wire coils of Figs. 443–6 with numerous turns, bar winding with a few large active conductors per group becomes suitable.

With a number of bars in slots in a single layer, it is evident that by means of simple curved end-connectors, riveted and soldered to the bars, they can be coupled up to imitate concentric coils

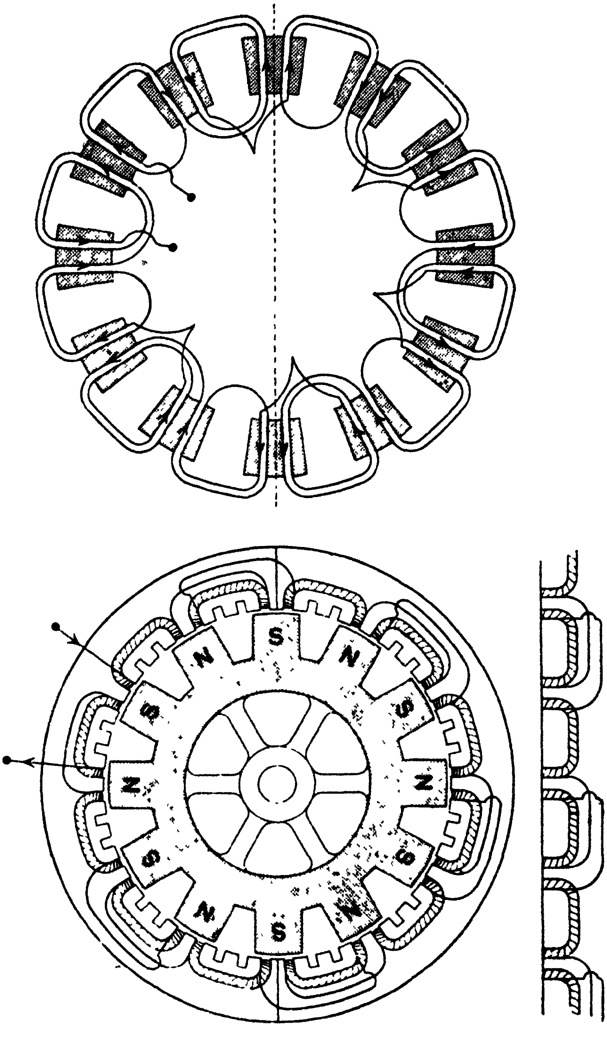

FIG. 446.—The same as Fig. 444, but with belt per pole divided between 2 slots.

(Fig. 447), but there must then be as many different lengths of such end-connectors as there are sheaths in the composite concentric coil. Thus a 3-turn coil demands 3 lengths at the back-end, 2 at the front, and a wave-connector from coil to coil.

Still keeping to a coil-side divided between two or more slots in a single layer, it has been already shown in Fig. 97 that while retaining the wave-connexion between the complete coils, their component members may themselves be lap-wound instead of being concentric. This applies equally to wire-coils as to bars: in the former case, a twist is given to the end-connexions which enables them to pass by one another (Figs. 448 and 449); in the latter case the end-connexions are of V butterfly shape (Fig. 450), similar to the involute end-connectors of a continuous-current machine, but, the stator being external, with the apex of the V pointing outwards.

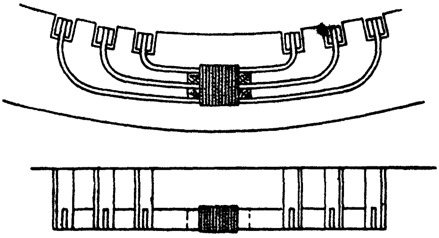
FIG 447.—End-connexions for bar concentric coils.

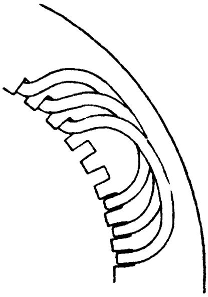
FIG. 448.
Lap-connected wire coils

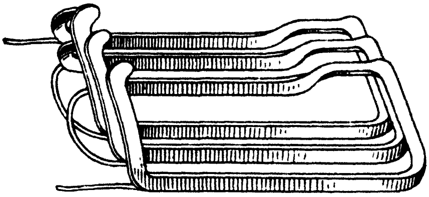
FIG. 449.
Lap-connected strip coils.

As soon as we pass to a *two-layer winding*, then with open slots the component coils, whether wave or lap-wound, can in either case be cheaply wound on a former exactly as for a barrel-wound continuous-current armature, with a bend at the nose of the V end-connexions to effect the passage from an upper to a lower layer. The advantage gained is that the coils are enabled thereby to pass one another easily. So far as single-phase work is concerned, this advantage is of little account. Thus *e.g.* with divided coils Fig. 451 is the analogue of Fig. 444, the layer of coil-sides at the bottom of each slot being represented in the right-hand portion by dotted lines. The two coil-sides per slot being arranged in two layers might suit the slot dimensions better, but otherwise no advantage is gained. The diagram is however introduced, since from it the possibility of another set of coils passing the first is seen, and the real advantage will be found to arise with several phases, (*a*) in the freedom which the double layer gives as regards the pitch of

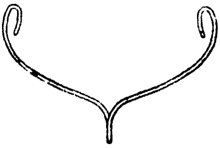
FIG. 450.
Bar connector.

the coils reckoned in slots, and (*b*) as rendering possible closed windings similar to those of a continuous-current armature.

With a two-layer bar winding several other variants become

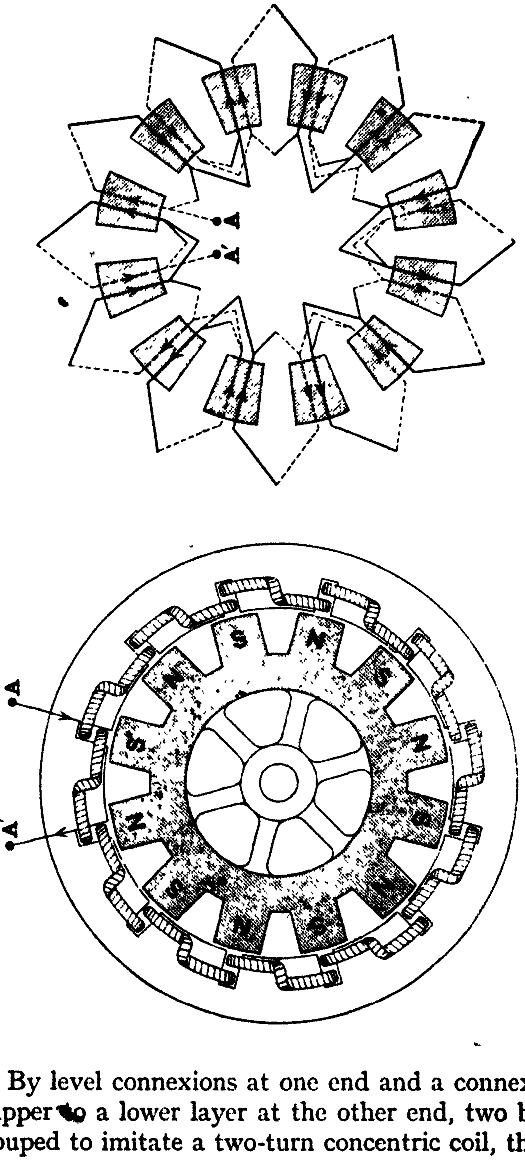

Fig. 451.—Two-layer single phase winding.

possible. By level connexions at one end and a connexion passing from an upper to a lower layer at the other end, two bars per slot can be grouped to imitate a two-turn concentric coil, the two turns being mounted one above the other (Fig. 452) ; a higher voltage can be thus reached with but small difference of potential between bars in the same slot.

In a wave-wound armature with one slot per pole and two bars per slot, after completing one tour of the armature, it is possible instead of continuing the wave progression in the same sense (as in Fig. 96 which only suits a single layer) to reverse[1] the direction (at *B*, Fig. 453) by means of a connector passing from one slot to another in the same layer, and so to fill up the slots on the return

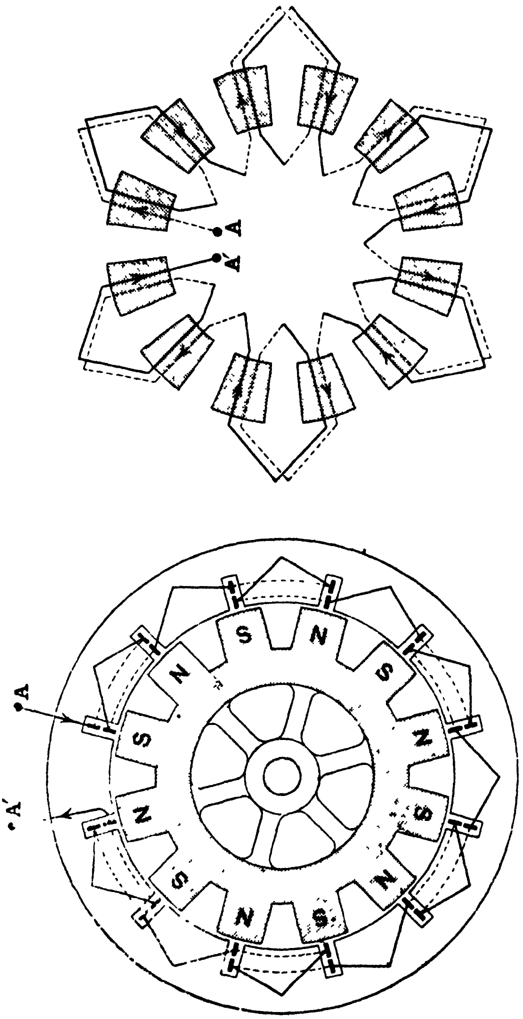

FIG 452 —Two-layer bar wave-winding

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[1] With a number of slots which is uneven and therefore not a multiple of the number of poles, continuous progression would be possible, but such numbers of slots would not be convenient for standard machines.

tour. With two slots per pole and two bars per slot, after completing two tours in the same direction, an upper and a lower bar in the same slot can be coupled together, and thence two complete

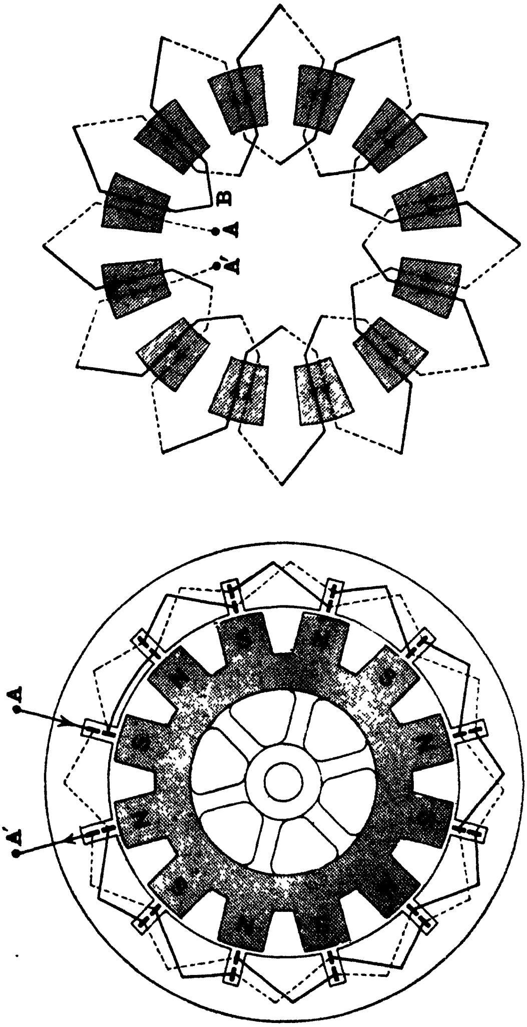

Fig. 453.—Two-layer double-tour bar wave-winding.

tours are made in the reverse direction (winding of Pichler); the connexions to the terminals then come out from opposite sides of the stator core (Fig. 454). In the diagram the rear connexions are omitted for the sake of clearness. Only a single length of end-connector is required.

If the number of pole-pairs is even, it is always possible to give different numbers of slots to alternate pairs of belts; thus with undivided coils let coil 1 occupy 2 slots per coil-side, and coil 2, the next in series, occupy 1 slot per coil-side; then we have in effect a fractional number of $3/2 = 1\frac{1}{2}$ slots per pole. Or again

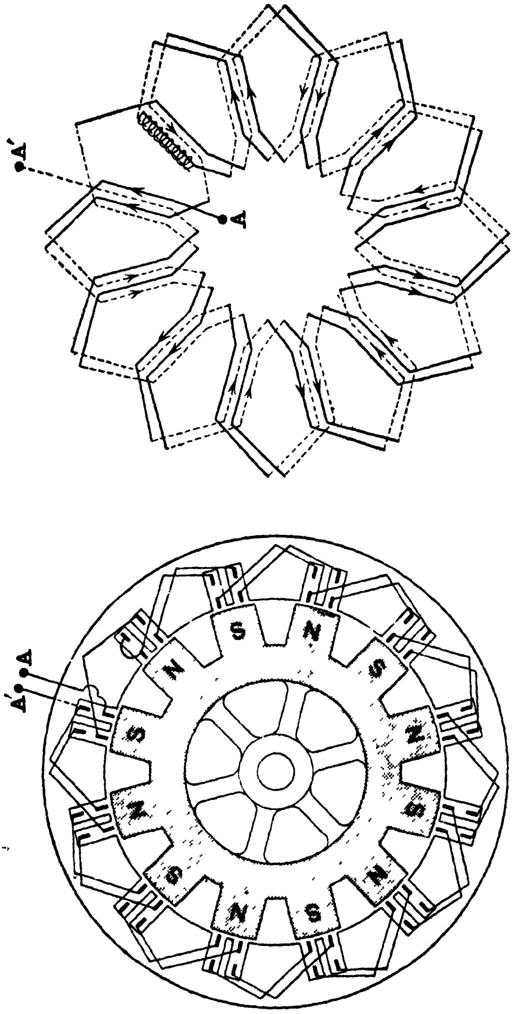

Fig. 454.—Two-layer four-tour bar wave-winding.

3 and 2 slots per coil-side in successive coils give $5/2 = 2\frac{1}{2}$ slots per pole, and 3 and 4 slots per coil-side give $3\frac{1}{2}$ slots per pole. Twice the double pole-pitch, *i.e.* a 4-pole machine, is in such cases the unit which is repeated so far as the winding is concerned, instead of the double pole-pitch. Three times the double pole-pitch could also be used as completing a unit in a machine with $p = 3$ or a multiple of 3.

Given a stator core stamped with 3 slots per pole for 3-phase work, it can be used for single-phase work if the coils are placed in the two outermost slots per pole with the central slot left empty, but the pitch of the coil is then, of course, only $\frac{2}{3}$rds of the pole-pitch. Or, with 6 slots per pole, 4 only are wound.

§ 3. The interlinking of phases in polyphase systems.—The relations of currents and voltages in the phases of the polyphase systems in everyday use are given in numerous text-books, so that it is only necessary here to remind the reader that in the *quarter-phase interlinked system* with two phases and E.M.F.'s in quadrature, one end of each phase-winding is interlinked at a common terminal and common transmitting line, and the two phases are otherwise independent; the virtual voltage between the unconnected pair of terminals is, then, $\sqrt{2}$ or 1·414 times the true phase voltage E between either of these terminals or outer lines and the common terminal or common transmitting line. But if alternatively the two phases are interlinked by a common junction at their centres, each is thereby divided into two halves each giving $e = E/2$ volts, and there are 4 terminals; the arrangement is then best regarded as a *star-connected four-phase system*; according as any two terminals belong to the same set of coils or to different sets, the virtual voltage between them is $2e = E$ or $\sqrt{2}e$.

With 3 phases 120 electrical degrees apart in time, the meaning of "a phase" of the generator winding is un-ambiguous; its virtual voltage being e, the voltage between any pair of terminals or transmitting lines on a balanced load is $\sqrt{3}e$ or 1·732 times e if the phases are *star- or Y-connected*, or e if they are *mesh- or Δ-connected*, the virtual current in a transmitting line being in the two cases i and $\sqrt{3}i$, where i is the current in one phase. With star-connexion, one generator phase is at any instant carrying outwards or away from the common junction at the centre of the star the sum of the currents which are flowing inwards towards the centre in the other two phases, or *vice versa*. The same phase-currents in the generator, if connected in mesh, result at any instant in the current in one phase being, say, in a clockwise direction round the mesh and the remaining pair of currents being both counter-clockwise.

Two particular instants in each period possess special interest from their simplicity and the ease with which they lend themselves to calculation; the two cases are (1) when one phase is carrying the maximum current **I** and the current in each of the other two phases is half of the maximum, and (2) at a moment differing from the above by a quarter of a period when the currents in two of the phases are equal and each is 0·866 **I**, one being towards the centre and the other away from it or in opposite directions round the mesh, and the current in the third phase is zero. Thus as regards the transmitting lines the relations at the two crucial instants are as follows—

Current in Y-connected armature.	Current in Δ connected armature.	Current in transmitting lines.
(1) Maximum in one phase, ½ max in the other two.	(2) Current in two phases equal, in 3rd phase zero	(1) Maximum in one line, ½ max. in the other two.
(2) Current in two phases equal, in 3rd phase zero	(1) Max. current in one phase, ½ max. in the other two	(2) Current in two lines equal, in 3rd line zero.

§ 4. Polyphase armature windings with concentric or lap-wound coils in a single layer.—The conversion of the preceding winding diagrams into forms suitable for quarter- or four-phase generators is easily effected by superposition upon the original set of coils of a second set exactly similar but displaced through an angle corresponding to half the pole-pitch; while for three-phase machines two new sets of coils must be added, displaced respectively through one-third and two-thirds of the pitch. Thus Fig. 455 is the quarter-phase equivalent of Fig. 443, with one coil per pair of poles per phase, and with the winding concentrated in one slot per pole per phase. Whether the winding be thus concentrated or be grouped in two or more slots per pole per phase, the stator can be split to enable the internal magnet to be lifted out, since at certain points at opposite ends of a diameter a radial section of the armature cuts no coils, and only the connecting wires between the coils need to be separated. Fig. 456 shows the quarter-phase equivalent of Fig. 444 with as many coils in each phase as there are poles; if the stator is split for convenience of carriage, two coils must be wound after assemblage *in situ*, and must be unwound if the upper half of the stator is to be removed. In the *B* set of coils of either Fig. 455 or Fig. 456, the end-connexions after clearing the slots are bent up at right angles and lie nearer to the sides of the stator; in the *A* set of coils the end-connexions after clearing the *B* set can then run practically straight across from slot to slot, care being taken that the length of the two kinds of coil is alike. Four terminals are shown, and the armature circuits may be either independent or interlinked as a star at their central points.

The three-phase equivalent of Fig. 443 with one undivided coil per pair of poles per phase permits of two alternative arrangements. Thus (1) if each of the two additional windings has a relative displacement of two-thirds of the pitch, although the stator cannot be split, the overhanging projections of the coil-ends occupy at each end two "ranges"[1] only, and the series of coils of any one phase

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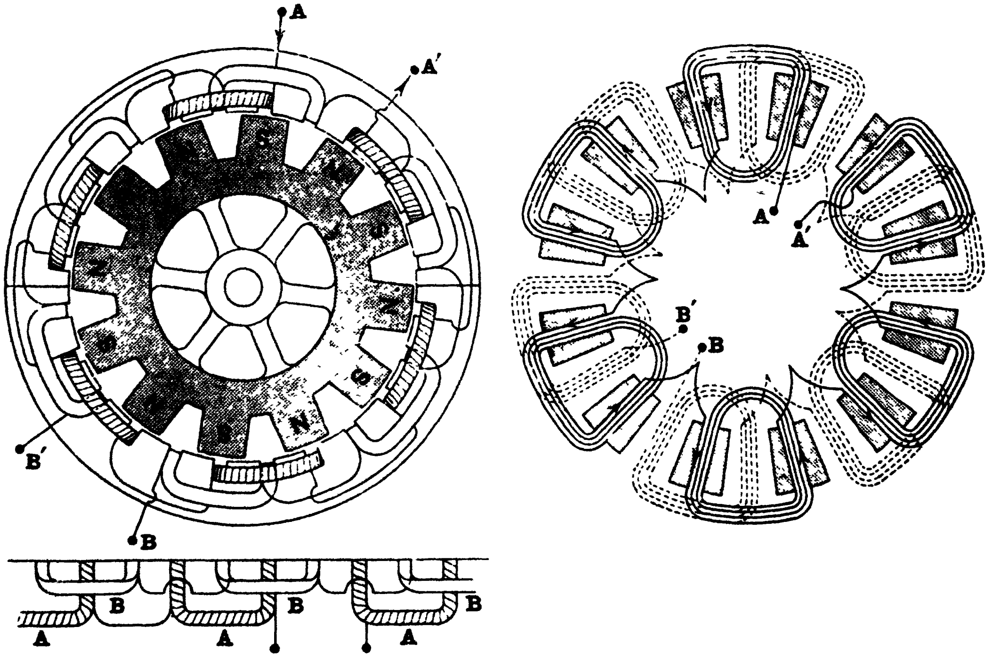
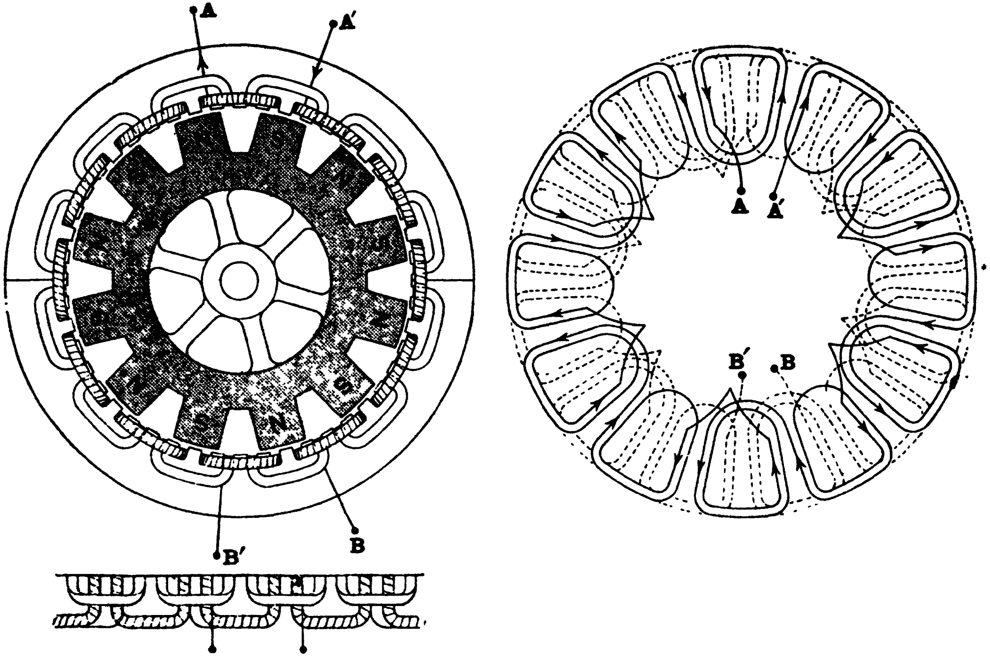
[1] Or "tiers" or "planes," as it is also expressed.



Developed plan of upper portion

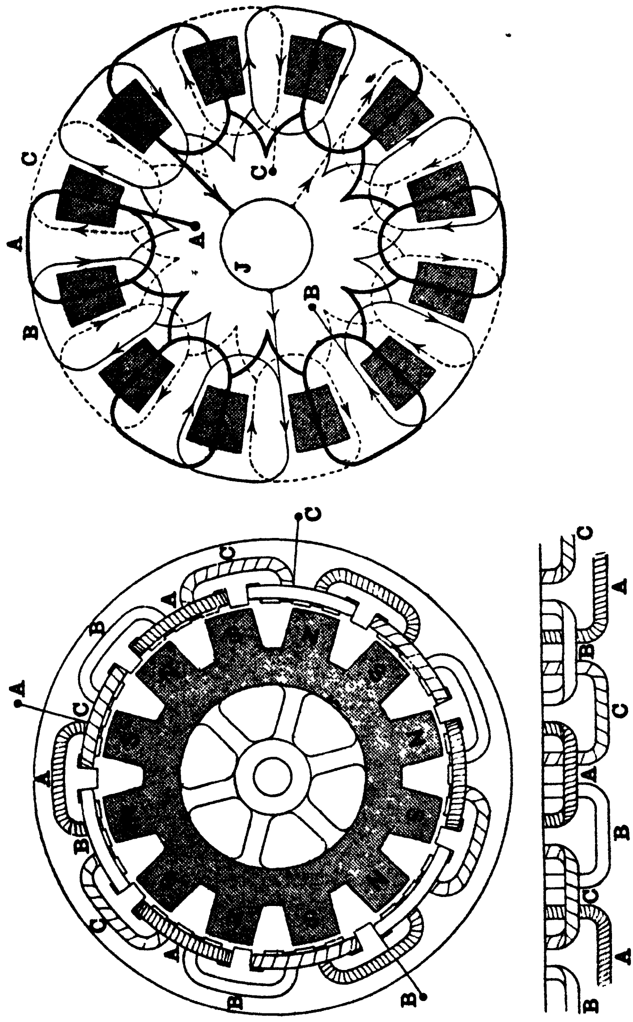
FIG. 455.—Quarter-phase winding, one coil per phase per pole-pair (*i.e.* undivided). The stator can be split.



Developed plan of upper portion.

FIG. 456.—Quarter-phase winding, one coil per phase per pole (*i.e.* divided). Two coils must be unwound or removed to divide stator.

consists alternately of straight and bent-up coils (Fig. 457); the appearance of such a three-phase armature is therefore exactly the same as that of the quarter-phase armature of Fig. 456, except in the number of sides of coils embraced within the pole-pitch. The moment chosen for illustration is that at which the current in phase *A*, shown in heavy lines, is at its maximum, and this is for simplicity assumed to occur when the coil-sides of phase *A* are immediately opposite to the centre of the poles; the currents in phases *B* and *C* have then half their maximum values. The E.M.F.'s are marked by arrows, and it will be seen that whereas



Developed plan of upper portion.

Fig. 457.—Three-phase winding, one coil per phase per pole-pair (*i.e.* undivided): coil-ends in 2 ranges, alternating in each set. Two coils must be unwound or removed to divide stator.

12 coil-sides of phase *A* are immediately under the action of the poles, the coil-sides of each of phases *B* and *C* are in the fringes of the poles, giving each only half the E.M.F. of phase *A* in correspondence with the fact that they are carrying half the maximum current. The winding is star-connected, as shown by the common junction or neutral point given by the connexion *J*. The current at the moment flows into the common junction from phase *A* and out by the two other phases to their terminals *B* and *C*. If the number of pole-pairs is uneven, the total number of coils on the machine, being $3p$, must also be uneven; the number of straight coils on the machine cannot then be the same as the number of bent-up coils. Hence one coil in one of the phases must be partly straight and partly bent (Fig. 458).

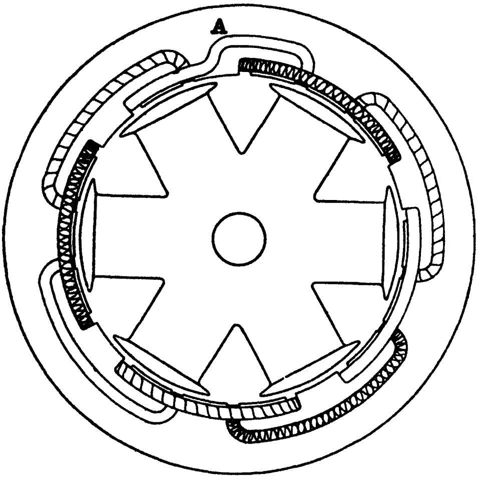
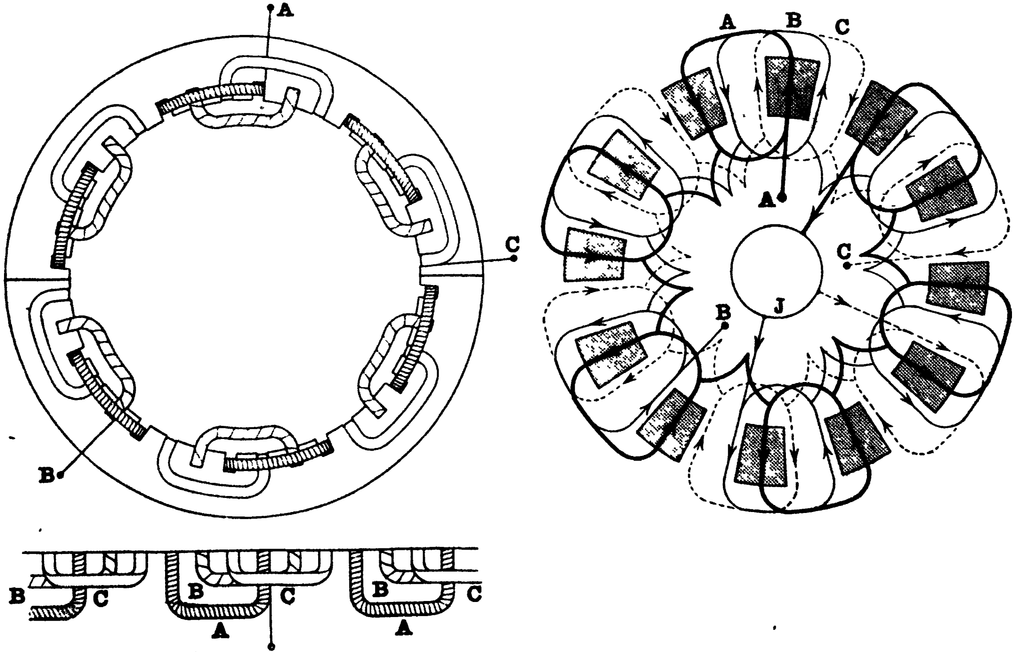


FIG. 458.—With p uneven, coil end *A* partly straight and partly bent.

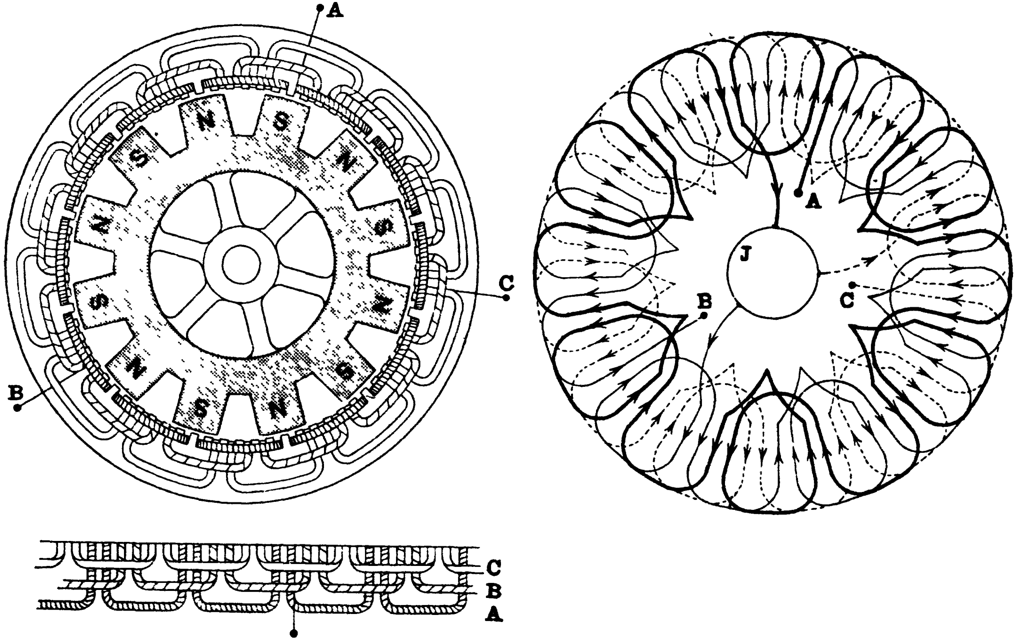
Next (2) if phases *A* and *C* are retained unaltered as in the right-hand part of Fig. 457, let the direction of winding phase *B* be reversed; the effect as shown by Fig. 459 is apparently to give a relative displacement of the three windings through one-third of the pitch or through 60 electrical degrees, but this appearance is merely due to the reversal in the hand of the winding of phase *B* and to the different position of the junction-end of *B* relatively to the other terminals. Electrically, therefore, no difference has been made, but the mechanical result is that although a section of the stator cuts through three coils at certain points, and therefore three ranges of coil-ends are required, there are other points where no coils are cut; hence the stator can be split on the horizontal diameter, and finally one of the 3 sets of coils, viz., the *B* set, can be bent inwards towards the centre.

The three-phase equivalent of Fig. 444, with as many coils per phase as there are poles, is shown in Fig. 460—again star-connected and at the same chosen instant. While there are 24 coil-sides of phase *A* immediately under the poles, there are only 12 in each of phases *B* and *C*, the remainder being outside the pole-edges, so



Developed plan of upper portion.

FIG. 459.—Three-phase winding, one coil per phase per pole-pair (*i.e.* undivided), but coil-ends in 3 ranges. The stator can be split, but the rotor cannot be drawn out.



Developed plan of upper portion.

FIG. 460.—Three-phase winding, one coil per phase per pole (*i.e.* divided) ; coil-ends in 3 ranges. Four coils must be unwound or removed to divide stator, but rotor can be drawn out.

that the E.M.F. of phases *B* and *C* is half that of *A*. With a stationary armature, it will be seen that the third or *C* set of coils would be most conveniently bent inwards in the opposite manner to the *B* set, but that they would then project in front of the magnet and, since the stator cannot be split, this would prevent its withdrawal sideways; they must therefore in practice be bent up outwards. Three ranges of coil-ends are therefore required (Fig. 460), and owing to this disadvantage coupled with the length of the coils and their numerous crossings, one coil per pole per phase is not so convenient or so often employed in 3-phase work as undivided coils with coil-ends in 2 ranges. The diagrams and the use of the expressions "one coil" or "undivided coil" here and in the following table must not be understood to preclude division of the *coil-side* between 2 or more slots; in practice a coil-side is usually so divided between a number of slots ranging from 2 to 12.

Thus to summarize the above coil-windings in tabular form—

Single-phase machines—		
One coil per pole-pair (*i.e.* undivided)	Concentrated (Fig. 443) or grouped (Fig. 445)	Stator can be divided without unwinding coils.
One coil per pole (*i.e.* divided)	Concentrated (Fig. 444)	Stator cannot be divided without unwinding coils.
	Grouped (Fig. 446)	Stator can be divided without unwinding coils.
Quarter-phase machines—		
One coil per phase per pole-pair	(Fig. 455)	Stator can be divided without unwinding coils.
One coil per phase per pole	(Fig. 456)	Stator cannot be divided without unwinding coils.
Three-phase machines—		
One coil per phase per pole-pair	Coil-ends in 2 ranges (Fig. 457)	Stator cannot be divided without unwinding coils.
	Coil-ends in 3 ranges (Fig. 459)	Stator can be divided without unwinding coils.
One coil per phase per pole	Coil-ends in 3 ranges (Fig. 460)	Stator cannot be divided without unwinding coils.

In a quarter-phase machine with divided coils or a 3-phase machine with divided coils, if each half of the divided coil is considered separately, it is evident that we virtually have machines with 4 and 6 phases respectively. The mean pitch of the half coils taken separately instead of being equal to the pole-pitch is $\pi(N'-1)/N'$ (where $N'=2N$ is the doubled number of phases, 4 or 6), and the pitch differential factor[1] is $\cos n\frac{\pi}{2}\left(1-\frac{N'-1}{N'}\right)=\cos n\frac{\pi}{2N'}$.

The belt differential factor is $\dfrac{\sin n\frac{\pi}{2N'}}{\frac{g}{N'}\sin n\frac{\pi}{2g}}$, so that the total differential factor is

$$\frac{\sin n\frac{\pi}{2N'}\cdot\cos n\frac{\pi}{2N'}}{\frac{g}{N'}\sin n\frac{\pi}{2g}}=\frac{\sin n\frac{\pi}{2N}}{\frac{g}{N}\cdot\sin n\frac{\pi}{2g}}$$

which is the same as the differential factor for the machines with the same number g of slots per pole, but with $N=N'/2$ phases and a mean coil-span equal to the pole-pitch. Their outputs are therefore the same, and hence by some writers[1] the ordinary N-phase machines with divided coils are regarded as obtained from a $2N$-phase winding with coils having a mean span less than the pole-pitch.

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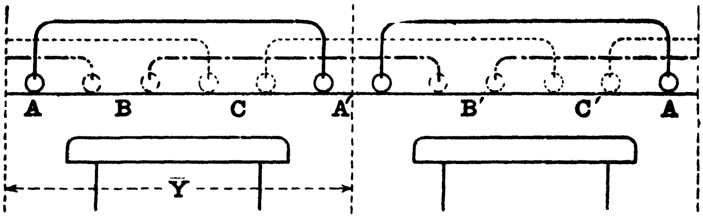
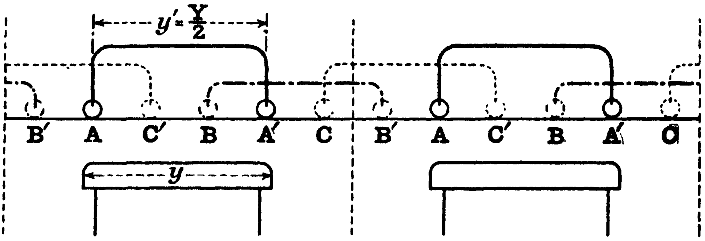
[1] This and the following terms will be explained in Chapter XXV.

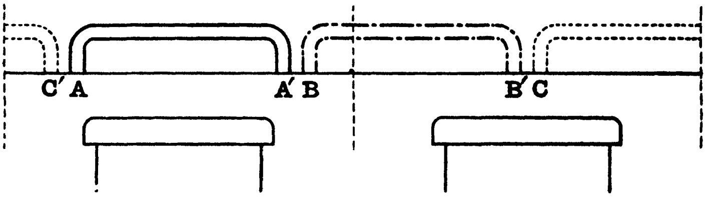
§ 5. Shortened coils in single layer with fractional pitch.—Of the methods derived immediately from single-phase windings by duplication or triplication,



(i) Long coils divided



(ii) Shortened coils.



(iii) Undivided coils, non-overlapping.

FIG. 461.—Development of shortened coils in 3-phase alternator.

some are more convenient than others, according to the nature of the output which the generator is to give. It has been said above (§ 4) that the forms with as many coils as there are poles, although utilizing the end-space well and having a good cooling effect and small inductance, yet involve a considerable amount of overlapping of the coils. This disadvantage may be lessened by the employment of *shortened* coils. The coils of the quarter-phase machine with a single layer winding are divided so that in each phase there are as many as there are poles; their span is then narrowed until their over-all width is only equal to half the pole-pitch, and there is no overlapping.

In the three-phase machine there are two stages of the shortening process. In Fig. 461 (i) there are 2 slots per pole per phase, which might be wound with long coils divided as shown, but a symmetrical winding is equally obtained if these coils are narrowed until their width is equal to half the pole-pitch (Fig. 461, ii). Next let the 2 slots per pole per phase be wound with an undivided coil, and let this be narrowed until its over-all width is no more than two-thirds of the pole-pitch, and we again reach entirely non-overlapping coils (Fig. 461, iii). The method is, however, not to be recommended, since either the pole-faces must be correspondingly narrowed until their width is about equal to the width of the coils, in order to minimize the differential action; or if retained of normal width, a coil never embraces all the lines of one field, and the E.M.F. coefficient K has a low value. The ends of the coils are still well utilized for cooling, yet the reduction of the effective pole-area is so far disadvantageous that entirely non-overlapping coils are seldom employed, any gain from the reduction in the mean length of an armature turn being of small importance.

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[1] Dr. S. P. Smith, *Journ. I.E.E.* (Vol. 55, pp. 23–25), and B. Hague, " Pressure Harmonics in Polyphase Systems and Windings," *Electrician*, Vol. 78, pp. 765–769, 1917, March 30.

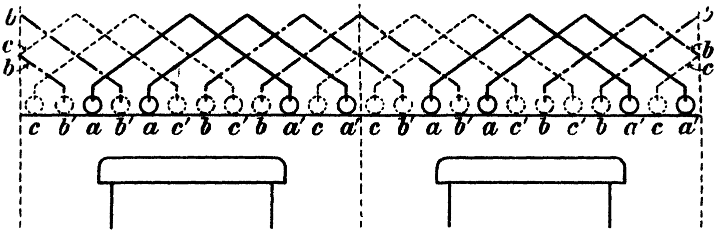


FIG. 462 —Three-phase winding with shortened coils in one layer and even number of slots per pole.

Now it is obvious that in Fig. 461 (ii) each of the slots may be doubled, trebled, quadrupled, etc, yielding 4, 6, 8, etc, slots per pole per phase, and the coils of a phase corresponding to a pole-pitch may be either concentric or lap-wound; in either case, their *mean* width is equal to half the pole-pitch. The objection to all such cases where the *mean* pitch of the two sides expressed as a fraction of the pole-pitch is $\lambda_c = \frac{1}{2}$, is that the fundamental E M.F. is so much reduced. But if the interlacing of the phases is not objected to, this disadvantage is to some extent overcome and a reduction is effected in some harmonics by making the pitch of the coils an *odd* number of slots and less than $g - 1$, where g is the number of slots per pole—e g $\frac{7}{12}$ in Fig 462. The angular pitch of the two or more slots forming one coil-side is thus doubled, and the amount of chording may be made what we please. The reduction of the fundamental remains, however, so disadvantageous that the arrangement is seldom used in practice.

The particular arrangements shown in Fig. 461 (ii) or Fig. 462 demand an even number of slots per pole, 6, 12, 18, 24. But by making the coils of a group within one pole-pitch exceed by one those of the same phase within the adjacent pole-pitch, so that it takes the coils under a pair of poles to form a unit group representative of the whole, it becomes possible to use a single-layer winding with shortened coils with an odd number of slots per pole, 9, 15, 21, etc., and the coil-pitch when measured in slots still remains odd. Thus in Fig. 463 with 9 slots per pole, the A groups consist alternately of two component coils and of one coil, each with pitch $= \frac{5}{9}$ when expressed in slots; the winding may be either lap (Fig. 463, i) or concentric (Fig. 463, ii). Such an arrangement has more in its favour than the previously considered shortened methods.

In the above consideration of shortened coils, only single-layer windings have been in view. But it is often feasible and convenient to employ a two-layer winding with divided coils, and in such cases, as already shown, the two sides of any one component coil appear respectively at the top and at the bottom of a slot, just as in a barrel-wound slotted continuous-current armature. The advantage is then gained that the designer is not limited to full-pitch coils or shortened coils whose mean pitch expressed as a fraction of the pole-pitch is $\lambda_c = \frac{1}{2}$, but may also employ intermediate *fractional* pitches.

Thus in both single and two-layer windings a great deal may be done by a suitable choice of pitch towards reducing or entirely eliminating objectionable harmonics in the E.M.F. wave which would otherwise be caused by higher harmonics present in the E.M.F. of a component loop.

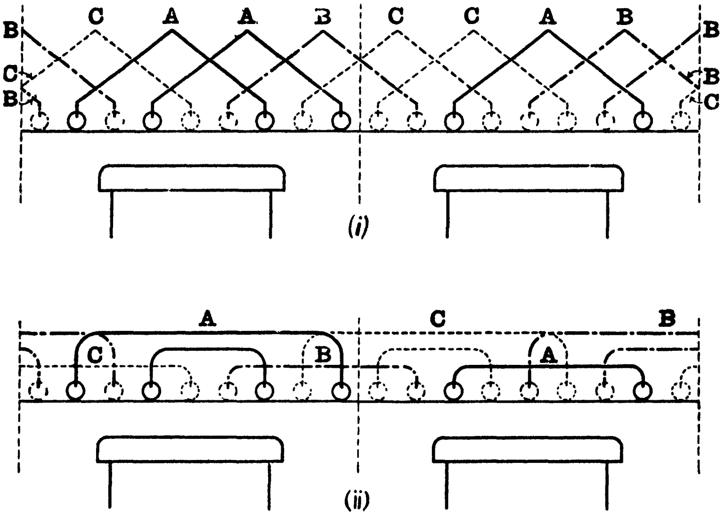


FIG. 463 —Three-phase winding with shortened coils in one layer and an uneven number of slots per pole; (i) lap, (ii) concentric.

§ 6. **The star connexion.**—In star-connected windings after completing one phase it is immaterial whether the start of the second and third phases is made from a common junction of three points lying close beside each other on the armature (*cp.* Fig. 464) or from a connexion to other points similarly placed relatively to poles of the correct sign. A common ring joining points approximately 120 mechanical degrees apart reduces the difference of potential between adjacent coils to a minimum, and is on this account theoretically preferable; it is, therefore, shown in Fig. 457 and elsewhere. For the same reason each ring or terminal of the Δ-connexion unites preferably two points approximately 120° apart,[1] if the winding is not in itself re-entrant. In practice it is, however, often more convenient for the terminals of the three phases to be near to one another, and therefore for the junction of the star to be made at three coils near to one another.

---

[1] If the number of poles is itself divisible by 3, the junction points cannot be exactly 120° apart.

§ 7. Rules for connecting up three-phase armatures.—In order to connect the coils of a three-phase machine correctly it is convenient to consider the instant when the active conductors of one phase are situated centrally under the poles, and to assume that they are then carrying the maximum current; if the machine is to be *Y*-connected, let one end of the phase in question be connected to the common junction, and let it be assumed that the current is flowing into the common junction. The direction of the current in any group of active conductors of the first phase is thus fixed, and the direction in the groups on either side in the two other phases must be the same under the same pole-piece. But the current in the two other phases must be flowing away from the common junction, so that their ends must be connected to the common junction in such a way that their E.M.F. agrees with the assumed direction of the current. If the machine is to be Δ-connected, and the ends of the first phase which is centrally under the poles are connected to two terminals, *A* and *B*, let it be assumed that the current flows from *A* to *B*. The current in the two other phases must also be flowing from *A* to *B*, passing through *C* on the way. The second phase must therefore be connected to *A* and *C* in such wise that the current flows from *A* to *C*, and the ends of the third phase must be connected to *C* and *B* in such wise that the current flows from *C* to *B*, the directions of their currents being the same under the same poles as that assumed in the first phase.

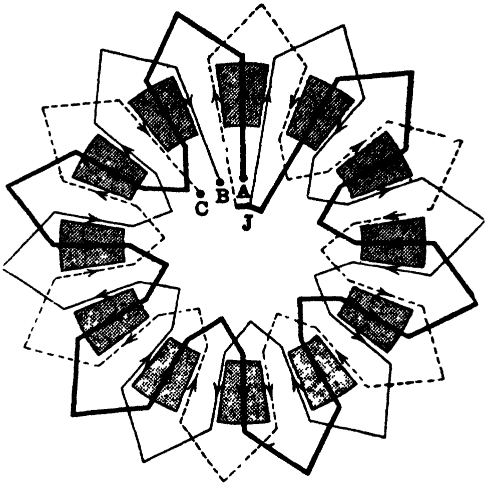


FIG. 464.—Three-phase bar winding for low pressures.

§ 8. Polyphase wave-wound armatures in single or double layer.—A bar winding is shown in Fig. 464, which is the three-phase equivalent of the simple wave-wound armature of Fig. 95. While suitable for low pressures, triplication of the wave and lap windings of Fig. 96 or Fig. 97 is required for higher 3-phase pressures; in the former of these the open-ended wave-winding has the front pitch shortened by one at the end of every tour.

Simple triplication of such an armature as that of Fig. 96, of course, leads to a number of bars $Z/3$ in one phase which is a multiple of the number of poles. But this limitation is not necessary when p is other than 3 or a multiple of 3, since ***multipolar simplex wave-winding*** following the same laws as an ordinary continuous-current wave-wound drum can then be used. If y be the total pitch of a loop $= y_B + y_F$, we have $Z = py \pm 2$, or $y = \frac{Z \pm 2}{p}$ as in a continuous-current machine, *e.g.* as in Fig. 465 for an 8-pole machine $Z = 3 \times 14 = 42 = 4 \times 10 + 2$, and the component pitches y_B

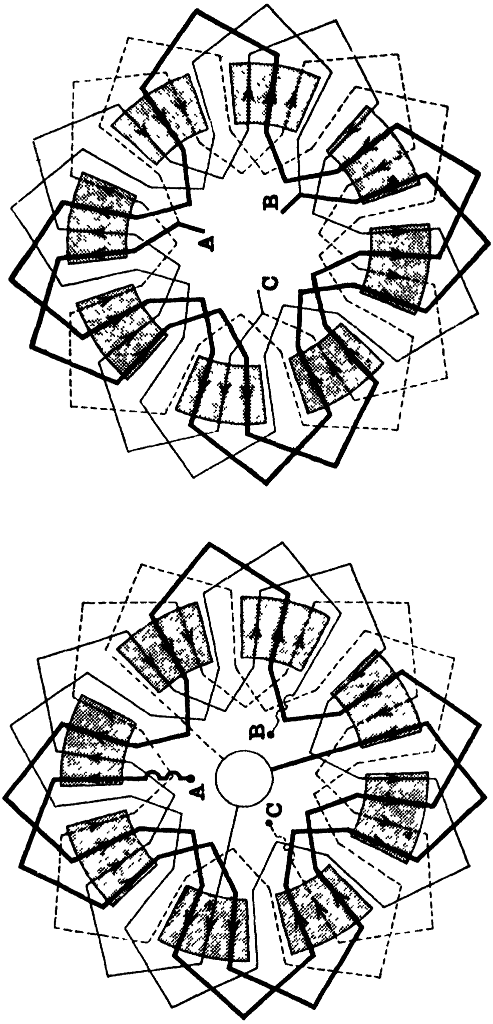


Fig. 465.—Three-phase re-entrant wave-winding, used either (i) open or (ii) closed.

ii. Delta.

i Star.

and y_F are each made 5. Such a winding can be used either open and Y-connected as in Fig. 465 (i) or closed and Δ-connected as in Fig. 465 (ii).

It will be seen that the winding of Fig. 465 is re-entrant, and even if used open, that it could always be transformed into a closed circuit; although shown as a single layer, if the slots are halved in number, so that each contains two coil-sides, the winding becomes an ordinary double-layer continuous-current wave-winding. It only remains to be added that for delta-connected machines in which the winding must necessarily form a closed circuit, any continuous-current winding can equally be used, provided that there are as many equidistant points in the windings as there are phases, N_{ph}, accessible so that connexions can be brought from them to N_{ph} collecting rings.

For 3-phase work the number of bars in such windings must, strictly speaking, be a multiple of 3 as assumed above, in order to secure exactly 120° phase-differences between the phases; but as this condition somewhat limits the possible numbers of bars and slots, and entirely rules out simplex wave-winding for 6-pole machines or those with p a multiple of 3, it may occasionally under due precautions become permissible with a large number of bars to make one phase greater than the other two to the extent of one coil or a pair of additional bars. But apart from this somewhat undesirable procedure, the disadvantage under which machines with $p = 3$ or a multiple of 3 suffer is lessened by the fact that in them the open-ended single- or double-layer winding with the front pitch shortened by one after every tour as in Fig. 96 can always be employed when $S =$ a multiple of $3p$.

§ 9. The simplex wave-winding opened.—When the conductor positions of all the coils in such a winding as that of Fig 465 are referred to a double pole-pitch as a standard, it will be found that the positions of, say, the leading sides of all the coils of a phase cover approximately 120° (electrical) or two-thirds of the pole-pitch. When arranged in a single layer with one conductor per slot, two consecutive conductors in a phase which follow one another *under poles of the same sign* differ in their relative position in the magnetic field by twice the amount of the creep measured on the commutator (if such were present), *i.e.* by $y - \frac{U}{p} = \pm \frac{2}{p}$ of a slot-pitch, positive when the winding is progressive, negative when retrogressive, which in the case of Fig. 465 is 2/4 = half a slot-pitch. Taking the first coil-side of a phase as a zero starting-point, when the remaining $\frac{C}{3} - 1$ corresponding coil sides of the phase are considered, their total divergence in the field adds up to $\frac{2}{p}\left(\frac{C-3}{3}\right)$ slot pitches. Since each slot-pitch represents $(360° \times p)/S$ electrical degrees, and $S = 2C$, the virtual width of the total phase-band is

$$\frac{2}{p}\left(\frac{C-3}{3}\right) \times \frac{360° \times p}{S} = 120°\left(1 - \frac{3}{C}\right)$$

In the case of Fig. 465, the width is 103° nearly, and it gradually approaches 120° as C is increased. When arranged as a double-layer winding with two conductors per slot or $c = 1$, the same electrical creep between the first sides of two consecutive loops is $\pm\frac{1}{p}$ of a slot-pitch, since the number of slots is halved and a slot-pitch is doubled, *i.e.*

$$(y_B{}^1 + y_F{}^1) - \frac{S}{p} = \pm\frac{1}{p}\text{, since } y_c - \frac{cS}{p} = \pm\frac{1}{p} \text{ and } y_c = c(y_B{}^1 + y_F{}^1)$$

But S then $= C$, so that the same final expression as above results for the virtual width.

When c is > 1, the front pitch reckoned in slots varies, since if $(y_B - 1)$ is exactly divisible by $u > 2$, $(y_F + 1)$ cannot be so divisible; *e.g.* if $u = 2c = 6$ and $y_F = 85$, $(y_F + 1)/u = 86/6 = 14\frac{1}{3}$, implies that the front pitch in slots is 14 in two adjacent cases, followed by 15 in the next case. The full expressions therefore become more complicated. For instance, if $c = 2$, the front pitch in slots alternately exceeds and falls short of the average mean value by half a slot-pitch. The averaged divergence per coil traversed being $\frac{1}{2p}$, its value for every alternate step in advance is reduced by half a slot-pitch. The number of coils per phase is even, and the averaged total divergence would be $\frac{1}{2p}\left(\frac{C}{3} - 1\right)$, but at the front end of the complete row of a phase $\left(\frac{1}{2p} - \frac{1}{2}\right)$ yields a divergence in the opposite direction and at the other end renders the divergence of the conductor less than that of its predecessor; the actual total therefore is only

$$\frac{1}{2p}\left(\frac{C}{3} - 2\right) + \left(\frac{1}{2} - \frac{1}{2p}\right) = \frac{1}{2p} \times \frac{C + 3(p-3)}{3}$$

slot-pitches and in electrical degrees the width of the phase-band is

$$\frac{1}{2p} \times \frac{C + 3(p-3)}{3} \times \frac{360^\circ p}{S} = 120^\circ \left\{1 + \frac{3}{C}(p-3)\right\}$$

To secure this, after passing through the first back-pitch from the beginning of a phase, it is essential that there must follow the shorter front-pitch if the winding is progressive or the longer front-pitch if it is retrogressive.

But in either the single-layer or double-layer case if the complete winding is opened or left open-ended in 6 places so as to divide it as nearly as possible into 6 equal sections, then by coupling up sections separated by one-third of the winding from one another into pairs, the winding of each phase is caused to overlap itself more closely in the fields of a standard double pole-pitch; its spread is thus reduced exactly or nearly to half its former values, *i.e.* approximately to 60° (electrical) or one-third of the pole-pitch, and this greater concentration results in an E.M.F. higher by 15 per cent. (*Cp.* the values (Table XXIII) for the differential factor with winding uniformly distributed over $\frac{1}{3}$ and $\frac{2}{3}$ of the pole-pitch, viz., 0·955 and 0·827.)

To secure the desired effect as fully as possible, it is essential that the back pitch, $y_B{}^1$, in slots should be the whole number nearest to the pole-pitch, $S/2p$, also reckoned in slots; hence in Fig. 466 when converted into a double layer with two conductors per slot (Table XIX), the back pitch in elements would require to be made $y_B = 7$, and the front pitch $y_F = 3$, so that $y_B{}^1$ may be 3, the nearest whole number to 21/8.

The pair of sections when joined in series and forming a phase can be star-connected (Fig. 466) or mesh-connected, as shown diagrammatically in Fig. 467 (i) and (iii). When the two sections of a phase contain an equal number of coils, they can also be connected in parallel, and again be connected in star or mesh (Fig. 467 (ii) and (iv)). To join two sections in series, either both finishes or both starts must be joined together; to join them in parallel, the start of one is connected to the finish of the other, and *vice versa*.

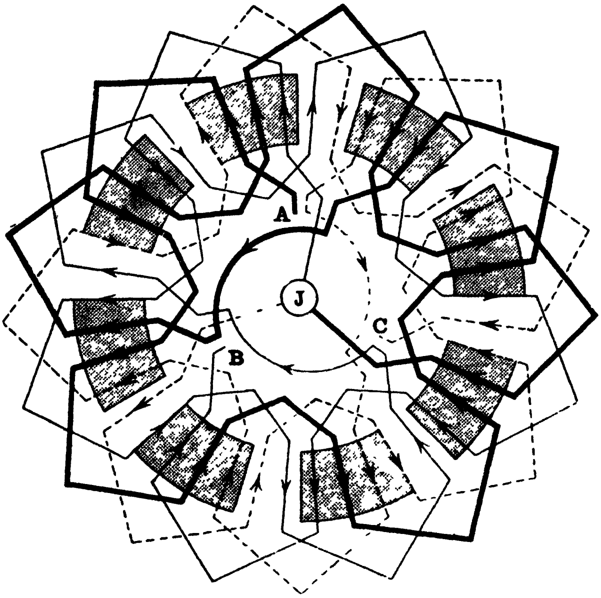
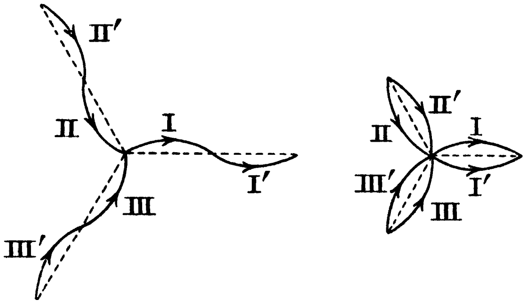


FIG. 466.—Re-entrant wave-winding left open in 6 places and Y-connected. $U = 42 \quad y_B = y_F = 5$.

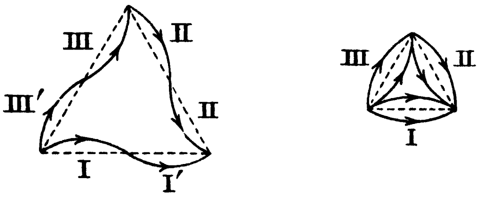
In general, if a closed double-layer winding in which the angular width of the belt of a phase under each pole is $2/N_{ph}$ of the pole-pitch, *i.e.* is $2\pi/N_{ph}$ electrical radians, is opened at $2N_{ph}$ points which are equidistant when the winding is traced out, it can be divided into N_{ph} pairs of sections, each pair with an angular width of belt of only π/N_{ph} radians.

Opening of a closed double-layer winding is usually confined to wave-windings, since with lap-windings the cross-connectors become too numerous.

Consideration of a few simple cases of wave-winding for 3-phase work with a small number of slots, each containing two coil-sides, will render the arrangement and its effect clearer. Let the sequence of the slots traced out by the winding be written down in 6 rows corresponding to the 6 sections; the number of columns will be twice the number of coils in a section, since each coil falls into two slots. When the total number of slots is divisible by 6 (Case B), each row will appear in all the columns. But if only divisible by 3 (Case C), when the sections are made equal, each will contain a half coil, so that it ends on the side of the stator opposite to that on which it started. The connexions to place the sections in series or parallel must then pass over the stator core from front to back To obviate this highly inconvenient arrangement, alternate sections are made to differ in length by one coil ; only pairs of coil-sides, *i e* complete wave-loops, then enter into the question, and all the connexions between sections can be made at the front end. Hence in case C, with the above usual arrangement, the intermediate rows of slot numbers will be shorter than the remainder by two columns, and correspondingly the two unequal sections of a phase cannot be placed in parallel, but must be in series—a larger section with a smaller section.



(i). Star, series **(ii). Star, parallel.**



(iii). Delta, series. **(iv). Delta, parallel**

FIG. 467.—Possible groupings of 3-phase opened wave-windings.

Since one double pole-pitch corresponds to S/p, and all loops are similar, electrical phase differences of 120° correspond to slot-positions distant by $S/3p$ and $2S/3p$ slot-pitches from the starting-slot of phase I. Slots in such positions will not be found, but their equivalents are given by adding some whole number of double pole-pitches, the lowest multiples of S/p which, when added to $S/3p$ and $2S/3p$, yield a whole number, give the slot-pitches which, when added to No. 1 with a progressive winding or deducted from it with a retrogressive winding, fix the starting-points of phases II and III.

Example of Case B (p odd, S even).

$$p = 5 \quad S = 24 \quad y_c = \frac{24+1}{5} = 5 \qquad y_B = 5 \qquad y_F = 5$$

$$C = 24$$

$$24/6 = 4 \text{ coils in each section} \qquad y_B{}^1 = \frac{y_B - 1}{2} = 2 \quad y_F{}^1 = \frac{y_F + 1}{2} = 3$$

Winding progressive

$$\frac{24}{3 \times 5} + \frac{24 \times 3}{5} = 16. \quad \text{Start of phase II} = \text{slot } 17$$

$$\frac{2 \times 24}{3 \times 5} + \frac{24}{5} = 8. \quad ,, \qquad ,, \quad \text{III} = ,, \quad 9$$

TABLE XVII

		Top.	Bottom.	Top.	Bottom.	Top	Bottom	Top.	Bottom.
I.	S →	1	3	6	8	11	13	16	18
		21	23	2	4	7	9	12	14
II.	S ←	17	19	22	24	3	5	8	10
	J	13	15	18	20	23	1	4	6
III.	S ←	9	11	14	16	19	21	24	2
		5	7	10	12	15	17	20	22

The junction point of the 3 phases when star-connected is marked *J*, and the arrow heads assume the maximum current to be entering phase I at slot 1. It will be noticed that every slot number must appear twice, and in each phase half the slot numbers repeat (but in different layers), showing the reduction of the angular width of the belt of a phase, which would otherwise correspond to two adjacent rows. The actual width is discoverable by considering, *e.g.* the divergence of alternate slots 1, 6, 11, 16, 6, 1, 20, 15 of the first and fourth rows which are under poles of the same sign from the values 1, 5·8, 10·6, 15·4, and 20·2, which they should have for coincidence in the 2-pole reference field (one double pole-pitch being equal to $S/p = 4{\cdot}8$ slot-pitches). The total width when referred to a 2-pole field is, therefore, exactly a slot-pitch, made up of + 0·6 on one side, and − 0·4 on the other side of slot 1 (Fig. 468). Since one slot-pitch corresponds to 360° × 5/24 (electrical), the virtual width is 75°, so that the phases partially overlap.

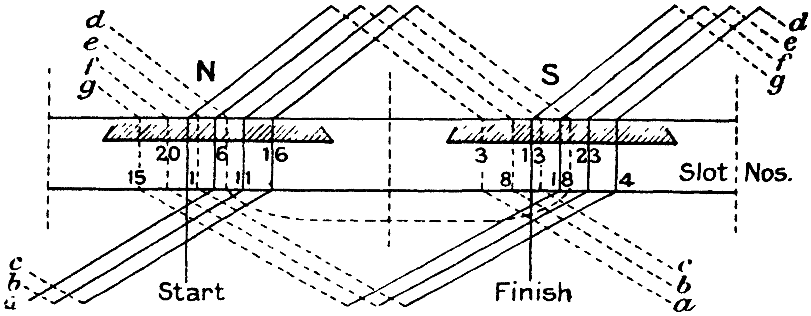


FIG. 468 —The 10-pole winding of phase I of Table XVII referred to a two-pole field.

All the pairs of slot numbers in an upper and lower row of one phase differ by $S/2$, *i.e.* the slot-positions differ by half the circumference, or half the pole-pitches; consequently each slot in the second half of a phase when not actually repeated is matched by another slot in the first half exactly similarly situated under a pole of opposite sign, so that, if desired, rows 1 and 4 could be placed in parallel, and similarly rows 2 and 5 or 3 and 6.

1st Example of Case C (with p and S odd).

$$p = 5 \quad S = 21 \quad y_c = \frac{21-1}{5} = 4 \quad y_B = 5 \quad y_F = 3$$

$$C = 21 \quad y_B{}^1 = 2 \quad y_F{}^1 = 2$$

21/6 = 3½, so that the division of coils giving the nearest approach to 6 equal sections will be 3 sections of 4 loops and 3 sections of 3 loops, 4 and

3 loops following alternately. Each section of 4 loops is then to be coupled in series with a section of 3 loops.

Winding retrogressive

$$\frac{21}{3 \times 5} + \frac{21 \times 3}{5} = 14. \quad \text{Start of phase II} = 1(=22) - 14 = \text{slot } 8$$

$$\frac{2 \times 21}{3 \times 5} + \frac{21}{5} = 7. \quad ,, \quad ,, \quad \text{III} = 1(=22) - 7 = \text{slot } 15$$

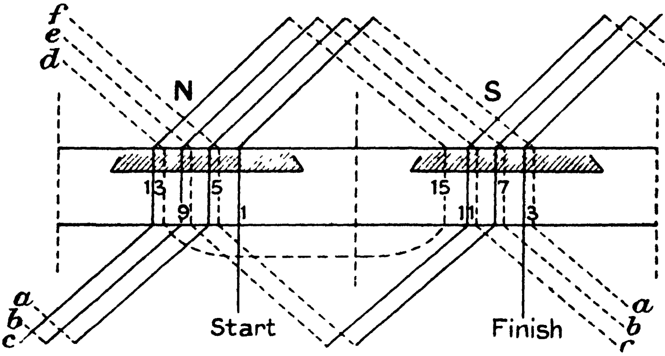


FIG. 469.—The 10-pole winding of phase I of Table XVIII referred to a two-pole field.

TABLE XVIII

		Top.	*Bottom.*	*Top.*	*Bottom.*	*Top*	*Bottom*	*Top*	*Bottom*
I.	*S* →	1	3	5	7	9	11	13	15
				17	19	21	2	4	6
II.	*S* ←	8	10	12	14	16	18	20	1
			J	3	5	7	9	11	13
III.	*S* ←	15	17	19	21	2	4	6	8
				10	12	14	16	18	20

The slot numbers of a smaller section are now exactly repeated in those of a larger section of the same phase (see Fig. 469), but in each phase a larger and a smaller section differing by one coil can only be placed in series

2nd Example of Case C (with p even, S odd).

$p = 4 \quad S = 21 \quad y_c = \frac{21-1}{4} = 5 \quad y_B = 7 \quad y_F = 3$

$C = 21 \quad 21/6 = 3\frac{1}{2}$, *i.e.* 4 and 3 loops $\quad y_B{}^1 = \frac{7-1}{2} = 3 \quad y_F{}^1 = \frac{3+1}{2} = 2$

Winding retrogressive

$$\frac{21}{3 \times 4} + \frac{21}{4} = 7. \quad \text{Start of phase II} = 1(=22) - 7 = \text{slot } 15$$

$$\frac{2 \times 21}{3 \times 4} + \frac{21 \times 2}{4} = 14. \quad ,, \quad ,, \quad \text{III} = 1(=22) - 14 = \text{slot } 8$$

TABLE XIX

		Top	Bottom	Top.	Bottom	Top.	Bottom	Top.	Bottom.
I	S →	1	4	6	9	11	14	16	19
				21	3	5	8	10	13
II.	S ←	15	18	20	2	4	7	9	12
			J	14	17	19	1	3	6
III.	S ←	8	11	13	16	18	21	2	5
				7	10	12	15	17	20

Four of the slot numbers in a smaller section are now seen to be repeated in the slot numbers of the larger section with which it has been placed in series, implying greater concentration of the belts of a phase (Fig. 470).

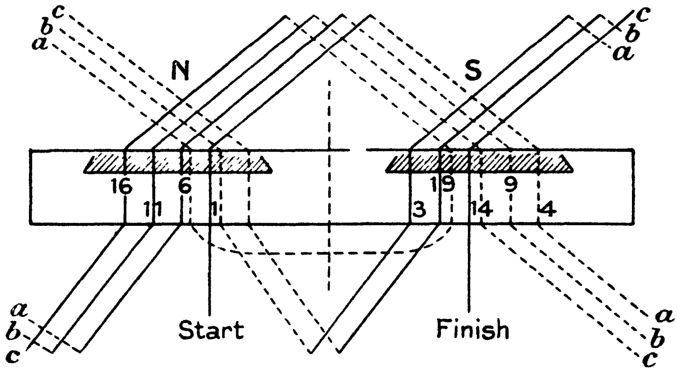


FIG. 470.—The 8-pole winding of phase I of Table XIX referred to a 2-pole field.

In connecting the 1st and 4th, 2nd and 5th, 3rd and 6th sections respectively as pairs in series, it has only to be borne in mind that when the winding is followed out in sequence as tabulated above, or is stretched out as in Fig. 471, the second section must in each case not continue onwards in the same direction as the first section, but must return on it as shown in Fig. 471; all the connecting leads are then at the same end of the stator, and that is the end at which are situated the terminals of the phase. The reason which necessitates the following of the second section of a pair backwards when they are in series is evident from a consideration of the slot numbers that appear in both the rows of a phase; the conductors in such slots in a fourth row are then in a different layer from those of the first row, and their end-connexions are the reverse. Consequently they must be followed through backwards to yield an E.M.F. in series throughout. The winding of a phase upon leaving the end of a row thus in all cases re-enters in its second row a slot which appears in the first row, but in a different layer, so that if the E.M.F. of the one row is from left to right, that of the other is from right to left.

The virtual width of the belt of conductors forming the one side of the double coil representing one phase when the conductors are referred to a standard two-pole field (Figs. 468–470), varies appreciably according to the circumstances of the case. When $c = 1$, the number of slots or of coils by Table XXII is $3pn_1 \pm (p - 1)$ for $p = 1, 4, 7, \dots$ and is $3pn_1 \pm (p + 1)$

for $p = 2, 5, 8, \ldots$ In either case the pitch $y_c = \frac{C \pm 1}{p} = 3n_1 \pm 1$, the upper sign corresponding to a progressive and the lower to a retrogressive winding for the former series or *vice versa* for the latter series of pole-pairs. When the number of slots is divisible by 6 (case B, Table XVII, p. 31; p is then 1, 5, 7, 11, . . .), the first row of that table gives a total "creep" of

$$\left\{\left(y_B^1 + y_F^1\right) - \frac{S}{p}\right\}\left(\frac{C}{6} - 1\right) = \pm\frac{1}{p}\left(\frac{C}{6} - 1\right)$$

according as the winding is progressive or retrogressive. At the left end of the 4th row exactly $S/2p$ slot-pitches have been gained or lost, *i.e.* exactly one pole-pitch; consequently the conductor is there situated relatively to a S. pole exactly as No. 1 to a N. pole, and the advance to the next conductor in the 4th row which is under a N. pole yields a divergence from No. 1 of $\left(y_B^1 - \frac{S}{2p}\right)$ of a slot-pitch in the two-pole reference field. But this divergence is in the reverse direction to that of the 1st row, and further it is the maximum in that direction. To add it, therefore, to the "creep" of the 1st row, its sign must be reversed, and the total is

$$\pm\frac{1}{p}\left(\frac{C}{6} - 1\right) + \frac{S}{2p} - y_B^1$$

According as the winding is progressive or retrogressive, $S/2p$ is less than or exceeds $y_c/2$, and y_c being in the present case necessarily odd ($= 3n_1 \pm 1$, where n_1 has an even value as required by Table XXII), the y_B^1 nearest to $S/2p$ is $\frac{y_c}{2} \mp \frac{1}{2}$. Thence if $y_B^1 = \frac{y_B - 1}{2} = \frac{y_c}{2} \mp \frac{1}{2}$

$$\begin{aligned} y_B &= y_c && \text{if the winding is progressive} \\ &= y_c + 2 && \text{,,} \quad \text{,,} \quad \text{retrogressive} \end{aligned}$$

$$\text{and } y_B^1 = \frac{y_c \mp 1}{2} = \frac{\frac{C \pm 1}{p} \mp 1}{2} = \frac{C}{2p} \pm \frac{1}{2p} \mp \frac{p}{2p}$$

Since $C = S$, the second part of the above expression for the total "creep" reduces to $\pm\frac{1}{p}\left(\frac{p-1}{2}\right)$

and the whole to

$$\frac{1}{p}\left(\frac{C}{6} - 1 + \frac{p-1}{2}\right) = \frac{1}{6p}\left\{C + 3(p - 3)\right\} \text{ slot-pitches.}$$

If the number of slots is only divisible by 3 (cases C), since $S = C$ is odd, the "creep" over the 1st row amounts to $\pm\frac{1}{p}\left(\frac{C}{6} - \frac{1}{2}\right)$ and over the first three rows up to the conductor at the left end of the 4th row is $\pm\frac{1}{p}\left(\frac{C}{2} + \frac{1}{2}\right)$, of which $C/2p$ corresponds exactly to a pole. The latter conductor therefore starts with a divergence of $\pm\frac{1}{2p}$, and the next conductor in the 4th row, which is under a pole of the same sign as that opposite to No. 1, has a divergence of $\mp\frac{1}{2p} + \frac{S}{2p} - y_B^1$.

In the first case when p is odd, $y_c = \frac{C \pm 1}{p} = 3n_1 \mp 1$, which is even.

The y_B^1 which is nearest to $S/2p$ is then $1\frac{1}{2}n_1 \mp \frac{1}{2}$, or $y_B = 3n_1$ or $3n_1 + 2$ according as the winding is progressive or retrogressive. In either case, $y_B^1 = \frac{S}{2p} \pm \frac{1}{2p}$, and there is no further divergence in the 4th row. There is left, therefore, only that of the first row, viz.,

$$\frac{1}{6p}(C - 3)$$

In the remaining case when p is a multiple of 2 but not of 3, y_c is again odd, and the nearest $y_B{}^1$ to $S/2p$ is $\frac{y_c}{2} \mp \frac{1}{2} = \frac{C \pm 1}{2p} \mp \frac{1}{2} = \frac{C \pm 1 \mp p}{2p}$

The divergence of the second conductor in the 4th row is then

$$\mp \frac{1}{2p} + \frac{C}{2p} - \frac{C}{2p} \mp \frac{1}{2p} \pm \frac{p}{2p} = \pm \frac{1}{p}\left(\frac{p-2}{2}\right)$$

and the total is

$$\frac{1}{p}\left(\frac{C}{6} - \frac{1}{2} + \frac{p-2}{2}\right) = \frac{1}{6p}\left\{C + 3(p-3)\right\} \text{ as in the B case.}$$

Multiplying by 360° × p/C, the angular width of a slot-pitch, the new phase-spread in the first and third cases becomes

$$60^\circ \left\{1 + \frac{3}{C}(p-3)\right\}$$

and in the second case,

$60^\circ\left(1 - \frac{3}{C}\right)$, *i.e.* half the width of the phase when the winding is not opened.

When $p = 1$, the former expression reduces to $60^\circ\left(1 - \frac{6}{C}\right)$

With $c = 2$ and $C/6$ even, the "creep" at the end of the first row is $\frac{1}{2p}\left(\frac{C}{6} - 2\right)$ in the positive direction; at the left-hand end of the 4th row, an exact pole-pitch has been passed, so that the second conductor of that row starts with a negative divergence $y_B{}^1 - \frac{S}{2p}$ and the 4th conductor from the left end of the 4th row has a negative divergence $\left(y_B{}^1 - \frac{S}{2p}\right) + \left(y_B{}^1 + y_F{}^1 \text{ short} - \frac{S}{2p}\right)$ The 6th conductor after the interposition of the back and longer front pitch ($y_B{}^1 + y_F{}^1$ long) obviously has a lesser negative divergence and so on to the end of the row. Consequently the maximum negative divergence is $\left(y_B{}^1 - \frac{S}{2p}\right) + \left(y_B{}^1 + y_F{}^1 \text{ short} - \frac{S}{p}\right)$

Since $y_B{}^1 - \frac{S}{2p} = \frac{C+1-p}{4p} - \frac{C}{4p}$ and $\left(y_B{}^1 + y_F{}^1 \text{ short} - \frac{S}{p}\right) = \frac{C+1-p}{2p} - \frac{C}{2p}$ this negative maximum is

$$= -\left(\frac{1}{4} - \frac{1}{4p}\right) - \left(\frac{p-1}{2p}\right) = -\frac{1}{2p}\left(\frac{3p-3}{2}\right)$$

which will also cover the lesser negative divergence at the left-hand end of the first row, viz. - $\left(\frac{1}{2} - \frac{1}{2p}\right) = \frac{1}{2p}(p-1)$

The total is therefore

$$\frac{1}{2p}\left\{\frac{C}{6} - 2 + \frac{3p-3}{2}\right\} = \frac{1}{2p}\left\{\frac{C + 3(3p-7)}{6}\right\}$$

or in electrical degrees

$$\frac{1}{6p} \times \frac{C + 3(3p-7)}{2} \times \frac{360^\circ p}{S} = 60^\circ\left\{1 + \frac{3}{C}(3p-7)\right\} \quad . \quad (225)$$

With $c = 2$ and $C/6$ odd, the conductor at the left end of the 4th row has diverged more than a pole-pitch by half a slot-pitch; the divergence of its next neighbour is therefore $\frac{1}{2} + y_B{}^1 - \frac{S}{2p}$ in the positive direction, which is less than the positive divergence of the first row, viz., $\frac{1}{2} - \frac{1}{2p} = \frac{1}{2p}(p-1)$.

Thence along the 4th row the divergence increases negatively up to a maximum at its end, where it is

$$\begin{aligned}\frac{1}{2} + y_B' - \frac{S}{p} + \left(y_B' + y_F' \text{ short} - \frac{S}{p}\right)\frac{C-6}{12} + \left(y_B' + v_F' \text{ long} - \frac{S}{p}\right)\frac{C-18}{12}\\ = \frac{1}{2} - \left(\frac{1}{4p} + \frac{1}{4}\right) - \left(\frac{1}{2} + \frac{1}{2p}\right)\frac{C-6}{12} + \left(\frac{1}{2} - \frac{1}{2p}\right)\frac{C-18}{12}\\ = -\frac{1}{2p}\,\frac{C + 3(p-3)}{6}\end{aligned}$$

This exceeds the negative divergence of the first row, so that the only addition from that row is its positive divergence $\frac{1}{2} - \frac{1}{2p}$, and the total is

$$\frac{1}{2p}\left(\frac{C + 3(p-3)}{6} + p - 1\right) = \frac{1}{2p}\left\{\frac{C + 3(3p-5)}{6}\right\}$$

or

$$60^\circ\left\{1 + \frac{3}{C}(3p-5)\right\} \quad . \quad . \quad . \quad . \quad . \quad (226)$$

§ 10. The bars and joints to be left open.—The determination of the joints of the winding to be left open and of the portions to be connected in series (or, it may be, in parallel, when they are of equal length) is easily made from simple considerations. Let the winding be imagined to be stretched out into a row of loops without any overlapping as indicated by the scheme of Fig. 471. If then one coil-side (say in the upper layer when there are 2 layers) and its joint which is to be left open as the terminal of phase I be numbered 1 as a starting-point, when an advance through x complete loops is made and is reckoned in coil-sides, the $(1 + xy - y_F)$th coil-side in a lower layer is reached, and the next loop begins with the $(1 + xy)$th coil-side in the upper layer, where $y =$ the total pitch, $= y_B + y_F$. In general, if we start from coil-side No. n, the finish of x loops and the start of the next loop are the coil-sides numbered $(n + xy - y_F)$ and $(n + xy)$ respectively. Every time that $n + xy - y_F$ or $n + xy$ exceeds the total number of coil-sides U, a new tour of the armature has begun, and U must be deducted from the number. *E.g.* in the C cases described above, coil-side $(1 + 50) =$ coil-side 9, or coil-side 43 = coil-side 1. When the second of these cases is arranged for a single-layer winding, as in Figs. 466 and 471 with $y_B = y_F = 5$, $y = y_B + y_F = 10$, $4y - 5 = 35$, $3y - 5 = 25$. The bars to be left open-ended are then as follows—

$$\left.\begin{array}{r} 1\\ 1 + 35 = 36\\ 36 + 5 = 41\\ 41 + 25 = 66(-42) = 24\\ 24 + 5 = 29\\ 29 + 35 = 64(-42) = 22\\ 22 + 5 = 27\\ 27 + 25 = 52(-42) = 10\\ 10 + 5 = 15\\ 15 + 35 = 50(-42) = 8\\ 8 + 5 = 13\\ 13 + 25 = 38\end{array}\right\}\quad \begin{array}{c}\text{As shown by the}\\ \text{third row of numerals}\\ \text{in Fig. 471}\end{array}$$

Usually, however, it is rather the numbers of the joints to be left open that is required to be known, and use is then made of $y_c = y/2$. An advance of x complete loops from the starting-joint numbered 1 brings us to the $(1 + xy_c)$th joint, or in general from joint j to joint $(j + x y_c)$, and every time that $j + xy_c$ exceeds the total number of joints $C = U/2$, a new tour of the armature has begun, and C must be deducted, so that in our case joint 22 = joint 1, and so on. Since $4y_c = 20$ and $3y_c = 15$, the joints to be left open are then as follows—

$$\left.\begin{array}{r}
1 \\
1 + 20 = 21 \\
21 + 15 = 36(-21) = 15 \\
15 + 20 = 35(-21) = 14 \\
14 + 15 = 29(-21) = 8 \\
8 + 20 = 28(-21) = 7 \\
7 + 15 = 22(-21) = 1
\end{array}\right\} \text{As shown by the fourth row of figures in Fig. 471}$$

The phases are then as follows—

Phase	I.	1*a* to 21*b*	connected to	8*b* to 14*a*
,,	II.	15*a* to 14*b*	,,	1*b* to 7*a*
,,	III.	8*a* to 7*b*	,,	15*b* to 21*a*

When connected in star, 14 *a*, 7 *a*, and 21 *a* form the star-connexion, and 1 *a*, 15 *a*, and 8 *a* the terminals (Fig. 467 (i)). To connect them in delta, 14 *a* is joined to 15 *a*, 7 *a* to 8 *a*, and 21 *a* to 1 *a* (Fig. 467 (iii)).

To combine the information in regard to joints and coil-sides, let the positions of the coil-sides in the *same* slot in the upper and lower layers respectively of a double-layer winding be marked as

$$\begin{array}{llll}
1 & 3 & 5 \ldots\ldots\ldots\ldots & (u-1) \\
2 & 4 & 6 \ldots\ldots\ldots\ldots & u
\end{array}$$

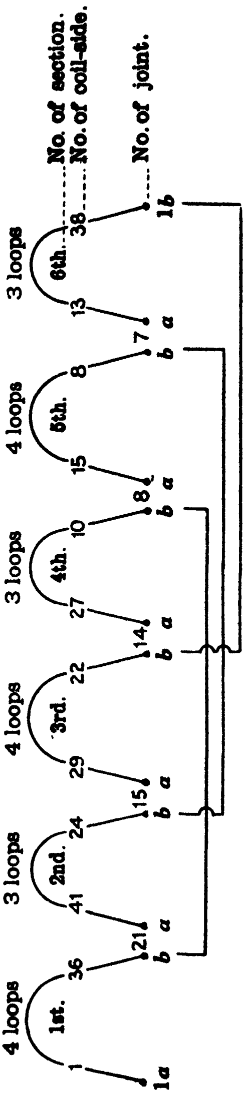


FIG. 471.—Scheme of winding and connexions for opening the wave-winding of Fig. 466.

Then if j is the number of an open joint calculated as above, the numbers of the coil-sides which would meet at the opened joint are $(2j - 1) - y_F$ in the lower layer and $(2j - 1)$ in the upper layer, and if these numbers are exactly divisible by u, the conductor is the last in the slot whose number is the quotient, while the numerator in the remainder, if any, indicates its position in the slot bearing the next higher number to the whole part of the quotient. Thus in the same case as above, when arranged for two layers[1] as in Table XIX,

Bottom coil-side ⟶	*Joint No*	⟵ *Top coil-side.*
$(2j-1)-y_F$	j	$(2j-1)$
40	1	1
38	21	41
26	15	29
24	14	27
12	8	15
10	7	13

An example with more than 2 coil-sides per slot will better illustrate this, *e.g.* a 4-pole machine with 57 slots and 6 coil-sides per slot, *i.e.* $C = 171$.

$$p = 2 \qquad y_c = \frac{171-1}{2} = 85 \qquad y_B = 85 \qquad y_F = 85$$

$$y_B{}^1 = 14$$

$171/6 = 28\tfrac{1}{2}$, therefore sections alternately of 29 and 28 coils

$$\frac{57}{3 \times 2} + \frac{57}{2} = 38. \quad \text{Start of phase II} = 1 (= 58) - 38 = \text{slot } 20$$

$$\frac{57 \times 2}{3 \times 2} = 19. \quad ,, \quad ,, \quad ,, \quad \text{III} = 1 (= 58) - 19 = \text{slot } 39.$$

Since $29\, y_c = 2465$ $(= 171 \times 14 + 71) = 71$, and $28\, y_c = 2380$ $(= 13 \times 171 + 157) = 157$, the joints and coil-sides to be left open are as follows (the joints are not here tabulated in the centre between the top and bottom coil-sides as in the preceding table) --

Joint No j.	Top coil-side.		Bottom coil-side	
	No. $(2j-1)$	*Position and slot*	*No.* $(2j-1)-y_F$	*Position and slot*
1 .	1	1 in slot 1	258	6 in slot 43
$1 + 71 = 72$.	143	5 ,, 24	58	4 ,, 10
$72 + 157 = 229 = 58$.	115	1 ,, 20	30	6 ,, 5
$58 + 71 = 129$.	257	5 ,, 43	172	4 ,, 29
$129 + 157 = 286 = 115$.	229	1 ,, 39	144	6 ,, 24
$115 + 71 = 186 = 15$.	29	5 ,, 5	286	4 ,, 48

In both the above cases there has been no common factor higher than 1 between the number of slots and the number of pole-pairs. If there is a common factor $f > 1$, the machine is a derivative from a parent machine having $S/f = S'$ slots and $p/f = p'$ pole-pairs. This portion of the derived machine must then be divided into six sections, and the similar f sets of pairs can be placed in series or parallel. *E.g.* in an 8-pole machine with 114 slots, 6 coil-sides per slot, 342 coils, the H.C.F. of the slots and pole-pairs is $f = 2$. Hence $171/6 = 28\frac{1}{2}$ gives 29 and 28 coils alternately, and the machine is the same as the last but with one repeat of poles and winding inserted.

---

[1] When so arranged, y_F for the reason given at foot of p. 28 is made 3, so that while the numbers of joints and of the top coil sides to be left open remain the same, the bottom coil-side numbers are higher by 2.

It is then of interest to see how the table of openings continues;[1] thus $29 y_c = 342 \times 7 + 71 = 71$ and $28 y_c = 342 \times 6 + 328 = 328$.

Joint No. j.	Top coil-side		Bottom coil-side.	
	No. $(2j-1)$.	*Position and slot.*	*No.* $(2j-1)-y_F$	*Position and slot.*
1 .	1	1 in slot 1	600	6 in slot 100
1 + 71 = 72 .	143	5 ,, 24	58	4 ,, 10
72 + 328 = 400 = 58 .	115	1 ,, 20	30	6 ,, 5
58 + 71 = 129 .	257	5 ,, 43	172	4 ,, 29
129 + 328 = 457 = 115 .	229	1 ,, 39	144	6 ,, 24
115 + 71 = 186 .	371	5 ,, 62	286	4 ,, 48
186 + 328 = 514 = 172 .	343	1 ,, 58	258	6 ,, 43
172 + 71 = 243 .	485	5 ,, 81	400	4 ,, 67
243 + 328 = 571 = 229 .	457	1 ,, 77	372	6 ,, 62
229 + 71 = 300 .	599	5 ,, 100	514	4 ,, 86
300 + 328 = 628 = 286 .	571	1 ,, 96	486	6 ,, 81
286 + 71 = 357 = 15 .	29	5 ,, 5	628	4 ,, 105

§ 11. Division into sections in wave-windings for machines with $p = 3$ or a multiple of 3.—When a simplex wave-winding is employed in a machine having $p = 3$ or a multiple of 3 in spite of the inaccuracy in the phase-differences which has already been pointed out, the inaccuracy also extends to its division into sections for the purpose of lessening the phase-spread. The quotient of the coils divided by 6 has a fractional remainder other than $\frac{1}{2}$, and one pair of sections in series must contain a coil more or less than others. *E.g.* in a 6-pole machine with 140 coils, 4 coil-sides per slot, and 70 slots, $140/6 = 23\frac{1}{3}$, which will necessitate (24 + 23) coils in each of two phases and (24 + 22) in the third phase.

But truly equal phases become possible when the winding is of the open-ended type with the front pitch shortened by one after every tour, after the fashion of Fig. 96. This type of winding is again divisible into class *D*, when the two sections paired together are equal, and class *E* when one is longer than the other by one coil.

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[1] The opening of double-layer wave-windings has been thoroughly treated in Dr. S. P. Smith's papers on "The Shape of the Pressure Wave" and "The Theory of Armature Windings," *Journ. I.E.E.*, Vol. 53, p. 237, and Vol. 55, pp. 27, 33. In the same author's article on "The Theory of Alternating-current Generators" in *Papers on the Design of Alternating Current Machinery* by Hawkins, Smith, and Neville (Pitman & Sons), there will be found (pp. 59–61) the scheme of connexions and openings for a wave-wound 24-pole machine with $S = 117$, $f = 3$.

When the sequence of slots is written down, if the number of columns is maintained as the number of coil-sides in a section as previously, the shortening of the pitch at intervals is apt to be overlooked; it is, therefore, more convenient to employ $2p$ columns as representing a tour, the pitch between the end slot of one row and the first slot of the next row being uniformly less by one. The table then takes the following form—

Example of Class D (*slots divisible by* 6; Table XXII. III).

$p = 3 \quad S = 36 \quad y_B{}^1 = 6 \quad y_F{}^1 = 6$

$C = 36$ and 5 at end of every tour

Winding retrogressive

$\frac{1}{3}$ of $\frac{36}{3} = 4$. Start of phase II $= 1(= 37) - 4 = 33$

„ „ III $= 1(= 37) - 8 = 29$

TABLE XIX

	Top	*Bottom.*	*Top.*	*Bottom.*	*Top.*	*Bottom.*
I. S ⟶	1	7	13	19	25	31
	36	6	12	18	24	30
	35	5	11	17	23	29
	34	4	10	16	22	28
II. S ⟵	33	3	9	15	21	27
	32	2	8	14	20	26
J	31	1	7	13	19	25
	30	36	6	12	18	24
III. S ⟵	29	35	5	11	17	23
	28	34	4	10	16	22
	27	33	3	9	15	21
	26	32	2	8	14	20

In each of the 3 pairs of equal sections in series, the same slot numbers repeat.

Example of Class E (*slots divisible by* 3, *but not by* 6; Table XXII. III).

$p = 3 \quad S = 27 \quad y_B{}^1 = 4 \quad y_F{}^1 = 5$, and 4 at end of every tour

Winding retrogressive—

$\frac{1}{3}$ of $\frac{27}{3} = 3$. Start of phase II $= 1(= 28) - 3 =$ slot 25

$\frac{2}{3}$ of $\frac{27}{3} = 6$. „ „ III $= 1(= 28) - 6 =$ „ 22

TABLE XX

	Top.	*Bottom*	*Top.*	*Bottom.*	*Top.*	*Bottom.*
I. S ⟶	**1**	5	10	14	19	23
	27	4	9	13	**18**	22
	26	3	8	12	17	21
II. S ⟵	**25**	2	7	11	16	20
	24	1	6	10	**15**	19
	23	27	5	9	14	18
III. S ⟵	**22**	26	4	8	13	17
	21	25	3	7	**12**	16
	20	24	2	6	11	15

Here the starts of the sections are marked in heavier figures.

When re-arranged in 6 rows, as in previous Tables, we have—

		Top		*Top.*		*Top.*		*Top.*		*Top.*	
I.	*S* ⟶	1	5	10	14	19	23	27	4	9	13
				18	22	26	3	8	12	17	21
II	*S* ⟵	25	2	7	11	16	20	24	1	6	10
			J	15	19	23	27	5	9	14	18
III.	*S* ⟵	22	26	4	8	13	17	21	25	3	7
				12	16	20	24	2	6	11	15

It will be seen that each larger group contains all except 2 of the slots of the smaller group in series with it.

§ **12. Summary of three-phase windings.**—Adopting the indicating letters used by Prof. Miles Walker,[1] let the leading types of three-phase windings be classified as follows—

SINGLE-LAYER. *Concentric, or lap-wound* (Fig. 97) (*or wave-wound when* $S = 6p$), (A). *Simplex wave-winding* in a single layer, divisible into either (b) 6 equal sections or (c) 3 pairs of sections, each pair containing a larger and a smaller group.

DOUBLE-LAYER. *Full-pitch or fractional pitch* (based on Fig. 451) *open-ended winding* with wave-connected coils, (a).

Parent simplex wave-windings as for a continuous-current machine with all end-connexions at any one end of the same pitch, either closed-circuit or divisible into either (B) 6 equal sections, the total number of slots being a multiple of 6, or (C) 3 pairs of sections.

Parent duplex wave-windings, i.e. two independent wave-windings, obtainable from the last-mentioned class by doubling the numbers of slots, transferring every top conductor of the single wave winding to the top of a new slot and then placing a second set of conductors at the top of the old and bottom of the new slots. The second independent winding is thus not interleaved in succession to the first, but alternately above and below it.[2] The component pitches must be so chosen that the first winding occupies the top, say, of every slot of odd number and the bottom of every slot of even number; the table of slots of the second winding then simply repeats that of the first winding, except that " Bottom " and " Top " appear in reverse order. The two windings with their number of slots appearing on the left-hand side of Table V (p. 248, Vol. I) only in fact give repetitions of E.M.F. owing to the grouping of the conductors in slots ; if distributed uniformly over the periphery, the one winding would follow in succession to the other.

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[1] *Specification and Design of Dynamo-Electric Machinery*, p. 101, and Table VII, p. 109. A few modifications and additions of minor importance are made in the following classification.

[2] Hence it is not available to give a symmetrical duplex continuous-current winding when closed and attached to a commutator in which the sectors are interleaved.

Such windings are again divisible into either (B_2) 12 equal sections, or (C_2) 6 equal pairs which can be coupled in series or parallel.

As an instance, let the first example of case C above (p. 31) be doubled to give C_2; $y_c = \frac{42-2}{5} = 8$, and $y_B{}^1$ must be made 5 and $y_F{}^1 = 3$. Then—

TABLE XXI

Top	*Bottom.*	*Top.*	*Bottom.*	*Top.*	*Bottom.*	*Top.*	*Bottom.*
1	6	9	14	17	22	25	30
		33	38	41	4	7	12
15	20	23	28	31	36	39	2
		5	10	13	18	21	26
29	34	37	42	3	8	11	16
		19	24	27	32	35	40

Each slot has received one conductor, and the second winding is the reverse.

Bottom.	*Top.*	*Bottom.*	*Top*	*Bottom.*	*Top*	*Bottom.*	*Top.*
1	6	9	14	17	22	25	30

Etc., etc.

Derivative multiplex wave-windings obtained from parent simplex windings and indicated by the letters B_f or C_f as containing f repeats. Here f is the H.C.F. between S and p and is the numeral in Table XXII before the bracketed value of the slots which correspond to a machine having $p' = p/f$ pole-pairs. If used closed as in a rotary converter and tapped, *e.g.* for 3 phases, each of the 3 slip-rings would tap f points. Such windings are given in full in the following Table for $u = 2$ for completeness' sake, although many would seldom be required. The process has not been followed further for $u > 4$.

Open-ended wave-windings with the front pitch shortened by one slot at the completion of each tour (Fig. 96), divisible again into either (D) 6 equal sections or (E) 3 equal pairs of sections.

With $S = 6\ p\ n_1$ where n_1 is any whole number, A is always possible, *whatever the number of pole-pairs*, and n_1 is simply the number of slots per pole per phase. Similarly with $S = 3\ p\ n_1$ and a double layer, (although the pitch may not always be advisable), a is always possible, since the second side of each coil falls in a second layer. Other possibilities in slots and windings are dependent upon the number of pole-pairs and fall into three distinct classes, as set forth in the following Table ; in the columns of windings in this Table have also, for completeness, been included the above-mentioned A and a cases. The method of deriving the slot numbers for B and C have already been given in Table VII (p. 259, Vol. I).

TABLE XXII. THREE-PHASE ALTERNATOR WINDINGS

$n_1 =$ a whole number.

I. The number of pole-pairs is a prime number other than 2 or 3.

No. of pole-pairs p.	Number of slots, S											
	Single-layer.			Double-layer.								
	A.	$u = 1$.		$u = 2$						$u = 4$.	$u = 6$.	
		n_1 even b.	n_1 odd c.	n_1 even a or D	n_1 odd a or E	n_1 even B	n_1 odd C.	n_1 even B_2.	n_1 odd C_2.	n_1 even or odd B.	n_1 even C.	n_1 odd. B
1	$6n_1$	$6n_1$		$3n_1$		$3n_1$		$2(3n_1)$		$3n_1$	$3n_1$	
5	$30n_1$	$30n_1 \pm 12$		$15n_1$		$15n_1 \pm 6$		$2(15n_1 \pm 6)$		$15n_1 \pm 3$	$15n_1 \pm 3$	
7	$42n_1$	$42n_1 \pm 12$		$21n_1$		$21n_1 \pm 6$		$2(21n_1 \pm 6)$		$21n_1 \pm 3$	$21n_1 \pm 9$	
11	$66n_1$	$66n_1 \pm 24$		$33n_1$		$33n_1 \pm 12$		$2(33n_1 \pm 12)$		$33n_1 \pm 6$	$33n_1 \pm 15$	
13	$78n_1$	$78n_1 \pm 24$		$39n_1$		$39n_1 \pm 12$		$2(39n_1 \pm 12)$		$39n_1 \pm 6$	$39n_1 \pm 9$	
17	$102n_1$	$102n_1 \pm 36$		$51n_1$		$51n_1 \pm 18$		$2(51n_1 \pm 18)$		$51n_1 \pm 9$	$51n_1 \pm 45$	
19	$114n_1$	$114n_1 \pm 36$		$57n_1$		$57n_1 \pm 18$		$2(57n_1 \pm 18)$		$57n_1 \pm 9$	$57n_1 \pm 51$	
.....	.....	.....		.....		.....		.....		.....	.....	

II. The number of pole-pairs is a multiple of 2 and not of 3.

No. of pole-pairs p.	Number of slots, S.								
	Single-layer.		Double-layer.						
	A.	$u = 1$.	$u = 2$.					$u = 4$.	$u = 6$.
		n_1 even or odd c.	n_1 even or odd a or D.	n_1 even or odd C	n_1 even B_f	n_1 odd C_f	n_1 even or odd C_f.	n_1 even or odd B_f.	n_1 even or odd C.
2	$12n_1$	$12n_1 + 6$	$6n_1$	$6n_1 + 3$	$2(3n_1)$		—	$2(3n_1)$	$6n_1 + 3$
4	$24n_1$	$24n_1 \pm 6$	$12n_1$	$12n_1 \pm 3$	$4(3n_1)$		$2(6n_1 + 3)$	$4(3n_1)$	$12n_1 \pm 3$
8	$48n_1$	$48n_1 \pm 18$	$24n_1$	$24n_1 \pm 9$	$8(3n_1)$		$4(6n_1 + 3)$ $2(12n_1 \pm 3)$	$8(3n_1)$	$24n_1 \pm 3$
10	$60n_1$	$60n_1 \pm 18$	$30n_1$	$30n_1 \pm 9$	$10(3n_1)$ $2(15n_1 \pm 6)$		—	$10(3n_1)$ $2(15n_1 \pm 3)$	$30n_1 \pm 3$
14	$84n_1$	$84n_1 \pm 30$	$42n_1$	$42n_1 \pm 15$	$14(3n_1)$ $2(21n_1 \pm 6)$		—	$14(3n_1)$ $2(21n_1 \pm 3)$	$42n_1 \pm 9$
16	$96n_1$	$96n_1 \pm 30$	$48n_1$	$48n_1 \pm 15$	$16(3n_1)$		$8(6n_1 + 3)$ $4(12n_1 \pm 3)$ $2(24n_1 \pm 9)$	$16(3n_1)$	$48n_1 \pm 21$
.....	.....	.....	.....	.....	.....		.....	.....	.....

III. The number of pole-pairs is 3 or a multiple of 3.

No. of pole-pairs p.	Number of slots, S.										
	Single-layer A.	Double-layer.									
		$u = 2$.						$u = 4$.	$u = 6$.		
		n_1 even a or D	n_1 odd a or E.	n_1 even E^*.	n_1 odd D^*.	n_1 even B_f	n_1 odd C_f	n_1 even or odd B_f.	n_1 even C_f.	n_1 odd B_f.	
3	$18n_1$	$9n_1$		$9n_1 \pm 3$		$3(3n_1)$		$3(3n_1)$	$3(3n_1)$		—
9	$54n_1$	$27n_1$		$27n_1 \pm 9$		$9(3n_1)$		$9(3n_1)$	$9(3n_1)$		—
15	$90n_1$	$45n_1$		$45n_1 \pm 15$		$\begin{cases} 15(3n_1) \\ 3(15n_1 \pm 6) \end{cases}$		$\begin{cases} 15(3n_1) \\ 3(15n_1 \pm 3) \end{cases}$	$\begin{cases} 15(3n_1) \\ 3(15n_1 \pm 3) \end{cases}$		—
.....	.....	.....		.....		.....		.....	.....		.....
	A.	n_1 even or odd a or D.		n_1 even B_f.	n_1 odd C_f.	n_1 even or odd C_f.		n_1 even or odd B_f.	n_1 even C_f.	n_1 odd B_f	n_1 even or odd C_f
6	$36n_1$	$18n_1$		$6(3n_1)$		—		$6(3n_1)$	$6(3n_1)$		—
12	$72n_1$	$36n_1$		$12(3n_1)$		$\begin{cases} 6(6n_1 + 3) \\ 3(12n_1 \pm 3) \end{cases}$		$12(3n_1)$	$12(3n_1)$		$\begin{cases} 6(6n_1 + 3) \\ 3(12n_1 \pm 3) \end{cases}$
18	$108n_1$	$54n_1$		$18(3n_1)$		$9(6n_1 + 3)$		$18(3n_1)$	$18(3n_1)$		$9(6n_1 + 3)$
.....	.....	.....		.....		.....		.....	.....		.....

NOTE.—Any B or C, B_f or C_f case is available equally for a closed-circuit wave winding.

* The slot numbers under the starred columns E^* and D^* (and similarly D^* numbers $18n_1 \pm 6$, $36n_1 \pm 12$, and $54n_1 \pm 18$ for $p = 6$, 12, and 18 respectively) do not yield phase-differences strictly of 120°, since there are no slot positions corresponding thereto; the inaccuracy is always one-third of a slot-pitch or $120/(3n_1 \pm 1)$ electrical degrees, where n_1 has the same value as in the expression for S. They only therefore become permissible after careful calculation has shown that the reactance will limit the circulating current to an amount that will not be harmful.

CHAPTER XXV

THE E.M.F. OF THE HETEROPOLAR ALTERNATOR

§ 1. The scope of the chapter.—Throughout the present Chapter, *the loop* is assumed as the E.M.F.-giving unit. In a stator winding if the loops are all of similar pitch, each must give the same periodic time-curve of E.M.F., or, if wound concentrically, they are convertible by an imaginary re-arrangement of their end-connexions into a number of similar overlapping loops of full pitch (Fig. 472), and the time-curve of E.M.F. of a coil or of a phase of many loops must be obtainable from the time-curve of E.M.F. of one such imaginary full-pitch loop. It is therefore presupposed throughout this Chapter either (1) that the time-curve of the E.M.F. of a single loop, acting as a specimen of the whole winding, is known, having been determined experimentally by a test coil; further that this curve has been analysed into a single Fourier series of fundamental and uneven harmonic E.M.F.'s, or (2) that when the machine has not been built and a loop cannot yet be tested on it, the fundamental and chief harmonic E.M.F.'s of a specimen loop have been calculated.

The present chapter is, therefore, solely concerned with the simple addition of E.M.F.'s regarded as vectors with different amplitudes and frequencies, and obtained from the actual or assumed E.M.F. wave of the loop. The *raison d'être* for the resolution of the E.M.F. of the loop into its sinusoidal components is, of course, that in relation to the fundamental or any harmonic, the reduction in the total E.M.F. of one phase as compared with the E.M F. of one loop multiplied by the number in series in the phase can be readily calculated either graphically or trigonometrically. The present treatment will require to be further supplemented by an examination of the basis E.M.F. wave of the single loop, and an elementary analysis of this is reserved for the following chapter. It need only here be added that provisionally with sufficient accuracy for purposes of design (subject to certain limitations to be mentioned later) the fundamental and chief harmonic E.M.F.'s of the specimen loop can be obtained on the assumption of a flux-density curve, constant in time and smoothed out as in fact would only be the case if the armature surface were smooth and the pole-pieces unslotted. In view of this some further conclusions are therefore added which are strictly and universally true only of the smooth armature.

§ 2. The differential factor in general, and its components.—In a two-pole winding or in a multipolar winding referred to a standard field of double pole-pitch, let the loops considered at any one moment when the machine is running occupy q_1 different loop-positions relatively to the pair of poles. The same number will, of course, also hold if they are considered when the machine is at rest in any position relatively to the pole-pair. A loop-position will be defined by the position of the axis of the loop, or, since the loops are similar, by the position of, say, their front edges, or again by the slot-positions occupied by these edges if, as is usually the case in practice, the stator is toothed.[1] In a double-layer winding, the total possible number of such loop-positions as fixed by the slot-positions is $S' = S/f$, where f is the H.C.F. of S and p.

If a number q_1 of loops, each giving a sinusoidal E.M.F. of amplitude $\mathbf{e}$ and of virtual value e, and each occupying a separate loop-position, are connected in series, the composition of their E.M.F.'s will yield a resultant sine-curve of amplitude $k_s\, q_1\mathbf{e}$ or virtual value $k_s\, q_1 e$, where k_s is the differential factor. If n be the order of a harmonic E.M.F., we are virtually dealing with a $2n$-pole machine, and any relative displacement of the loop-positions in the standard fundamental field is magnified n times in relation to the harmonic standard field with its own double pole-pitch. If then the q_1 loops or their vectors are successively displaced from each other by the electrical angle ψ_e in relation to the fundamental two-pole field, the differential factor in relation to fundamental or harmonic is in every case (Chapter IX, § 12)

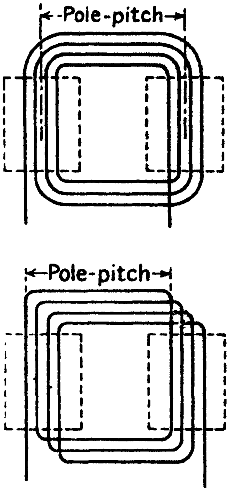

FIG. 472.—Equivalence of full-pitch loops to concentric coil.

$$k_{sn} = \frac{\sin q_1 n \frac{\psi_e}{2}}{q_1 \sin n \frac{\psi_e}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

Often it is convenient to split this up into component factors dealing with portions of the whole winding. Thus let the loops considered be those of a composite coil with its coil-side forming a belt or band corresponding to a pole, or in general the successive loops of any part of a phase winding. Then the particular value of k_{sn}' is the ***belt differential factor*** or the ***distribution factor***, as it is also termed, which will be given a fuller form in § 7, eq. 228. It defines the extent to which the actual E.M.F. is reduced owing to the loops being distributed over some portion of the armature surface, as compared with the arithmetic sum $q_1 e$ which would result from the same number q_1 of loops concentrated in a single pair of small slots. Other similar loops in series with them may or may not occupy exactly the same relative slot-positions under other pole-pairs or in other parts of the winding. In the latter case corresponding to the relative displacement of each successive group in electrical degrees, their joint vectors of E.M.F. will be similarly displaced with the same successive phase-differences. When the vectors of m groups successively displaced by ψ_e''' electrical degrees are compounded together, the *group differential factor* for the nth harmonic is

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[1] The E.M.F.'s of loops embedded in the same slots of a toothed armature may be considered as practically coinciding in phase, the difference between loops in different layers or at different sides of a slot being negligibly small.

$$k_{sn}''' = \frac{\sin mn \dfrac{\psi_e'''}{2}}{m \sin n \dfrac{\psi_e'''}{2}} \quad . \quad . \quad . \quad . \quad . \quad (227)$$

and the total differential factor is $k_{sn}' \times k_{sn}'''$.

In a closed-circuit winding with two or more loops in each coil in the same slots, if the phase-connexion is attached to the centre of a coil instead of to the junction of two coils, the belt of the phase is widened, and in addition to the vectors of the whole coils within the belt, there is at each end a vector of half the length representing the E.M.F. from half a coil; thence there results a " mid-coil differential factor," as it has been termed by Prof. Comfort A. Adams,[1] which forms an additional multiplier.

It only remains to mention that it is often possible to take the vectors in different order, belt and group factors being interpreted in agreement with the change, since there is no intrinsic difference in their natures. To take a simple example, in a star-connected 3-phase alternator, let there be 12 slots per pole with concentric coils in a single layer; between a pair of terminals two phases are in series, and 8 of their slots in each of the p double pole-pitches are adjacent and form a continuous belt: there are therefore 8 vectors to be considered, mutually displaced from one another by $\pi/12$. In relation to the fundamental, the differential factor is $\sin(8 \times 7{\cdot}5°) \div 8 \sin 7{\cdot}5° = 0{\cdot}866/1{\cdot}044 = 0{\cdot}83$, and if e_a is the fundamental pressure of a component full-pitch coil, the interlinked fundamental pressure is $0{\cdot}83 \times 8 \times p\,e_a$ But alternatively the belt differential factor of a phase is $\sin(4 \times 7{\cdot}5°)/4 \sin 7{\cdot}5° = 0{\cdot}5/0{\cdot}522 = 0{\cdot}958$, and the fundamental pressure E_a of a phase is $0{\cdot}958 \times 4\,pe_a$. The two resultant vectors of the two phases are separated by an angle of $120° = 2\pi/3$, but as in the case of the two sides of a loop, it is the deficiency from 180° that causes differential action here, so that the group differential factor is $\sin 60°/2 \sin 30° = 0{\cdot}866$; the interlinked fundamental pressure is then, as is well known, $0{\cdot}866 \times 2E_a = 0{\cdot}83 \times 8\,pe_a$. The total differential factor for the two phases in series is, therefore, as above $0{\cdot}866 \times 0{\cdot}958 = 0{\cdot}83$.

§ 3. The reduction factor, k_{rn}.—If $\mathbf{e}_1$, $\mathbf{e}_3$, $\mathbf{e}_5$ . . . . are the amplitudes of the fundamental and harmonic E.M.F.'s of the single loop which has been presupposed as the starting-point, since in a phase there are $Z/2qN_{ph}$ such loops, the total instantaneous E.M.F. of the phase is

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[1] " Electromotive Force Wave-shape in Alternators," *Trans. Amer. I.E.E.*, Vol. 28, Part II, p. 1064.

$$e_a = \frac{Z}{2qN_{ph}} \left\{ k_{s1}\, \mathbf{e}_1 \sin(\omega_e t) + k_{s3}\, \mathbf{e}_3 \sin(3\omega_e t + \theta_3) + k_{s5}\, \mathbf{e}_5 \sin(5\omega_e t + \theta_5) + \ldots \right\}$$

where t is reckoned from the moment when the resultant E.M.F. of fundamental frequency passes through zero from a negative to a positive value. Let x be the fraction which the amplitude of any harmonic E.M.F. is of the fundamental E.M.F. of the loop, so that $\mathbf{e}_3 = x_3\mathbf{e}_1$ or $\mathbf{e}_n = x_n\,\mathbf{e}_1$. Thence

$$e_a = \frac{Z}{2qN_{ph}}\, k_{s1}\, \mathbf{e}_1 \left\{ \sin(\omega_e t) + x_3 \frac{k_{s3}}{k_{s1}} \sin(3\omega_e t + \theta_3) + x_5 \frac{k_{s5}}{k_{s1}} \sin(5\omega_e t + \theta_5) + \ldots \right\}$$

$$= \frac{Z}{2qN_{ph}}\, k_{s1}\, \mathbf{e}_1 \left\{ \sin(\omega_e t) + x_3\, k_{r3} \sin(3\omega_e t + \theta_3) + x_5\, k_{r5} \sin(5\omega_e t + \theta_5) + \ldots \right\}$$

This shows that if the amplitude of any harmonic in the loop E.M.F. is 100 x_n per cent. of the amplitude of the loop's fundamental E.M.F., the amplitude of that harmonic in the phase E.M.F. is $100x_n \times k_{sn}/k_{s1}$ per cent. of the amplitude of the fundamental. The quotient k_{sn}/k_{s1} is the *reduction factor*, k_{rn}, and it will be seen that it is *larger* than k_{sn}, when the coil-side has any width or k_{s1} is less than unity. If the amplitude of any harmonic in the E.M.F. wave of the loop is x_n of the amplitude of the fundamental, the value of the same harmonic in the final E.M.F. of the phase expressed as a percentage of the fundamental E.M.F. can thus at once be predicted as

$$100\, x_n \times k_{rn}$$

§ 4. **The smooth-surface armature.**—In the special case of an armature with smooth surface, in which the shape and content of the flux curve is unvarying in time, the matter can be pushed much farther. There is then for a given excitation only one flux-density curve to be resolved into a Fourier series. The two sides of a loop can be treated separately, and if y' be the distance between them, there arises a *pitch or loop-span differential factor* when y' is different from the pole-pitch, Y (*cp.* Fig. 60). When the general form of expression for vectorial addition is converted to suit the case of the two sides of a loop, the E.M.F.'s of which are additive when they are of opposite sign, the pitch differential factor is given for fundamental or uneven harmonics as in Chapter IX § 12 by the expression

$$k_{sn}'' = \cos n \frac{\pi}{2}(1 - \lambda_c) = \cos n \frac{\varepsilon}{2} \quad . \quad . \quad . \quad (33)$$

where λ_c is the pitch of the loop expressed as a fraction of the pole-pitch or ε is the angle by which the pitch or span of the loop exceeds or falls short of the pole-pitch. For the fundamental, and for this only, is the pitch differential factor equal to $\sin\left(\frac{\pi}{2} . \lambda_c\right)$.

The total differential factor for a band of several loops is then in relation to the fundamental $k_{s1} = k_{s1}' \, k_{s1}'' \, k_{s1}'''$ and for the nth harmonic $k_{sn} = k_{sn}' \, k_{sn}'' \, k_{sn}'''$.

Further in the smooth armature the passage from loop-positions to conductor-positions can legitimately and easily be made, and lastly the E.M.F. becomes expressible through the amplitudes of the fundamental and uneven harmonics B_{g1}, B_{g3}, B_{g5} . . . . of the unvarying *flux-density* curve. If x be the fraction which the amplitude of any harmonic in the flux-density curve is of the fundamental, that curve is expressible as

$$B_{g1} (\sin a \pm x_3 \sin (3a + \theta_3) \pm x_5 \sin (5a + \theta_5) \pm . . . .)$$

the fractions x_3, x_5, etc., not being given any sign in themselves. Choosing as the zero of time the moment when the real or imaginary conductor occupying the exact centre of a belt corresponding to a phase passes through zero field, if $c = (Z/N_{ph}\, q)Lv \times 10^{-8}$ and $v = \pi DN/60$ in cm. per sec., the total instantaneous E.M.F. of a phase on the smooth armature by eq. (28) Chapter IX, § 11 is

$$\begin{aligned} e_a = c \, \{ & k_{s1} B_{g1} \sin (\omega_e t) \pm k_{s3} B_{g3} \sin (3\omega_e t + \theta_3) \\ & \pm k_{s5} B_{g5} \sin (5\omega_e t + \theta_5) \pm . . . . \} \\ = c \, B_{g1} k_{s1} \Big\{ & \sin (\omega_e t) \pm x_3 \frac{k_{s3}}{k_{s1}} \sin (3\omega_e t + \theta_3) \\ & \pm x_5 \frac{k_{s5}}{k_{s1}} \sin (5\omega_e t + \theta_5) \pm . . . . \Big\} \end{aligned}$$

Thence by the same steps as in § 3 it can be predicted for the smooth armature that if the amplitude of any harmonic in its steady flux curve is x_n of the amplitude of the fundamental, the value of the harmonic of the same order in the final E.M.F. wave of a phase expressed as a percentage of the fundamental E.M.F. will be $100 \, x_n \times k_{rn}$, where k_{rn} is the reduction factor, k_{sn}/k_{s1}.

The fundamental amplitude of the flux-density, B_g, may also be expressed as $yB_{g\,max}$, where y may be greater than, equal to, or less than unity. At no load at the middle of the half wave,

$$B_{g\,max} = B_{g1} - B_{g3} + B_{g5} - B_{g7} + \dots$$

but it must be remembered that any one or more of the harmonic amplitudes may itself be a negative quantity. The average value of the flux-density is

$$B_{g\,av} = \frac{2}{\pi}\left(B_{g1} + \tfrac{1}{3}B_{g3} + \tfrac{1}{5}B_{g5} + \tfrac{1}{7}B_{g7} + \dots\right)$$

§ 5. The smooth-surface and toothed armatures contrasted.—It is in the inapplicability of the pitch differential factor to toothed armatures that their difference from smooth-core armatures in the present connexion lies. This factor is related to the spacial flux-density, and it is not possible to express the E.M.F. or time-rate of change of linked flux through a spacial rate of change of flux when the shape or content or both of the spacial flux curve vary in time.

The true instantaneous E.M.F. wave of the loop in the presence of teeth and slots is in fact a complex phenomenon. On the smooth core the spacing undulations, instances of which have been given in Figs. 72 and 73, are greatly modified by the placing of several loops in different slots in series for the same reason that tooth ripples in the E.M.F. wave of a loop are modified when several loops occupying different loop positions are placed in series. But in their origin the two remain differentiated by the fact that with spacing ripples the E.M.F., when rising on the whole, never shows subordinate falls, and when falling on the whole, never shows subordinate rises, although the rate of the rise or fall alters.

It is not even the case that the toothed armature with its varying shape of flux-density curve must necessarily yield an E.M.F. wave in a loop with tooth ripples (*cp.* Chapter XXVI, § 16), or that the fundamental E.M.F. of the loop is strictly that given by consideration of loop and smoothed-out flux curve in their mutual relation. It can only be said that the values obtainable from the flux curve supposed constant and smoothed out are close to the truth for the fundamental E.M.F. and even for the chief harmonics from harmonics of low order in the flux curve until we come to harmonics of frequency connected with the presence of teeth. Such values suffice, therefore, for the determination of k_s (eq. 37) or of K (eq. 38*b*) for the purposes of design.

When, therefore, conclusions are given below that are only strictly true for smooth armatures as involving the pitch differential factor, use is made of the smaller type, since they are to be regarded as guides only, upon which implicit reliance cannot be placed when toothed armatures are in question.

§ 6. The symbols employed for the general case of the toothed armature.—In connexion with toothed armatures the symbols now to be employed need some further explanation.

Let $f =$ the H.C.F. between the total number of slots, S (including any empty slots), and the total number of pole-pairs, p; then, as explained in Chapter XII, § 5, and using the terms of that section if the winding repeats when the slot-positions repeat, the actual machine is equivalent to a parent or unit machine having $S/f = S'$ slots and $p/f = p'$ pole-pairs, multiplied f times.

All of the slots are not necessarily wound, but it may at once here be stated that in a polyphase machine if the correct phase-difference between the vectors of the phase E.M.F.'s (*e.g.* 120° in the 3-phase machine) is to be maintained, the minimum number of empty slots that may occur at equal distances round the stator within *each* of its f sections is N_{ph}, and higher multiples $x\,N_{ph}$ are also permissible. The total number of slots in the whole machine may therefore be written in general as

$$S = fS' = 2pg_0 + x\,N_{ph}f$$

where $g_0 =$ the number of slots per pole which are wound; x may be zero or any whole number, $x\,N_{ph}f$ being the total number of empty slots, if any.

The number of slots in each of the f sections of the machine is thus

$$S' = 2p'g_0 + x\,N_{ph}$$

which must be a whole number. Whether of the parent machine or of the actual derivative machine,

the number of slots per pole-pair $= 2g = 2g_0 + \dfrac{x\,N_{ph}}{p'}$

the number of slots per pole $= g = g_0 + \dfrac{x\,N_{ph}}{2p'}$

and *the number of slots per pole per phase* $= g' = \dfrac{g}{N_{ph}} = \dfrac{g_0}{N_{ph}} + \dfrac{x}{2p'}$

$$= g_0' + \frac{x}{2p'}$$

all of which may be fractional.

The distinction between the wound slots per pole per phase or g_0', and the total number when divided by the poles and phases or g' must be noted, and for convenience of reference, the symbols are again summarized below—

Slots per pole	g	Wound slots per pole . .	g_0
Slots per pole per phase .	g'	Wound slots per pole per phase	g_0'

§ 7. The value for the belt differential factor with grouped distribution.—If there are g slot-positions in the toothed armature per pole, and g' per pole per phase, of which g_0' are actually wound, and if these are adjacent to one another so as to form a single belt (as is the case with long undivided coils), then the number of vectors of E.M.F. to be added together is $q_1 = g_o'$. The total number of slots, including any that may be empty, being uniformly spaced over a width $1/N_{ph}$th. of the pole-pitch where N_{ph} is the number of phases, their angular distance apart in terms of a bipolar machine is $\gamma_e = \frac{\pi}{g} = \frac{\pi}{N_{ph}g'}$ electrical radians. Their displacement is, therefore, $\psi_e = \gamma_e$ in relation to the fundamental field, and in relation to any harmonic of order n is $\psi_e = n\gamma_e = n\frac{\pi}{g} = n\frac{\pi}{N_{ph}g'}$. Substituting this value in the expression (32) for the ratio of the vectorial to the arithmetical sum, the ***belt differential factor*** or the ***distribution factor*** is

$$k'_{sn} = \frac{\sin g_o' n \frac{\pi}{2g}}{g_o' \sin n \frac{\pi}{2g}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (228)$$

When there are no empty winding spaces, $g_o' = g'$, and since $g = N_{ph}g'$ this becomes

$$k'_{sn} = \frac{\sin n \frac{\pi}{2N_{ph}}}{g' \sin n \frac{\pi}{2N_{ph}g'}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (228a)$$

Thus so far as the fundamental is concerned, in the 3-phase alternator with long coils, if there are two slots per pole per phase, $k_s' = 0{\cdot}9659$; if there are 3 slots per pole per phase, $k_s' = 0{\cdot}96$, and so on as the number of slots is increased. But even when the coil-side is uniformly distributed over the whole $1/N_{ph}$th of the pole-pitch, there is no very great reduction, the minimum value in the 3-phase alternator for uniform distribution over one-third of the pole-pitch being $\frac{\sin 30^\circ}{\pi/6} = \frac{3}{3{\cdot}14} = 0{\cdot}955$.

With the mid-coil arrangement, the value of k_{sn}' must be multiplied again by $k_{sn}'''' = \cos n \frac{\pi}{2N_{ph}g'}$.

Next with shortened coils in a single layer and with a mean pitch = half the pole-pitch, although the angular pitch of the slots remains as before $\gamma_e = \frac{\pi}{N_{ph}g'}$, yet q_1 is now $= \frac{g'}{2}$. Therefore the belt differential factor becomes in this particular case

$$k_{sn}' = \frac{\sin n \frac{\pi}{4N_{ph}}}{\frac{g'}{2} \cdot \sin n \frac{\pi}{2N_{ph}g'}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (228b)$$

But if, as in Fig. 462, with an even number of slots per pole, the pitch reckoned in slots is made *odd*, the angular pitch of the slots *of one phase* is twice that of the slots, *i.e.* $\gamma_e = \frac{2\pi}{N_{ph}g'}$; and the belt differential factor is

$$k_{sn}' = \frac{\sin n \cdot \frac{\pi}{2N_{ph}}}{\frac{g'}{2} \cdot \sin n \frac{\pi}{N_{ph}g'}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (228c)$$

i.e. the same as for half the number of slots in a two-layer winding.

With a single-layer winding and an odd number of slots per pole (Fig. 463), the belt differential factor returns to value (228*a*), since the E.M.F.'s of the coils of one phase under one pole exactly fit in between the E.M.F.'s of the coils of the same phase under the adjacent pole and form in effect a single wide belt.

§ 8. Differential and reduction factors for (A) uniform distribution.—The values of the differential and reduction factors for different harmonics vary widely according to the grouping of the loops or of the coils in toothed armatures. But first, as a general standard of reference with which other cases may be compared, the case of perfectly uniform distribution over some arc of the field as given by an infinite number of slots will be considered. In this ideal case when the number of vectors within the given arc is very large, the length of each on such a diagram as Fig. 77 is correspondingly small; the sine of the angle in the denominator of k_{sn}' in eq. (228*a*) then becomes equal to the angle in radians, and its product with the very large number of vectors is equal to half the angle $n\chi$ over which the uniform distribution extends; the differential factor then becomes "chord/arc" as stated in eq. (32*a*).

In the fundamental field the belt of conductors or the coil-side can never extend over more than 180° (electrical), but when it is placed in a harmonic field of np pole-pairs, the possibility that the denominator in the expression "chord/arc" may include one or more entire circumferences in the vectorial diagram must be recognized. For in the nth harmonic field, let $n\,\chi$ be expressed as $(x \times 2\pi + \delta)$ electrical radians, where x may be zero or any whole number, and δ is less than 360°. Then

$$\frac{\sin n\frac{\chi}{2}}{n\frac{\chi}{2}} = \frac{\sin(x\pi + \delta/2)}{x\pi + \delta/2}$$

The angle $x\pi$ being zero or 180° or a multiple of 180°, this is equal to

$$\pm\frac{\sin \delta/2}{x\pi + \delta/2}$$

Multiplying numerator and denominator by $2R$,

$$\pm\frac{2R \sin \delta/2}{x \times 2\pi R + R\delta} = \frac{\text{chord of } \delta}{x \times 2\pi R + \text{arc of } \delta}$$

If $\sigma = 1/N_{ph}$ = the fraction of the pole-pitch over which the uniform distribution of one phase extends, the denominator of (228a) becomes $= n\sigma\pi/2$, and the expression for the series representing the differential factor becomes for uniform distribution

$$k_{sn}' = \frac{\sin n\sigma\frac{\pi}{2}}{n\sigma\frac{\pi}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (229)$$

and

$$k_{rn}' = \frac{\sin n\sigma\frac{\pi}{2}}{n \sin \sigma\frac{\pi}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (230)$$

The same expressions are applicable equally to a single-phase alternator having an open winding with the fraction σ of the pole-pitch wound, or to closed windings in two layers, in which case $\sigma = 2/N_{ph}$. An extended list of the values of k_{sn}' and k_{rn}' for uniform distribution are given in Tables 1 and 2 of the paper by Dr. S. P. Smith and Mr. Boulding,[1] on "The Shape of the Pressure Wave in Electrical Machinery." The curve for the belt factor for uniform distribution over one-third of the pole-pitch is given at the foot of Fig. 474, and it is seen to be a *damped* curve.

In the single-layer 3-phase case with long coils, whether divided or undivided, $\sigma = \frac{1}{3}$, and for the fundamental $k_{s1} = \frac{\sin \pi/6}{\pi/6} = \frac{6 \times 0{\cdot}5}{3{\cdot}14}$ $= 0{\cdot}955$, as already mentioned, and in the two-layer closed-circuit 3-phase case $\sigma = \frac{2}{3}$, so that k_{s1} sinks to $\frac{\sin \pi/3}{\pi/3} = \frac{3 \times 0{\cdot}866}{3{\cdot}14} = 0{\cdot}827$. An interesting point in the latter case is that the damped curve crosses the zero line at the 3rd, 9th, 15th . . . harmonics, showing that they will not be found in the phase E.M.F. owing to the numerator of the expression (229) becoming zero for $n = 3$ or any uneven multiple of 3.

SINGLE-LAYER WINDINGS

But the number of slots must in practice be finite, so that we next pass to the general case of a finite number of slots, for which it will be found that the critical question is the number of slot-positions, S'.

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[1] *Journ. I.E.E.*, Vol. 53, p. 215. *Cp.* also Dr. S. P. Smith, "Theory of Armature Windings," *Journ. I.E.E.*, Vol. 55, p. 23, and especially "The Properties of the Distribution Factor of Armature Windings," by B. Hague and S. Neville, *Journ. I.E.E.*, Vol. 60, p. 861, and Dr. A. E. Clayton, *Journ. I.E.E.*, Vol. 61, p. 754, where convenient curves for use are given.

§ 9. (B) Whole number of slots per pole, i.e. *$g =$ an integer.*— The condition stated at the head of the section can be reached in two ways—

(1) *The number of wound slots per pole per phase, $g_o' =$ a whole number*, and $x = 0$ or $2p'$ or a multiple of $2p'$.

Since $g_o' =$ a whole number, and the number of phases N_{ph} must be a whole number, $N_{ph}g_o'$ must be a whole number, and, therefore, to comply with requirement (B), x must be zero or have the values given above. That is, either all slots are wound, and $g_o' = g'$, or if there are any empty, there is one or more *per phase* in each pole-pitch. This latter condition would in practice only occur in the case of a single-phase machine for which a stock frame slotted for 2 or 3 phases is used, since in a polyphase machine it would be wasteful of valuable space.

The pole-pitch being an exact multiple of the slot-pitch, this gives in a single-layer winding the usual case of coils with a mean span equal to the pole-pitch, *i.e.* in each coil there are two belts of conductors of which the centres are separated by the distance of a pole-pitch or of π electrical radians. The coils, if wound concentrically, are therefore equivalent to a number of full-pitch loops, as shown in Fig. 472.

Since successive groups corresponding to successive pole-pairs occupy the same relative positions at each moment when referred to a standard 2-pole field, the group differential factor is uniformly 1, and there is only left the belt or distribution factor of eq. (228), *i.e.*

$$k_{sn} = k'_{sn} = \frac{\sin g_o' n \dfrac{\pi}{2g}}{g_o' \sin n \dfrac{\pi}{2g}} \quad . \quad . \quad . \quad . \quad . \quad (228)$$

It is then important to examine the effect of giving all odd values to n for each value of g_o' from 2 upwards. Dr. S. P. Smith and Mr. Boulding[1] in the above-mentioned paper have pointed out that in this case the belt factor for harmonics periodically rises to the same maximum value as for the fundamental every time that n passes a multiple of $2g$, the number of slots per pole-pair. The physical explanation is simple; if placed *e.g.* in a $2(2g)$-pole field, the divergence of phase between the loops would be exactly annihilated, since $(\pi/g) \times 2g = 360°$ exactly, but if placed in a $2(2g \pm 1)$-pole field, their phase-differences in relation to adjacent poles *of the same sign* in the imaginary harmonic field-system of many poles departs from 360° by exactly the amount of their divergence in relation to a pole of the original fundamental field. Mathematically, whether g be odd or even, $2g$ must be even and $2g \pm n'$ odd, if n' be any odd integer; hence $2g \pm 1$, $2g \pm 3$ . . . . $2g \pm n'$ and in general $M2g \pm n'$ (where M is any whole number including zero), are possible orders of harmonics that may be present. Inserting then $n = M2g \pm n'$ in equation (228).

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[1] *Journ. I.E.E.*, Vol. 53, p. 218.

$$k_{s\,(M2g \pm n')} = \frac{\sin\ (M2g \pm n')g_o' \frac{\pi}{2g}}{g_o' \sin\ (M2g \pm n') \frac{\pi}{2g}}$$

$$= \frac{\sin\left(\pm n'g_o' \frac{\pi}{2g}\right)}{g_o' \sin\left(\pm n' \frac{\pi}{2g}\right)} = + k_{sn}' \text{ or } - k_{sn}'$$

i.e. the *same* factor as for the harmonic of order n'.

This implies that in the series for k_{sn} the factors for values of n up to $2g - 1$ continually recur without any diminution, so that the curves obtained in the present case (B1) by plotting k_s in relation to n for any given number of wound slots and slots per pole (*cp.* Figs. 473, 474) are *undamped* and pass continually through the same cycle of values as n is increased. In the 3-phase case when all slots are wound, by equation (228*a*),

$$k_{sn} = k'_{sn} = \frac{\sin n\ 30^\circ}{g' \sin n \frac{30^\circ}{g'}} \quad . \quad . \quad . \quad . \quad . \quad (231)$$

and a complete table[1] of k'_{sn} and k'_{rn} up to $n = 65$ when g' is given values from 2 to 10, has been given by Dr. Smith and Mr. Boulding in Tables 3 and 4 of the above-mentioned paper.

The fundamental belt factor only decreases very slightly with increasing values of g', and it is the recurrence of this maximum value which is specially disadvantageous (Figs. 473, 474). Thus, if $g' = 2$ in a 3-phase alternator, *i.e.* $g = 6$ and $2g = 12$, the same factor holds for $n = 11$, 13; 23, 25; 35, 37; 47, 49, etc., as for the fundamental, and the same factor holds for $n = 9$, 15; 21, 27, etc., as for $n = 3$.

Harmonics in the loop E.M.F. (or in the field) of the orders $2g \pm 1$ are therefore specially to be avoided, since there is no reduction factor for the resulting harmonics in the phase E.M.F., and they will reappear with the same percentage value, *i.e.* in the same ratio that the harmonics in the original curve bear to its fundamental. They thus play the most important part in "tooth-ripples," and to a lesser degree the harmonics of the orders $2g \pm 3$, both of which are found to cause telephonic disturbance.[2]

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[1] Reprinted for k'_{sn} in Table II, *Papers on the Design of Alternating-current Machinery* (Pitman & Sons), p. 27.

[2] Owing to the slot frequency falling within the limits of speech frequencies. See especially S. C. Bartholomew, "Power Circuit Interference with Telegraphs and Telephones," *Journ. I.E.E.*, Vol. 62, p. 824, with bibliography.

As the number of slots per pole per phase is increased, the angular width of the belt of t conductors of one phase, when the mean pitch of the coils is not shortened, gradually approaches $2\pi/2N_{ph} = \pi/N_{ph}$ electrical radians, since in each pole-pitch there must be the sides of N_{ph} coils. This is the limiting figure in a single-layer winding, which corresponds to uniform distribution, *e.g.* in a 3-phase machine over $\frac{1}{3}$rd of the pole-pitch. Hence with increasing values of g' the reappearance of the same belt factor as for the fundamental is delayed to progressively higher harmonics, and for the lower harmonics the more nearly and the longer does the curve resemble that for *uniform* distribution over one-third of the pole-pitch.[1]

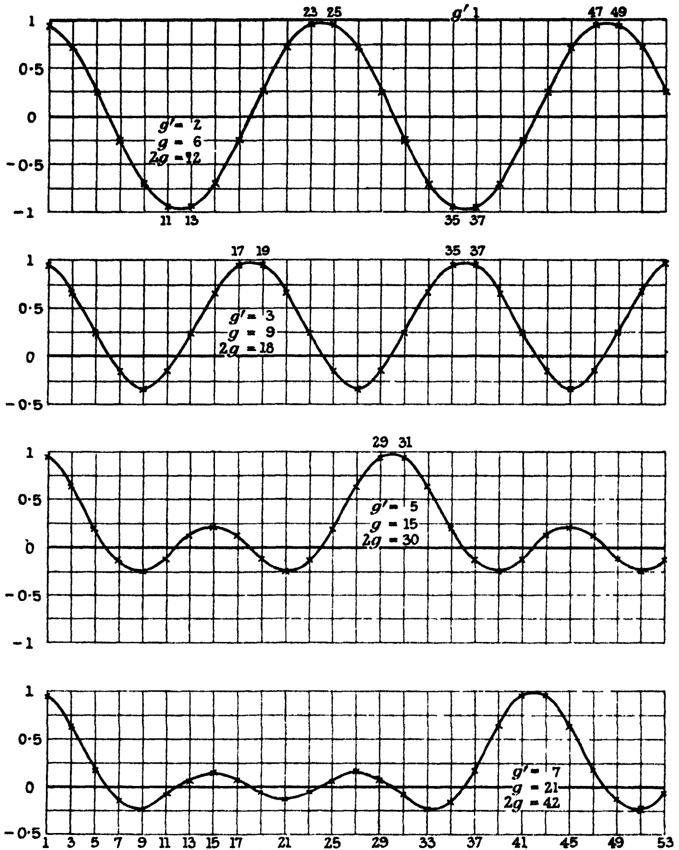

FIG. 473.—Belt differential factor for $g' = 1, 2, 3, 5, 7$.

Although it is customary to deduce and consider the differential factor which cannot exceed unity, yet since it has afterwards in use to be multiplied by $q_1 e$, it is even simpler to deal with the function $\sin q_1 n \frac{\gamma_e}{2} \Big/ \sin n \frac{\gamma_e}{2}$ and to multiply it by e, the E.M.F. of a pair of slots comprising a specimen coil. It is instructive to

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[1] *Cp.* the curves given by B. Hague and S. Neville, *Journ. I.E.E.*, Vol. 60, p. 861, and the explanation of the properties of the curves there set forth.

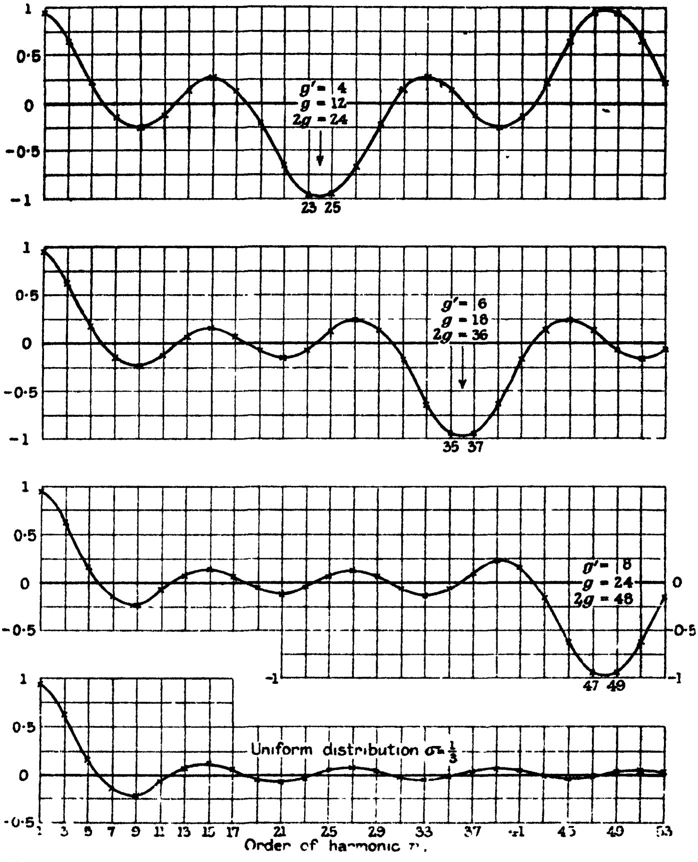

FIG. 474.—Belt differential factor for $g' = 4, 6, 8$, and for uniform distribution over one-third of pole-pitch.

plot this latter function in relation to g', the slots per pole per phase (Fig. 475). Were there no differential action, it would rise as an inclined straight line from 1 passing through 2, 3, 4 . . . . Actually for $n = 1$, it diverges gradually therefrom, and for higher harmonics it is seen to vary widely. The effect of $g' = 2$ in the cases of $n = 11$ or $n = 13$ will be evident; for $n = 17$ or 19, the curves would in the same way mount to the highest curve for $n = 1$ above $g' = 3$ on the abscissa axis, and again for $n = 23$ or 25 above $g' = 2$.

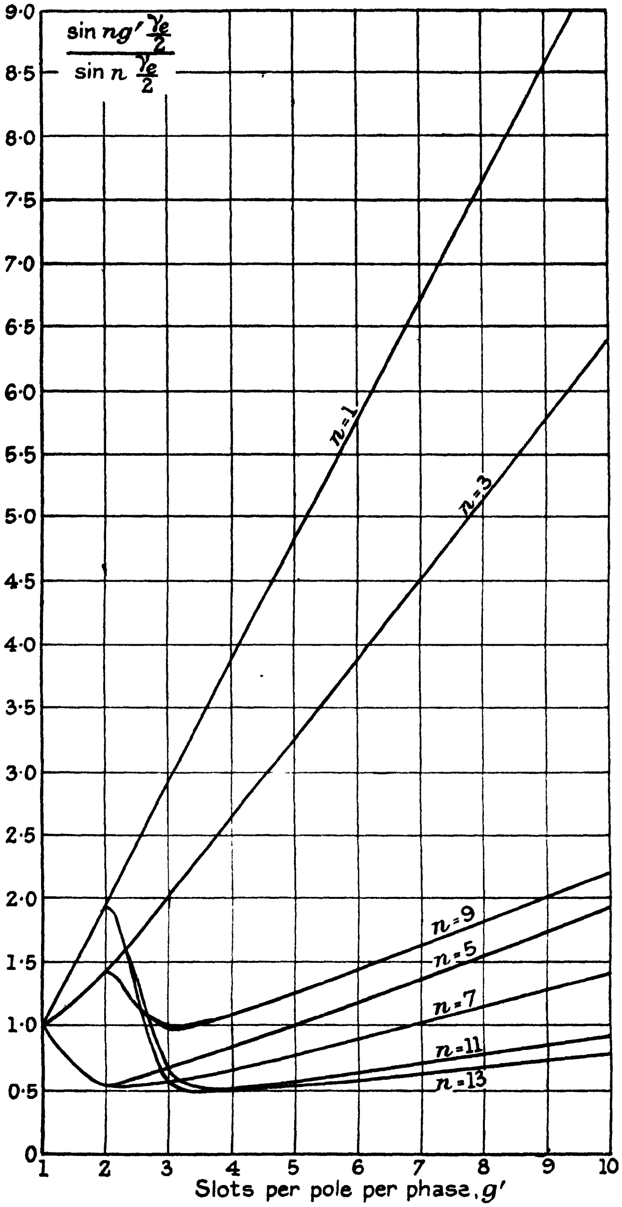

FIG. 475.—$g'k_s'$ plotted in relation to g' (without regard to sign) for a 3-phase alternator with a whole number of slots per pole.

The physical meaning of a negative distribution factor requires a word of explanation. Let the start of the positive half-wave of the fundamental at zero time correspond to a position of a single loop facing a N. pole, so that the change in time begins with a reduction in the number of lines from a N. pole and an increase in the number of lines from a S. pole, as linked with the loop. Let the fundamental E.M.F. wave of the loop or the fundamental field be subdivided into a number of half waves of harmonic E.M.F.'s or into harmonic fields, and first let these begin with a positive half-wave or N. pole, analogously to the fundamental. The order of the harmonic *n* being uneven, there will always be half-waves or harmonic poles of the same sign at each end of the fundamental pole-pitch, and we are at present merely assuming any harmonic to be at first positive. Now, whenever $n\gamma_e$ passes $360°/g'$, or a multiple of it which is other than 360° or a multiple of 360°,

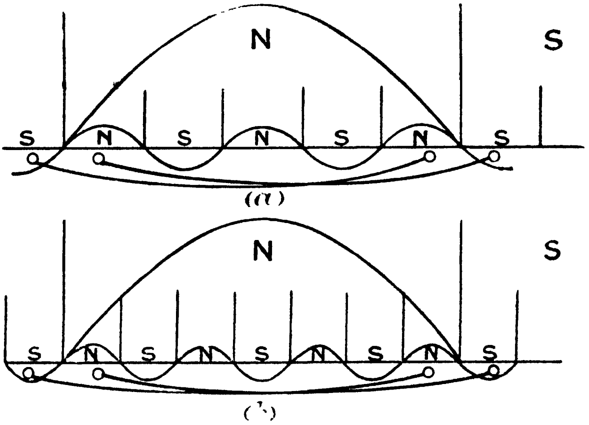

FIG. 476 —Two slots per pole per phase in relation to (*a*) 5th harmonics and (*b*) 7th harmonics.

the belt differential factor changes sign. As $n\gamma_e$ is increased, the harmonic vectors plotted for zero time first diverge and then draw together until they coincide or cross one another if $n\gamma_e$ be for the moment supposed to become 360° ; a further small divergence therefore yields the same result and the same sign as in similar positions before the vectors have crossed. *E.g.* let two slots per pole per phase be considered, for which $360°/g' = 180°$. For the 3rd and 5th harmonics, $n30°$ is less than 180° ; the two slots are then at the start similarly situated ± 45° or ± 75° (Fig 476*a*) ahead of or behind the interpolar line under a harmonic N and S. pole respectively. The fluxes of most of the harmonic poles cancel out, but the net result when the time-change begins is a reduction in the linked N flux and an increase in the linked S flux in the one full-pitch loop, and *vice versa* in the other loop ; of these, the former has the greater effect and yields the greater E.M F. since it occurs on rising flux-curves, while in the latter the flux-curves are falling towards zero. Consequently the resultant half-wave agrees in sign with the change in the fundamental linked flux in the first half-wave, and the belt differential factor is positive. When situated at ± 90°, *i.e.* with 180° phase-divergence of the two slots, the changes in the linked flux in the two loops would mutually cancel each other. But with the 7th and following harmonics (Fig. 476*b*), when $n\gamma_e$ exceeds 180°, the change in the second loop of reverse sign outweighs that in the first ; the resultant wave of harmonic E.M.F. begins by being negative, and the belt differential factor is negative. This continues until between the 17th and 19th harmonics, *i.e.* until the (even) harmonic divergence of the two slots is 540° or 270° on either side of the interpolar line of symmetry, and they are at the start in the middle respectively of poles of different signs from those first considered. Distribution factors of the same sign thus occur in groups for 3 uneven harmonics and larger groups for 6, when $n\gamma_e$ is 360° in the middle of larger group.

If the first amplitude of the harmonic is itself negative, a positive differential factor will imply a first half-wave of opposite sign to that of the fundamental. With uniform distribution over one-third of the pole-pitch, the sign changes every time $n60°$ passes 360°, 720° . . .

On the smooth-surface armature under the present heading, the pitch of a component loop with long coils, whether divided or undivided, is equal to a pole-pitch; therefore by eq. (33) the pitch differential factor k_{sn}'' is $\cos 0° = 1$ equally for the fundamental and for any uneven harmonic in the flux-density curve. Consequently it introduces no new element, and the total differential factor of the smooth armature reduces simply to the belt or distribution factor. The conductors on the two sides of a coil are situated similarly in relation to a N and a S pole so that they may be paired together, and there are virtually, even in the multipolar machine, only g_0' conductor-positions, all differential action as between the sides of a full-pitch coil or as between the resultant vectors from the two belts of conductors forming the coil-sides being absent.

With shortened coils in a single layer with a *mean* pitch $\lambda_c = 1/2$, the pitch differential factor for the fundamental or any harmonic is $\pm$ 0·707. When $g' = 2$, there is no further reduction in the uneven harmonics resulting from the flux curve as compared with the reduction in the fundamental With $g' = 4, 6, \ldots$ the fundamental is reduced even more than the harmonics.

If the spans of the component coils were specially selected, particular harmonics could be eliminated from the E M F. of the coil as a whole, but the unequal spacing of the slots which is thereby involved increases the difficulty of manufacture and the cost of the stampings; and, further, the position of the slots seldom lends itself conveniently to use in a quarter-phase or 3-phase machine, or conductors of different phases have to be placed in the same slots [1]

(2) *The number of wound slots per pole per phase g_0' is fractional,* but the number of slots per pole, g, remains whole either (a) owing to the winding of a single phase being repeated for another or other phases, *i.e.* when g_0' is multiplied by N_{ph}, or (b) owing to the presence of x empty slots per phase in each of the f sections, *i.e.* owing to the addition of $x/2p'$ empty slots per pole per phase to the number g_0' which are wound. The first case (a) is of some practical importance, and deserves to be considered.

In Chapter XXIV, § 2 (end), it was shown how in a machine with an even number of pole-pairs, *i.e.* with 4 or a multiple of 4 poles, 1½, 2½, 3½ . . . . slots per pole could be obtained for a single phase by the use, *e.g.*, of undivided coils having alternately 1 and 2, or 2 and 3, or 3 and 4 slots in each coil-side, or in general by pairs of belts alternately of 1 and 2, 2 and 3 . . . slots. In such a machine then the vector sum for a pair of undivided coils or 4 belts of conductors in a phase is the unit that is repeated; the H.C.F. between the slots and pole-pairs of the 4-pole machine is actually $f = 2$, but, the winding not being similar in each pole-pair, 2 pairs of poles are required to give a unit winding before repetition occurs. Thus the machine forms a special case in which a section must be interpreted to cover 4 poles.

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[1] See especially "E.M.F. Wave Forms," by E. A. Biedermann and J. B. Sparks, *Journ. I.E.E.*, Vol. 35, p. 493.

Now by the above arrangement, $g_o' = \frac{6}{4} = 1\frac{1}{2}$, or $\frac{10}{4} = 2\frac{1}{2}$ or $\frac{14}{4} = 3\frac{1}{2}$ and so on, but when the winding is duplicated for two phases, or in general when N_{ph} is even, $g_o' = g'$ for there are no empty slots, and $g'N_{ph} = g$ is a whole number in compliance with requirement (B).

As compared with a machine with the next lower whole number of slots per pole per phase, the arrangement benefits by the increase in the number of slot-positions per phase, so far at least as reduction of higher harmonics is concerned, but the fundamental factor is, of course, also reduced. Thus let $g' = g_1' + \frac{1}{2}$, where g_1' is the lower whole number; the coil with the larger number of slots $(g_1' + 1)$ per coil-side always contains slot-positions identical with all those of the coil with the smaller number of slots g_1', so that the latter coil does not itself add any new slot-positions. But to the g_1' slot-positions of the machine with g_1' slots is now added one new slot-position, reducing not only the higher harmonics but also the fundamental.

If e_l is the E.M.F.[1] on a toothed armature for any harmonic from one component *coil* out of a composite lap-wound coil having the large number of slots per coil-side and e_s is the corresponding E.M.F. for a component coil out of a coil with the smaller number of slots per coil-side, the E.M.F.'s of the two composite coils are respectively

$$e_l \frac{\sin\left\{(g' + \frac{1}{2})\, n \frac{\pi}{2g}\right\}}{\sin n \frac{\pi}{2g}} \quad \text{and} \quad e_s \frac{\sin\left\{(g' - \frac{1}{2})\, n \frac{\pi}{2g}\right\}}{\sin n \frac{\pi}{2g}}$$

and they are additive, since the pitch of the two groups is equal to a double pole-pitch.

On a smooth armature with introduction of the pitch differential factor, let e_n be the E.M.F. from one conductor-position for the nth harmonic; then the E.M.F.'s from a wide and a narrow belt respectively are the same as above with the substitution of e_n for e_l or e_s. The pitch factor is in each case unity, since the *mean* span of either coil is exactly a pole-pitch. The E.M.F. of the coil with the larger number of slots per coil-side is therefore

$$2e_n \frac{\sin\left\{(g' + \frac{1}{2})\, n \frac{\pi}{2N_{ph}g'}\right\}}{\sin n \frac{\pi}{2g}} = 2e_n \frac{\sin\left\{\frac{n\pi}{2N_{ph}} + \frac{n\pi}{4N_{ph}g'}\right\}}{\sin n \frac{\pi}{2N_{ph}g'}},$$

and of the coil with the smaller number of slots per coil-side

$$2e_n \frac{\sin\left\{(g' - \frac{1}{2})\, n \frac{\pi}{2N_{ph}g'}\right\}}{\sin n \frac{\pi}{2g}} = 2e_n \frac{\sin\left\{\frac{n\pi}{2N_{ph}} - \frac{n\pi}{4N_{ph}g'}\right\}}{\sin n \frac{\pi}{2N_{ph}g'}},$$

But the two are unequal, and the difference of phase between them corresponds to half a slot-pitch, *i.e.* $\frac{\pi}{2g}$.

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[1] Either virtual or maximum according to whether the resultant E.M.F. to be found is required to be the virtual or the maximum value.

§ 10. (C) Fractional number of slots per pole.[1]—(1) *The number of slots per pole-pair, $2g$, is an odd integer,* so that *the number of slots per pole, g = an integer + $\frac{1}{2}$.* This condition when the fractional part of g has the special value of $\frac{1}{2}$ may be reached in either of two ways, (*a*) and (*b*).

(*a*) *All slots wound,* and $g = N_{ph}g'$, where g'= a whole number + $\frac{1}{2}$ and N_{ph} is uneven. If the single-phase winding described in § 9 under B(2) is triplicated for a 3-phase machine, since g_0'= an integer + $\frac{1}{2}$, the number of slots per pole = $3g'$ is also an integer + $\frac{1}{2}$, or in general the required condition is fulfilled when the winding is multiplied N_{ph} times for a N-phase machine, *so long as N_{ph} is uneven.*

As regards higher harmonics, the arrangement benefits by the increase in the number of slot-positions per phase, as compared with a machine with the next lower whole number g_1' of slots per pole and phase. As in case B(2), the coil with the smaller number of slots g_1' does not itself add any new slot-positions. But to the g_1' slot-positions of the machine with g_1' slots are now added $1 + (g_1' + 1)$ new positions, making a total of $2\,(g_1' + 1)$ as against g_1', reducing not only the higher harmonics but also the fundamental. The E.M.F.'s on the toothed armature from coils with the larger and with the smaller number of slots per coil-side respectively have then the same expressions as under B(2).

To secure complete similarity of the three windings, the mean span of the coil with the larger number of slots per coil-side must be $g - \frac{1}{2}$, and of the coil with the smaller number must be $g + \frac{1}{2}$ Hence $\lambda_c = \frac{g + \frac{1}{2}}{g}$ and $1 - \lambda_c = \pm \frac{1}{2g}$. Proceeding therefore to the smooth armature case, the pitch factor is for either coil $k_{sn}'' = \cos\left(n\frac{\pi}{2}\cdot\frac{1}{2g}\right)$. Now $2g$, being odd, is a possible value of n. Inserting this value, $k_{sn}'' = \cos \pi/2 = 0$, and the same also holds for $n = 3(2g)$, $5(2g)$, etc. Consequently harmonics of the order $2g = S/p$ or of the orders $3(2g)$, $5(2g)$, etc., from the spacing of the conductors can never appear in the phase E.M.F. of the smooth armature under the present case: the curve of the total differential factor never rises again to the same value as for the fundamental, but is damped. The same conclusion must not, however, be pressed to hold true for the E M.F. of the loop in respect of tooth ripples due to the slots on the toothed armature.

Adding the pitch differential factor to the expressions of § 9 (B) 2 for the smooth armature, the E M.F. of the coil with the larger number of slots per coil-side is

$$2e_n \cos\left(n\frac{\pi}{2}\cdot\frac{1}{2N_{ph}g'}\right)\frac{\sin\left(\frac{n\pi}{2N_{ph}} + \frac{n\pi}{4N_{ph}g'}\right)}{\sin n\frac{\pi}{2N_{ph}g'}} = e_n\frac{\sin\left(\frac{n\pi}{2N_{ph}} + \frac{n\pi}{4N_{ph}g'}\right)}{\sin\left(n\frac{\pi}{2}\cdot\frac{1}{2N_{ph}g'}\right)}$$

and of the coil with the smaller number of slots per coil-side

$$2e_n \cos\left(n\frac{\pi}{2}\cdot\frac{1}{2N_{ph}g'}\right)\frac{\sin\left(\frac{n\pi}{2N_{ph}} - \frac{n\pi}{4N_{ph}g'}\right)}{\sin n\frac{\pi}{2N_{ph}g'}} = e_n\frac{\sin\left(\frac{n\pi}{2N_{ph}} - \frac{n\pi}{4N_{ph}g'}\right)}{\sin\left(n\frac{\pi}{2}\cdot\frac{1}{2N_{ph}g'}\right)}$$

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[1] A subject treated in detail by R. Richter, *Archiv. f. Elektrot,* Vol. 8, p. 214.

The E.M.F. of the two is again additive, since the pitch of the groups is equal to a double pole-pitch, and is, therefore,

$$2e_n \frac{\sin \dfrac{n\pi}{2N_{ph}}}{\tan\left(n\dfrac{\pi}{2}\cdot\dfrac{1}{2N_{ph}g'}\right)}$$

as compared with the arithmetical sum $4g'e_n$. Hence the total differential factor in the smooth armature case is

$$\frac{\sin \dfrac{n\pi}{2N_{ph}}}{2g'\tan\left(n\dfrac{\pi}{2}\cdot\dfrac{1}{2N_{ph}g'}\right)}$$

and with $N_{ph} = 3$

$$\frac{\sin n30^\circ}{2g'\tan\dfrac{n15^\circ}{g'}}$$

With $g' =$ only $1\frac{1}{2}$, this gives for the fundamental, 3rd and 5th harmonics values for the differential factor of 0·945, 0·577, 0·14; with $g' = 2\frac{1}{2}$, of 0·95, 0·616, 0·173, and as the number of slots is increased, the curve approaches still more nearly that for uniform distribution over $\frac{1}{3}$rd of the pole-pitch.

(*b*) *Per pole-pair* $2g_o$ *wound slots and* N_{ph} *empty slots,* so that $g = g_o + N_{ph}/2$ and N_{ph} is uneven.

Next let g_o be any whole number of slots per pole $= N_{ph}g_o'$, wound after any of the usual methods, and at equal angles within each double pole-pitch, let there be added N_{ph} extra slots where N_{ph} must be uneven, that is, in the general formulæ for the slots per pole-pair x is given the value p'. These additional slots are to be left entirely empty, and they are to be inserted between adjacent belts of different phases without interfering with the winding of the coils, save that their mean span in slot-pitches will be uniformly lengthened to $g_o + 1$ of the reduced slot-pitches (Fig. 477). There are thus $N_{ph}p'$ empty slots in each of the f sections and $N_{ph}p$ in all, so that the actual total number of slots is $2pg_o + N_{ph}p$. The number of slots per pole $g = g_o + N_{ph}/2$ is fractional (N_{ph} being uneven), but the number of slots per pole-pair $2g$ is an odd integer, so that the condition stated at the head of the section is fulfilled.

Evidently the present method for making g fractional cannot be applied to the quarter-phase machine or any case in which N_{ph} is even.

The real angle in radians between the slots in a belt is $\dfrac{2\pi}{2pg_o + N_{ph}p}$, and the electrical angle $\gamma_e = \dfrac{2\pi}{2g_o + N_{ph}}$. The belt differential factor is therefore

$$\frac{\sin\left(ng_o'\dfrac{\pi}{2g_o + N_{ph}}\right)}{g_o'\sin\left(n\dfrac{\pi}{2g_o + N_{ph}}\right)}$$

and any benefit from the reduction of harmonics can only be looked for in the loop E.M.F. by reason of the alteration of the span of the loop which is no longer full-pitch.

In the expression for the total number of added slots $= N_{ph}p$ there is no limitation in the value of p, so that it can be an even

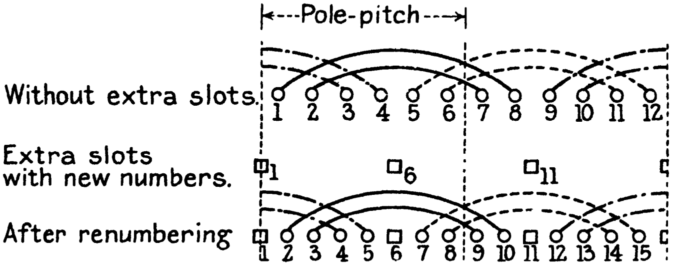

FIG. 477.—Additional empty slots, N_{ph} in number per pole-pair.

number, giving 6 extra slots in a 4-pole machine or a multiple of 6 in an 8-pole, 12-pole . . . machine. In these circumstances there is nothing to prevent the 3 pairs of slots in every set of 4 poles from being wound and one extra coil being added to each phase (Fig. 478 *cp*. Fig. 477). We thus return to the arrangement already

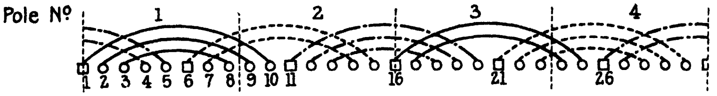

FIG. 478.—Additional slots wound

described under 1 (*a*) of this section, which may therefore be regarded as a machine with a whole number of slots per pole $N_{ph}g_1'$, and 6 extra slots which are wound, two being assigned to an additional coil in each phase in each double pole-pitch. Since $p = 2$ is the first case in which this arrangement is possible, the 4-pole machine is here the parent machine which is repeated so far as the winding which is to form a single layer is concerned.

The mean span of the coils being $g_0 + 1$ slot-pitches, and a pole-pitch containing $g_0 + \frac{N_{ph}}{2}$ slot-pitches, $1 - \lambda_c = 1 - \frac{g_0 + 1}{g_0 + N_{ph}/2} = \frac{N_{ph} - 2}{2g_0 + N_{ph}} = \frac{N_{ph} - 2}{2g}$ Proceeding to the smooth armature case, the pitch differential factor is thus

$$k_{sn}'' = \cos\left(n\frac{\pi}{2} \cdot \frac{N_{ph} - 2}{2g}\right)$$

As compared with the g' conductor-positions per phase without the extra slots, there are now $2g'$ conductor-positions per phase, the two sides of a loop not being similarly situated relatively to a N. and a S pole.

Since $2g$ is odd, it is a possible value of n. Inserting this value,

$$k_{sn}'' = \cos\frac{\pi}{2} = 0$$

since $N_{ph} - 2$ is uneven. Again therefore in the smooth armature, as in the previous case, harmonics of the order $2g = S/p$ or of the orders $3(2g)$, $5(2g)$, etc., cannot appear in the phase E.M.F.

The total differential factor for a pole-pair is for the smooth armature

$$k_{sn} = \cos\left(n\frac{\pi}{2}\cdot\frac{N_{ph}-2}{2g_o+N_{ph}}\right) \times \frac{\sin\left(ng_o'\frac{\pi}{2g_o+N_{ph}}\right)}{g_o'\sin\left(n\frac{\pi}{2g_o+N_{ph}}\right)}$$

which, when $N_{ph} = 3$, reduces to

$$k_{sn} = \frac{\sin\left(ng_o'\frac{\pi}{2g_o+3}\right)}{2g_o'\sin\left(n\frac{\pi}{2}\cdot\frac{1}{2g_o+3}\right)}$$

Each pole-pair repeats the effect of every other pole-pair,[1] and there is no further differential factor.

(2) *The number of slots per pole-pair or 2g is fractional.* As in (1), this condition may be reached in either of two ways. Either (*a*) with all slots wound $g = N_{ph}g'$ is a whole number + a fraction other than $\frac{1}{2}$, and therefore g' must be a whole number + a fraction other than $\frac{1}{2N_{ph}}$, or (*b*) g must be $= g_o + \left(\frac{xN_{ph}}{2p'}\text{ empty slots}\right)$, where $\frac{xN_{ph}}{2p'}$ is a fraction other than $\frac{1}{2}$. But in either case $2g$, $2g \pm 1$, $2g \pm 3$, $2g \pm 5$ . . . . are not whole numbers, and do not represent possible harmonics. The curves of the total differential factor are now damped, and with increasing values of g' or g_o' closely approach the curve for uniform distribution over one-third of the pole-pitch, due to the coil-sides of a phase occupying different positions in successive fields.

In the previous cases under (1), the number of slots per pole-pair having been an integer, the same slot-positions were repeated under every pole-pair, and so far as the slots were concerned the parent machine was one with $p' = 1$, although when the special winding of 1 (*a*) is taken into account, the parent machine in this case was one with $p' = 2$. But now with $2g$ fractional, a complete unit machine cannot be reached until p' is equal to the denominator of the fractional part of S/p in its lowest terms.

(*a*) *All slots wound* and $g = N_{ph}g'$, where $g' =$ a whole number + a fraction other than $1/(2N_{ph})$.

In a single-layer winding which is alone now under consideration, the required fractional value for g' may be obtained with divided coils, by making the slots in the belts belonging to a phase differ by one under certain poles from their value under the remaining poles in each of the f sections or units into which the whole machine may be divisible. Usually the slots in the belts of a phase under two pairs of poles, which are diametrically opposite to each other in the unit machine if closed upon itself, will exceed by one the slots of the same phase under the rest of the poles or will fall short of that number by one. Thus in a 3-phase machine with 78 slots and 5 pole-pairs, there is no common factor higher than 1 between the

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[1] So that the same result is also obtained from the formulæ given in the next section by making $p' = 1$ and $x = 1$.

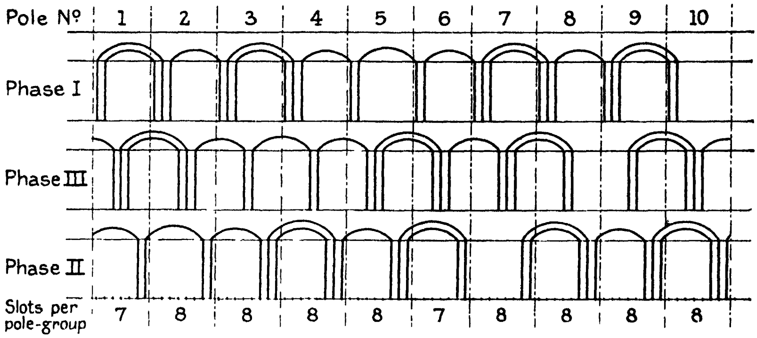

FIG. 479.—Slots per pole-pair fractional, and all wound ; 6 belts of 3 slots and 4 of 2 slots in a phase on a 10-pole machine.

slots and pole-pairs, so that $f = 1$, and we have a parent machine ; $78/5 = 15\frac{3}{5}$, and in each phase $2g' = 5\frac{1}{5}$, or $g' = 2\frac{3}{5}$. This number will be given by making 6 belts of 3 slots and 4 of 2 slots $\left(\frac{18+8}{10} = 2\frac{3}{5}\right)$, arranged thus (Fig. 479)—

No. of slots in belts.

Phase I . .	2	3	3	3	2	2	3	3	3	2	= 26
,, III . .	3	3	2	2	3	3	3	2	2	3	= 26
,, II . .	2	2	3	3	3	2	2	3	3	3	= 26
	7	8	8	8	8	7	8	8	8	8	= 78

As compared with a machine wound uniformly with the smaller number g_1' of slots per belt, giving g_1' slot-positions, there are now in each phase $p'g'$ slot-positions. But Fig. 479 might equally well represent half of a 20-pole machine with 156 slots, in which $f = 2$, and the same differential factor would hold for each of the two sections into which it is divisible. The value of the conception of the parent machine as given in Chapter XII is apparent in such a case.

The method has, however, some objections in practice, as also method C 1 (a) owing to the armature M.M.F. varying under different poles and also from difficulties in the construction unless the coils are hand-wound.

(*b*) *Per pole-pair* $2g = 2g_0$ *wound slots* + xN_{ph}/p' *empty slots,* where xN_{ph}/p' is fractional.

In place of adding N_{ph} extra empty slots to the number $2g_0$

within a double pole-pitch as in C.1(*b*), the fractional number of slots per pole-pair is more often obtained by adding N_{ph} empty slots to the number $2p'g_0$, *i.e.* at equal distances within *each* of the f sections of the stator. Higher multiples xN_{ph} are also permissible, so long as they secure the requisite fractional character for $2g$. Thus there must be at least 3 empty slots in a 3-phase machine as originally proposed by F. Punga,[1] and it must not be a 6-pole machine, while at least 4 empty slots must be added in a quarter-phase machine which must not be a 2-, 4-, or 8-pole machine. The phase-pitch in slots is

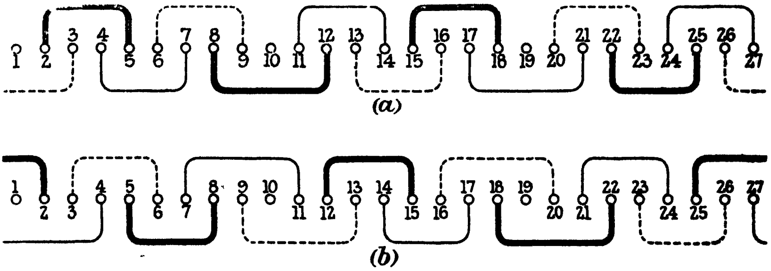

FIG. 480.—N_{ph} empty slots added to a parent machine.

$$y_{ph} = \frac{S}{fN_{ph}} = \frac{S'}{N_{ph}} = 2p'g_0' + xN_{ph}$$

As an illustrative example will be taken the case, cited by F. Punga, in which there is no common factor between the slots and pole-pairs higher than 1, viz. an 8-pole 3-phase alternator with a total of 27 slots, 1 slot per pole per phase and 3 empty slots. The winding table, which with 24 slots would be

Phase I	1	4	7	10	13	16	19	22
,, II	3	6	9	12	15	18	21	24
,, III	5	8	11	14	17	20	23	2

now becomes

Phase I	2	5	8	12	15	18	22	25
,, II	4	7	11	14	17	21	24	27
,, III	6	9	13	16	20	23	26	3

The 3 empty slots 1, 10, 19 are added at equal angles round the periphery (Fig. 480) and must occur between a pair of adjacent slots that belong respectively to phases I and II, II and III, and III and I. The difference between the angles of a slot-pitch electrically in the cases of 24 and of 27 slots is

$$\alpha = 2p\pi\left(\frac{1}{24} - \frac{1}{27}\right) = \frac{\pi}{27} = 6\tfrac{2}{3} \text{ degrees}$$

In the symmetrical case without empty slots, the pitch between any pair of adjacent coil-sides of one phase, being that of 3 slots, is uniformly 180° (electrical), and all the four coils occupy identical positions in a two-pole reference field. In the present case there is displacement, but it does not progressively increase, since the pitch between coil-sides of one phase, which is in the majority of cases 3 of the shorter slot-pitches, is lengthened at intervals to 4 of the shorter slot-pitches. If therefore one slot, say, No. 5 in Fig. 480*b*, be taken arbitrarily as a standard and be re-entitled No. 1, the relative angular displacements of the other slots of the same phase, reckoning on either side of this centre, are deduced as shown in Fig. 481 and in the table on opposite page, under the first columns headed **T** (trailing) and **L** (leading).

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[1] *E.T.Z.*, 1908, Vol. 29, p. 118. *Cp.* M. Seidner, *Elektrotechnik u. Maschinenbau*, Vol. 28, p. 785.

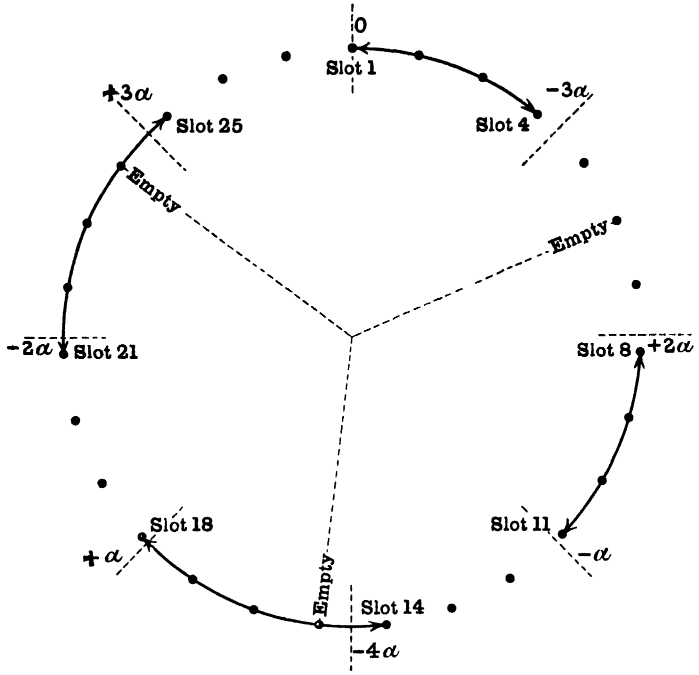

FIG. 481.—Displacements of coil-sides in Fig 480*b*.

The mean of the displacements of the two sides of a coil gives the displacement of its axis and, as first obtained, these may require to be adjusted by the addition of a fixed proportion of a to balance them against one another, and so to bring them into the closest alignment possible with the resultant vector of the whole (*cp* Fig. 482*b*). This done (by adding $+\frac{1}{2}a$ in our case), the final relative displacements of the sides of the 4 coils are—

$+\frac{1}{2}a$

$+3\frac{1}{2}a$ $-2\frac{1}{2}a$

$-1\frac{1}{2}a$ $+2\frac{1}{2}a$

$1\frac{1}{2}a$ $\frac{1}{2}a$

$-3\frac{1}{2}a$

There is thus, finally, only one new position added for the coil-axes, but there are two kinds of coils, making in all 4 pairs or 8 dissimilar conductor-positions for the coil-sides of one phase

Slot No.		Actual Angular Position in Electrical Degrees.		For Synchronism.		Relative Displacement of Coil-sides.					Mean for Coil-axis.	Axes and Coil-sides Adjusted.		
Fig. 480.	Fig. 481.								L	T			L	T
5	1	0°		0°		0°		=	0				$\frac{1}{2}a$	
											$-1\frac{1}{2}a$	$-a$		
8	4		160°		180°		$-20°$	=		$-3a$				$-2\frac{1}{2}a$
12	8	373·33°		360°		$+13·33°$		=	$+2a$				$2\frac{1}{2}a$	
											$+\frac{1}{2}a$	$+a$		
15	11		533·33°		540°		$-6·66°$	=		$-a$				$-\frac{1}{2}a$
18	14	693·33°		720°		$-26·66°$		=	$-4a$				$-3\frac{1}{2}a$	
											$-1\frac{1}{2}a$	$-a$		
22	18		906·66°		900°		$+6·66°$	=		$+a$				$1\frac{1}{2}a$
25	21	1066·66°		1080°		$-13·33°$		=	$-2a$				$-1\frac{1}{2}a$	
											$+\frac{1}{2}a$	$+a$		
2	25		1280°		1260°		$+20°$	=		$+3a$				$3\frac{1}{2}a$

The real angle in radians between the slots in a belt is $\frac{2\pi/f}{2p'g_o + xN_{ph}}$, and the electrical angle $\gamma_e = \frac{2\pi p'}{2p'g_o + xN_{ph}}$. The belt differential factor is then

$$k_{sn}' = \frac{\sin\left(ng_o' \frac{\pi}{2g_o + xN_{ph}/p'}\right)}{g_o' \sin\left(n \frac{\pi}{2g_o + xN_{ph}/p'}\right)}$$

applicable to each of the two different loop E.M.F.'s which form the basis of the E.M.F. of the toothed armature.

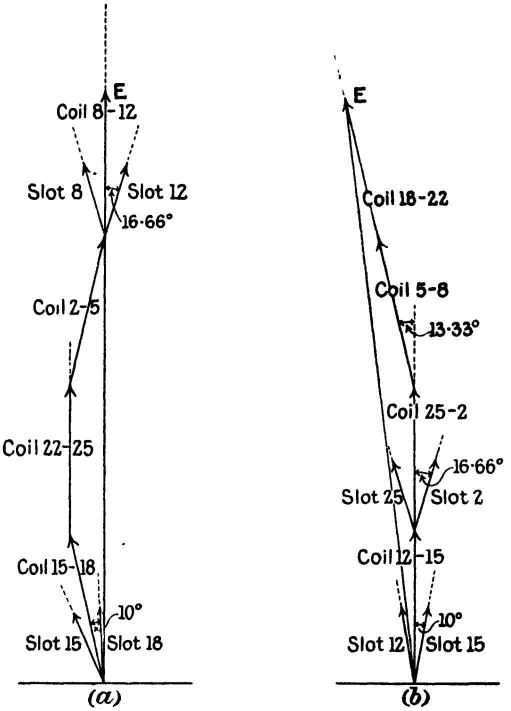

FIG. 482.—Addition of vectors for cases (*a*) and (*b*) of Fig. 480.

Only two formers are required for the two kinds of coils, but it is interesting to note that with the same slots occupied by each phase, its winding may be arranged either (*a*) to give 3 coils embracing 3 slot-pitches and one embracing 4 slot-pitches which gives least copper, or (*b*) 2 coils of each kind (Fig. 480).

It is instructive to compare the vector diagrams for the (*a*) and (*b*) arrangements and the calculation of the differential factor therefrom, as shown in Fig. 482, for the assumed case of a smooth armature. The following example will serve to show how the vectors of the several coils of a phase may generally be grouped to yield a simple mathematical expression without the need for a large-scale vector diagram from which the angles may be read off with accuracy.

One slot-pitch corresponds to $\frac{2\pi \times 4}{27} = 53{\cdot}33$ electrical degrees, and the pitch factor of a coil embracing 3 slot-pitches is cos $\left\{n\frac{\pi}{2}\left(1-\frac{24}{27}\right)\right\} = \cos n10^\circ$, while that of a coil embracing 4 slot-pitches is cos $n \left\{\frac{\pi}{2}\left(1-\frac{32}{27}\right)\right\} = \cos n16{\cdot}6^\circ$.

By the arrangement of Fig. 480 (*b*), coils 12–15 and 25–2 are separated by 13½ slot-pitches = 720 electrical degrees; and so also are coils 5–8 and 18–22. In each pair, therefore, the component vectors are in phase, and the sum for each pair is $2e_n(\cos n10^\circ + \cos n\,16{\cdot}6^\circ)$, where e_n is the E.M.F. of a coil-side. But the two pairs are at a distance from one another of 7 slot-pitches = 373·33°, or their relative displacement is 7 − 6¾ = one-quarter of a slot-pitch = 2α = 13·33° (electrical), and as they are equal, the resultant total vector will be displaced by half this amount = 6·66° (electrical) from the vector of either pair. Its value will therefore be

$$2e_n\ (\cos n\ 10^\circ + \cos n\ 16{\cdot}66^\circ) \times 2 \cos n\ 6{\cdot}66^\circ$$

Dividing by $8e_n$, the total differential factor is

$$\tfrac{1}{2} \cos n\ 6{\cdot}66^\circ(\cos n\ 10^\circ + \cos n\ 16{\cdot}66^\circ)$$

By arrangement (*a*), the distance between the centre of coil 22–25, and the adjacent 3-slot-pitch coils 15–18 and 2–5 in the same phase, one on either side, is in each case 7 slot-pitches = 373·33° (electrical), or their relative displacement is ± 13·33° (electrical), so that the resultant vector of the trio will correspond in position to that of the vector of the centre coil 22–25, as shown in Fig 482 (*a*), and its value is $2e_n\cos n10^\circ(1 + 2\cos n13{\cdot}33^\circ)$. It will next be seen that the centre of the remaining coil embracing 4 slot-pitches is separated from the centre of coil 22–25 by 13½ slot-pitches = 720° (electrical), so that its E.M.F. is simply additive to that of the trio, and its vector is in line with that of coil 22–25. The E M.F. of the phase is therefore

$$2e_n\cos n10^\circ(1 + 2\cos n13{\cdot}33^\circ) + 2e_n\cos n16{\cdot}66^\circ$$

and dividing by $8e_n$, the total differential factor is

$$\tfrac{1}{4}\left\{\cos n10^\circ(1 + 2\cos n13{\cdot}33^\circ) + \cos n16{\cdot}66^\circ\right\}$$

Although smaller than the previous value, the difference is practically inappreciable.

$$\text{Thence } k_{s1} = 0{\cdot}965 = 96{\cdot}5 \text{ per cent.}$$

$$k_{r3} = \frac{0{\cdot}7089}{0{\cdot}965} = 73{\cdot}4 \quad ,,$$

$$k_{r5} = \frac{0{\cdot}3169}{0{\cdot}965} = 32{\cdot}8 \quad ,,$$

$$k_{r7} = -\frac{0{\cdot}0366}{0{\cdot}965} = -\ 3{\cdot}79 \quad ,,$$

$$k_{r9} = -\frac{0{\cdot}2165}{0{\cdot}965} = -22{\cdot}4 \quad ,,$$

$$k_{r11} = -\frac{0{\cdot}192}{0{\cdot}965} = -19{\cdot}7 \quad ,,$$

which should be compared with the figures for uniform distribution

The real coil-span of the normal-pitch coils in radians is $\dfrac{\frac{2\pi}{f}g_o}{2p'g_o + xN_{ph}}$ and of the long-span coils is $\dfrac{\frac{2\pi}{f}(g_o + 1)}{2p'g_o + xN_{ph}}$ as compared with the pole-pitch $\dfrac{2\pi}{f\,.\,2p'}$, so that λ_c in the two cases respectively is

$$\frac{2p'g_o}{2p'g_o + xN_{ph}} \text{ and } \frac{2p'(g_o + 1)}{2p'g_o + xN_{ph}}$$

Hence

$$1 - \lambda_c = \frac{x}{2p'g' + x} \text{ or } \frac{x - 2p'/N_{ph}}{2p'g' + x}$$

and

$$k_{sn}'' = \cos\left(n\frac{\pi}{2} \cdot \frac{x}{2p'g' + x}\right) \text{ or } \cos\left(n\frac{\pi}{2} \cdot \frac{x - 2p'/N_{ph}}{2p'g' + x}\right)$$

The numbers of the two kinds of coils, their grouping, and their relative displacements which are required to be known in order to complete the differential factor, depend on the numbers of poles and of added slots, so that they are best found by setting out the slots occupied by one phase covering p' pole-pairs on the assumption that $g' = 1$ and that there are only $2p'N_{ph}$ slots; the additions are then inserted at their correct distances of $2p'/x$ slot-pitches relatively to the $2p'N_{ph}$ slots, beginning with an empty slot No. 1 ahead of the former slot 1 (arrangement *a*) or immediately after it, as No. 2 (arrangement *b*)

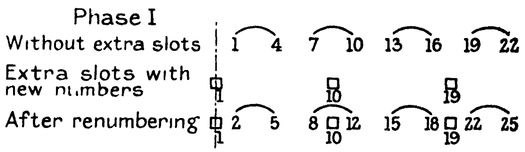

FIG. 483.—Method of setting out extra empty slots of Fig 480*a*.

Remembering that the added slots remain finally pitched at equal distances, it will be seen in which cases the coil-span in the reduced slot-pitches is unaltered and in which it is extended Thus Fig 483 shows arrangement (*a*) of Fig 480 for the 8-pole machine above considered.

Or again, let $p' = 5$, $N_{ph} = 3$, and $g_0' = 2$, with 3 empty slots; when the belt of the coil-side is reduced down to a single slot, the winding of a phase without the added slots would be as shown in the first line of Fig. 484, and the 3 empty slots might then occupy the relative positions shown in the second line, but would be Nos 1, 12, and 23. The third line then shows that there are 4 coils with a pitch of 3 slots and one with the longer pitch of 4 slots, and the same would also be found true of phases II and III

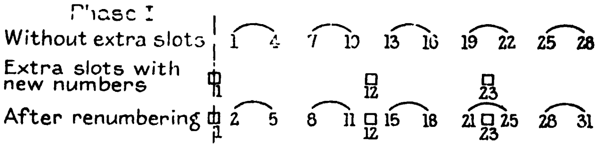

FIG. 484 —Method of setting out extra empty slots

The slot-pitches between the resultant vectors of the several coils are respectively 6, 7, 6½, 6½, and 7, reckoning from the centre of coil 2–5; and since 33/5 = 6·6 slot-pitches correspond to one pole-pair, the relative displacements of the 2nd, 3rd, 4th, and 5th coils in a standard reference field as compared with coil 2–5 are respectively − 0·6, − 0·2, − 0·3, and − 0·4 slot-pitches, or when balanced against one another by the addition of 0·3 of a slot-pitch are + 0·3, − 0·3, + 0·1, 0, and − 0·1. One slot-pitch in electrical degrees = 360°/6·6 = 54·5°, so that the displacements correspond to angles of 32·7°, 10·9°, 16·35°, and 21·8° from coil 2–5. The pitch-factor of a normal-pitch coil is in relation to the fundamental $\cos\left\{\frac{\pi}{2}\left(1 - \frac{3}{3 \cdot 3}\right)\right\} = \cos 8{\cdot}18°$ $= 0{\cdot}99$, and of the coil 21–25 with the longer span is $\cos\left\{\frac{\pi}{2}\left(1 - \frac{4}{3 \cdot 3}\right)\right\} =$ $\cos 19{\cdot}09° = 0{\cdot}945$. If therefore e is the E.M.F. of the belt of two slots forming a coil-side, taking into account the belt factor, the maximum E.M.F.

of a normal-pitch coil is $e_1 = 2e \times 0{\cdot}99$, and of the coil with longer span is $e_2 = 2e \times 0{\cdot}945$.

It will now be noticed from the displacements given above that the resultant vector of the 1st and 2nd coils must fall midway at $\frac{32{\cdot}7}{2} = 16{\cdot}35°$ from the vector of coil 2–5, and also the resultant of the 3rd and 5th coils at $\frac{10{\cdot}9 + 21{\cdot}8}{2} = 16{\cdot}35°$ from the same vector, and that both these angles coincide with the displacement of the 4th coil, so that the three resulting vectors are all in line lagging 16·35° behind that of the coil 2–5 which formed the starting-point, as shown in Fig. 485. That is, the 1st and 2nd, 3rd and 5th coils balance respectively on either side of coil 21–25. The fundamental of the E.M.F. of the phase is therefore

$$2e\,\{0{\cdot}99\,(2\cos 16{\cdot}35° + 2\cos 5{\cdot}45°) + 0{\cdot}945\}$$
$$= 2e \times 4{\cdot}816$$

Dividing by $10e$, the fundamental differential factor without taking into account the belt factor is 0·963.

Usually in the present method $x = 1$, since $N_{ph}f$ empty slots are sufficient to reach good reduction factors for harmonics above the third. But when $x = 2$, or a multiple of 2, the added slots can also be wound in pairs to form coils, and one or more pairs can be added to each phase. We thus return to case C. 2 (*a*), which will now be recognized as equivalent to a machine with a uniform number g_1' slots per pole per phase, and in addition to the total of these with $2N_{ph}$ extra slots which, instead of being left empty, are also wound. Unless the slots in C. 2 (*a*) exceed the number with a uniform g_1' by 6 or a multiple of 6 (*e.g.* $78 - 10 \times 3 \times 2 = 18$), the three vectors of the phase E.M.F.'s will not remain displaced by 120°.

It will also be recognized that the expressions of the present section contain implicitly the general case of (C) a fractional number of slots per pole which can now be formulated. The general formula for the slots per pole-pair is $2g = 2g_0 + \frac{xN_{ph}}{p'}$, but the $\frac{xN_{ph}}{p'}$ slots need not necessarily be empty and $\frac{xN_{ph}}{p'}$ need not necessarily be fractional. Returning backwards through more general to particular cases—

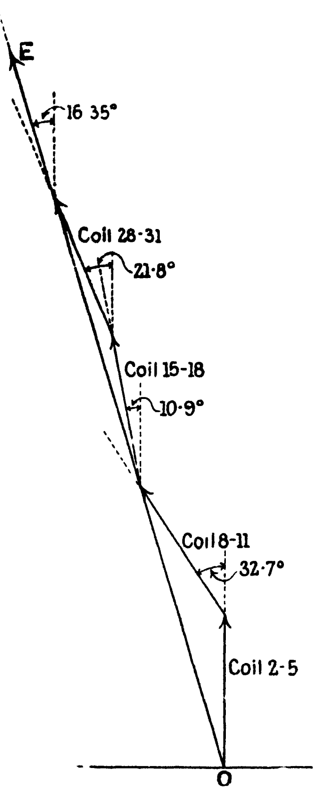

FIG. 485 —Addition of vectors of one phase of Fig. 484.

(*b*) *When they are empty*, $g_0 = N_{ph}g'_0$ and $2g = 2N_{ph}g'_0$ *wound* $+ \frac{xN_{ph}}{p'}$ *empty*. Then let xN_{ph}/p' be fractional and we have case C. 2 (*b*).

But when $p' = 1$, so that every pole-pair reproduces the effect of every other pole-pair, and $x = 1$, $g = N_{ph}g'_0 + N_{ph}/2$, and C. 1 (*b*) is reproduced, so long as N_{ph} is uneven. All the coils have then the long-span of the present section, and the second value of k_{sn}''.

(*a*) *When they are wound*, $g_0 = N_{ph}g_1'$ where g_1' is the lower value for the slots per pole per phase for the nearest symmetrical winding. Then $2g = (2N_{ph}g_1' + xN_{ph}/p')$, all wound.

Let $x = 2$, then $g' = g_1' + 1/p'$, and C. 2 (*a*) is reached, when p' is different from $2N_{ph}$. The total number of additional slots is $2p\frac{N_{ph}}{p'} = 2N_{ph}f$.

When $p' = 1$, let $x = 1$, then $g = N_{ph}g_1' + N_{ph}/2 = N_{ph}(g_1' + \frac{1}{2})$ and C. 1 (*a*) is reached so long as N_{ph} is uneven.

In either case of (*a*), if the differential factor for the g_1' portion of the winding be calculated by the formula of the last section, it must again be combined with that for the extra wound slots in proper proportion.

DOUBLE-LAYER WINDINGS

§ 11. (B′) Whole number of slots per pole, and two-layer winding.—Since the second sides of the coils in a double-layer winding are now no longer in the same layer with the first sides, the 2π electrical radians corresponding to one double pole-pitch need only be divided into N_{ph} parts, and the angular width of the belt of conductors of each phase with increasing numbers of slots per pole per phase approaches $2\pi/N_{ph}$, which is the limiting value for uniform distribution. Such a great width, however, introduces so much differential action within the coil-side, that, unless a commutator is present as in a rotary converter, it is, as already described in Chapter XXIV, § 9, usual to divide and re-group the coils so as again to reduce the angular width of belt in each coil-side to π/N_{ph}, and thus obtain the same E.M.F. as with a single-layer winding.

When the winding is in two layers, the coils are not necessarily full-pitch, but the span between the sides of a single component coil may be made any whole number of slots or of winding spaces on the smooth armature. The pitch differential factor then takes effect in the case of the smooth armature, and particular harmonics can with certainty be eliminated by proper choice of the pitch. But the same certainty cannot be stated in general of the toothed armatures, where, *e.g.* tooth ripples of the order $2g \pm 1$ may still persist for reasons to be explained in the next Chapter.

The leading case for the elimination of harmonics in the E.M.F. wave, in spite of their presence in the flux-density curve of the smooth armature or the smoothed-out curve of the toothed armature, is furnished by the chording of the winding to two-thirds of the pole-pitch, to eliminate the harmonic of triple frequency. The reduction factor due to the coil-pitch for the nth harmonic expressed as a percentage is

$$k_{rn}'' = 100 \times \frac{\cos\left\{n\frac{\pi}{2}(1-\lambda_c)\right\}}{\cos\left\{\frac{\pi}{2}(1-\lambda_c)\right\}}$$

Hence when $\lambda_c = \frac{2}{3}$, the numerator becomes $\cos n\frac{\pi}{6}$. Such a chording will therefore eliminate the 3rd, 9th, 15th, etc., harmonics from the phase E.M.F. and on this ground, in spite of its reduced fundamental E.M.F., it has been recommended for delta-connected armatures.

The action is easily followed by reference to Fig. 486, which shows a two-thirds pitch coil in relation to the fundamental and third harmonic of a flux curve. It will be seen that the span of the coil is exactly equal to a complete wave length of the third harmonic ; the two sides of the coil must then be cutting through fields of the same strength and sign (in other words, the coil can have no net flux linked with it) so far as the harmonic is concerned, and if triple-harmonic E.M.F.'s are credited to the coil-sides, they will neutralize one another. At the same time, the E.M.F.'s in the two sides due to the fundamental field are out of phase, and their total is appreciably reduced. Further, the fundamental and all harmonics, the frequencies of which are not multiples of 3, are reduced in the same proportion, viz., in the proportion of 0·866 to 1, so that the reduction factors of the harmonics which remain are unity.

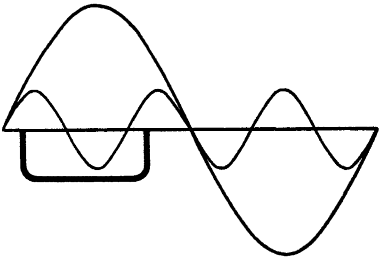

FIG. 486 —Pitch of 0·66 to eliminate harmonics which are multiples of 3.

The method of eliminating in the smooth armature case any particular harmonic from the E.M.F., by means of the pitch factor, may evidently be given a general expression ; it is only necessary that the mean pitch of the coil as a fraction of the pole-pitch or λ_c should be an even multiple of the reciprocal of the order of the harmonic. Thus if $\lambda_c = 2k/n$, where n is the order of the uneven harmonic and k is any whole number, the pitch factor becomes

$$\cos\left\{n\frac{\pi}{2}\left(1-\frac{2k}{n}\right)\right\} = \cos\left\{\frac{\pi}{2}(n-2k)\right\} = 0$$

since $n-2k$ must be odd.

On a smooth core, a fractional pitch[1] of about 0·833, as given, e g. by 5 out of 6 slots per pole, or 10 out of 12 slots per pole, would, by the combined action of the pitch and belt differential factors, reduce the 5th, 7th, 17th, and 19th harmonics to about one-quarter of their value in full-pitch coils ; if the multiples of 3 are cared for by star-connection, the 11th and 13th are left as the most prominent harmonics. A pitch of 7/9 greatly reduces the 5th harmonic and 8/9 the 7th harmonic, if these are especially to be feared from their amplitude in the flux-curve.

With high-speed alternators having few poles and coils of very wide span, a certain amount of chording has much in its favour.[2]

§ 12. (C′) Fractional number of slots per pole and two-layer winding.—Continuous-current wave-windings possess special importance, not only from their use as closed circuits on rotary converters, but also in general in alternators, as conducing to a smooth wave of E.M.F., owing to their fractional number of slots per pole. Thus in Fig. 465, when arranged in two layers, the slots would be $21/8 = 2\frac{5}{8}$ per pole and 7/8 per pole per phase ; in the example at the end of Chapter XXIV, § 10, $57/4 = 14\frac{1}{4}$ slots per pole and $4\frac{3}{4}$ per pole per phase. In fact g can be given a great variety of fractional values. But in all such cases of wave-windings, it is important to remember that the parent machine is one having $p' = p/f$ pole-pairs and $S' = S/f$ slots, and that all ripples present in the E.M.F. of one phase from the p' pole-pairs are present in the phase E.M.F. as a whole.

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[1] As recommended by Prof. Comfort A. Adams, *Trans. Amer. I.E.E.*, Vol. 28, p. 1077.

[2] *Cp.* J. Bache-Wiig, *Trans. Amer. I.E.E.*, Vol. 27, Part II, p. 1077.

When all the slots of a wave-wound armature are fully filled

$$\frac{cS}{p} = y_c \mp \frac{a}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

and y_c must be a whole number. After eliminating the case of $a = p$ when a lap winding is preferable, and the cases of series-parallel windings with $a/p > 1$ and equal to a whole number, the right-hand side of the above equation must be fractional. Since c is a whole number, it follows that S/p, the number of slots per pole-pair, is in all ordinary cases fractional, and is necessarily so when $a = 1$, as already mentioned in Chapter XI, § 22 (*a*).

If the wave-wound coils were spread out uniformly over the armature surface, the " creep " in sectors would be $\pm\, a/p$, and the displacement of successive coils referred to a two-pole field in electrical

$$\psi_e = \pm \frac{2\pi p}{cS} \times \frac{a}{p} = \pm\, 2\pi \frac{a}{cS} = \pm \frac{\pi}{cg} \times \frac{a}{p} \quad . \quad . \quad (232)$$

radians which also holds for the toothed armature with $c = 1$.

But with $c > 1$, there are two or more different front pitches in slots (*cp.* Chapter XXIV, § 9). The coils can then be divided into groups, the belt differential factor for each group calculated, and the belt differential factors for the two or more groups subsequently combined by means of the group differential factor. *E.g.* with $c = 2$, there are 2 groups of belts, the component coils of each belt being uniformly pitched at successive distances of ($2y_B{}^1 + y_F{}^1$ *long* $+\, y_F{}^1$ *short*) slot-pitches from each other, and the two groups being displaced ($y_B{}^1 + y_F{}^1$ *long* or *short*) slot-pitches. Or with $c = 3$ there are 3 groups, with component coils displaced successively ($3y_B{}^1 + 2y_F{}^1$ *short* $+\, y_F{}^1$ *long*) slot-pitches, and the 3 groups displaced ($y_B{}^1 + y_F{}^1$) slot-pitches from each other. But the effect of the fractional number of slots per pole-pair is usually so considerable that, even with no very large total number of slots, it becomes legitimate to consider the reduction factors as nearly the same as for a uniformly distributed winding.

§ 13. Differential and reduction factors for different numbers of slots in a 3-phase alternator.—It is evident that the fundamental and the various harmonics in the E.M.F. wave are affected in very different proportions by variations in the number of slots in the belt, *i.e.* by the belt or " distribution " factor. To illustrate this, the following Tables XXIII and XXIV are given for a 3-phase alternator; while universally true for harmonics in the *E.M.F. wave* of the component loop on a toothed armature, they are also incidentally true in relation to harmonics in the *flux-wave* of the smooth armature, when long coils are assumed virtually of full-pitch,

TABLE XXIII

Belt Differential Factors for Three-Phase Alternators

No. of Slots per pole per phase. g'.	Fund-amental	3rd harmonic	5th harmonic	7th harmonic	9th harmonic	11th harmonic	13th harmonic	
1	+ 1·00	+ 1·00	+ 1·00	+ 1·00	+ 1·00	+ 1·00	+ 1·00	
2	+ 0·966	+ 0·707	+ 0·259	− 0·259	− 0·707	− 0·966	− 0·966	
3	+ 0·960	+ 0·667	+ 0·218	− 0·177	− 0·333	− 0·177	+ 0·218	
4	+ 0·958	+ 0·654	+ 0·205	− 0·158	− 0·270	− 0·126	+ 0·126	
5	+ 0·957	+ 0·647	+ 0·200	− 0·149	− 0·248	− 0·110	+ 0·102	
6	+ 0·957	+ 0·644	+ 0·197	− 0·145	− 0·236	− 0·102	+ 0·092	
∞	+ 0·955	+ 0·636	+ 0·191	− 0·136	− 0·212	− 0·087	+ 0·073	Uniform distribution over $\frac{1}{3}$ pole-pitch
	+ 0·827	+ 0·000	− 0·165	+ 0·118	− 0·000	− 0·075	+ 0·064	,, $\frac{2}{3}$,, in 2 layers

TABLE XXIV

Reduction Factors corresponding to Table XXIII

2	—	+ 0·732	+ 0·268	− 0·268	− 0·732	− 1·00	− 1·00
3	—	+ 0·696	+ 0·227	− 0·184	− 0·347	− 0·184	+ 0·227
4	—	+ 0·682	+ 0·214	− 0·164	− 0·282	− 0·131	+ 0·131
5	—	+ 0·677	+ 0·209	− 0·156	− 0·259	− 0·115	+ 0·107
6	—	+ 0·674	+ 0·206	− 0·152	− 0·246	− 0·106	+ 0·096
∞	—	+ 0·666	+ 0·200	− 0·143	− 0·222	− 0·091	+ 0·077
	—	+ 0·000	− 0·199	+ 0·143	− 0·000	− 0·091	+ 0·077

whether wound over-lapping or concentrically, and whether in one or two layers.

It is, however, only misleading to add tabulated values for the *total* differential factor of two-layer chorded windings on a toothed armature, embodying and relying on the pitch factor of the loop—a factor truly applicable only in relation to the steady flux-wave of the smooth armature.

§ 14. The value of K in the $\dot{E}$.M.F. equation expressed in terms of Φ_a.—For purposes of design and with the adoption of equation (37) (Chap. IX, § 14) as the basis for the determination of the virtual E.M.F. of one phase, it must suffice to plot the smoothed-out curve of the flux-density as in Chaps. XVIII and XIX (*mutatis mutandis*) and thence to extract the fundamental B_{g1}. A first estimate can also be made therefrom of the earlier

TABLE XXV

Values of K

Uniformly distributed winding and long coils with allowance for fringe,
$l_g = \frac{1}{20}$*th pole-pitch.*

Ratio of width of belt of Coil-side to Pole-pitch.	Ratio of Pole-width to Pole-pitch.			
	0·5	0·6	0·7	0·8
0	1·18	1·125	1·08	1·05
0·1	1·16	1·112	1·06	1·035
0·2	1·135	1·09	1·05	1·008
0·3	1·11	1·075	1·025	0·983
0·4	1·083	1·05	1·00	0·955
0·5	1·046	1·015	0·967	0·92
0·6	1·00	0·963	0·92	0·87
0·7	0·944	0·91	0·87	0·82
0·8	0·88	0·843	0·803	0·764
0·9	0·812	0·78	0·745	0·707
1·0	0·745	0·716	0·685	0·654

Grouped winding and long coils with allowance for fringe.
$l_g = \frac{1}{20}$*th pole-pitch.*

One slot per pole per phase.	1·18	1·125	1·08	1·05
Two slots per pole per phase $\frac{1}{6}$th of pitch apart . . .	1·13	1·085	1·05	1·005
Two slots per pole per phase $\frac{1}{4}$th of pitch apart . . .	1·102	1·058	1·02	0·965

Grouped winding and long coils with allowance for fringe.
$l_g = \frac{1}{35}$*th pole-pitch.*

One slot per pole per phase.	1·235	1·162	1·105	1·058
Two slots $\frac{1}{6}$th of pitch apart	1·171	1·109	1·056	1·007
Three slots $\frac{1}{6}$th of pitch apart	1·158	1·096	1·0[illegible]	0·997

harmonics until we come to the pair on either side of $2g$. The process for every machine would be tedious, and under fixed conditions of similarity, normal values that may be expected for the percentage flux components, fundamental as well as harmonics, could be tabulated.

But an alternative expression for the virtual value of the phase E.M.F. has been given in equation 38*b* (Chap. IX, § 15) in terms of the total flux Φ_a of a pole-pitch and of a factor $K = k_f \, . \, k_d$. The values of this factor can only be tabulated upon the assumption of a steady smoothed-out flux-density curve, and this again only *upon the assumption of some particular air-gap* length as determining the distribution of the useful fringe of lines in the interpolar gap. The E.M.F.'s of component coils can then be added together graphically, and this has approximately been done in Table XXV opposite. It thus roughly serves the same purpose as would a summary of the values of $\left(\frac{1}{\sqrt{2}} k_{s1} \times B_{g1}\right.$ expressed as a percentage of $\left.B_{g\,max}\right)$, tabulated as suggested in the preceding paragraph, but it will theoretically include the small influence that harmonic E.M.F.'s have upon the virtual E.M.F.

The basis of Table XXV is the assumption that the length of the single

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TABLE XXVI

RATIO OF MAXIMUM TO VIRTUAL E.M.F.

Uniformly distributed winding and long coils with allowance for fringe.

Ratio of width of belt of Coil-side to Pole-pitch.	Ratio of Pole-width to Pole-pitch.			
	0·5	0·6	0·7	0·8
0	1·385	1·265	1·18	1·11
0·1	1·405	1·28	1·2	1·121
0·2	1·435	1·31	1·22	1·155
0·3	1·468	1·33	1·245	1·182
0·4	1·505	1·36	1·278	1·218
0·5	1·525	1·406	1·32	1·265
0·6	1·499	1·49	1·39	1·34
0·7	1·43	1·45	1·45	1·42
0·8	1·395	1·44	1·49	1·51
0·9	1·36	1·418	1·476	1·535
1·0	1·34	1·405	1·47	1·54

Grouped winding and long coils with allowance for fringe.
$l_g = \frac{1}{20}$ *pole-pitch.*

	0·5	0·6	0·7	0·8
One slot per pole per phase.	1·385	1·265	1·18	1·11
Two slots per pole per phase $\frac{1}{6}$th of pole-pitch apart .	1·442	1·315	1·22	1·155
Two slots per pole per phase $\frac{1}{4}$th of pole-pitch apart .	1·48	1·35	1·25	1·205

Grouped winding and long coils with allowance for fringe.
$l_g = \frac{1}{35}$*th pole-pitch.*

	0·5	0·6	0·7	0·8
One slot per pole per phase.	1·375	1·258	1·174	1·106
Two slots $\frac{1}{6}$th of pitch apart.	1·451	1·317	1·228	1·164
Three slots $\frac{1}{9}$th of pitch apart	1·467	1·333	1·24	1·176

air-gap is $\frac{1}{20}$th or alternatively is $\frac{1}{35}$th of the pole-pitch,[1] and that the smoothed-out curve of flux-density in the air-gap shades off from its full value under the pole to zero on the interpolar line of symmetry, as in Fig. 252. As the ratio of the pole-width to the pitch is reduced, the effect of the fringe becomes more marked, since it forms a greater percentage of the total number of useful lines entering the armature, but for ordinary values of the pole-ratio the results may be taken without much error even if the exact distribution of the fringe is in practice somewhat different.

The curve of the form-factor k_f in relation to the ratio of pole-arc to pole-pitch is throughout convex to the horizontal axis, while the curve of the differential coefficient k_d, which is straight for a single slot, is throughout concave for two or more slots per pole per phase. As soon as the distance between the outer slots of a coil-side approaches the width of the interpolar gap, the curve of k_d falls off rapidly; hence, in the curve of K as the product of $k_d \cdot k_f$, for two or more slots there is a point of inversion where it changes from convex to concave. This point lies farther up the curve the greater the distance between the outer slots, until for distributed winding covering the whole pole-pitch the curve of K becomes throughout concave to the horizontal axis.

Any influence which causes a concentration of the flux increases K; the smaller value of the air-gap in relation to the pole-pitch, therefore, increases K, since the flux is then not so widely spread out.

In ordinary cases the value of K lies within the limits of the value for a sinusoidal E.M.F., in a single bar or concentrated coil-side, namely, 1·11, and 1·25.

§ 15. The ratio of maximum to virtual E.M.F.

—In all cases as the winding becomes more distributed the value of k_{s1} or K decreases; but the reduction of the E.M.F. due thereto has its full force only on open circuit or no-load. As will be more apparent later, the reaction on the field when the full current is taken out of the armature is less in the distributed than in the concentrated winding, while incidentally a better exposure of the coils to the cooling action of the air is obtained. Hence it is advantageous to employ a distributed winding even at the sacrifice of some of the possible E.M.F. at no-load; at least two or three slots per pole per phase are the rule, and as many as twelve are met with in turbo-alternators. The shape of the curve of instantaneous E.M.F. also then more nearly resembles a sine curve, and this possesses considerable importance from its bearing upon the ratio of the maximum to the virtual E.M.F. The insulation of the conductors must be capable of withstanding the strain of the maximum E.M.F., but while with a concentrated winding the ratio of the maximum to the virtual E.M.F. of the phase increases rapidly from unity as the pole-ratio is decreased, with a uniformly distributed winding or with a large number of slots the ratio decreases from a maximum of 1·73 corresponding to a triangular wave of E.M.F. and a pole-width ratio of unity. The two curves for concentrated and distributed winding cross at a value of about 1·33 for a pole-ratio in the neighbourhood of 0·5. Any number of intermediate cases are therefore possible, but in general it may be said that the maximum strain that the insulation of the coil has to withstand from the action of its own phase E.M.F. is about 1½ times its virtual E.M.F. In slow-speed machines of high virtual E.M.F. the coil must needs be concentrated within one or a few slots per pole in order not to lose too much space in the insulation of the slots ; these must further be large, so that there is greater likelihood of higher harmonics in the E.M.F. curve.

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[1] The values for other proportions of l_g to pitch may be interpolated between the maxima figures given on page 80 and those which are tabulated in a paper by the writer (*Electr. Review*, Vol. 47, p. 655 ff.) for the hypothetical case of no fringe.

§ 16. Armature paths in parallel.—In many cases when the total current of a phase is large, it is necessary to divide it between two or more branches in parallel. For this to be successful it is necessary that the E.M.F.'s of each branch should be identical in magnitude and phase, and this necessitates that the component loops of each branch must be situated relatively to the fields identically with those of the other branches in parallel. It is not merely sufficient that they should occupy corresponding slots ; even their positions within those slots should be the same. Complete symmetry is therefore essential.[1]

In single-layer windings with g a whole number, the number of branches may be equal to any whole number which is divisible into $2p$ without remainder, and therefore can amount as a maximum[2] to $2p$. With a closed two-layer winding the number of branches for parallel connexion depends on the H.C.F. between the slots and pole-pairs, and may be $q = 2a$ when both the total number of slots and the number of pole-pairs are divisible by a without remainder, *i.e.* when S/a and also p/a are whole numbers, these being also the conditions for perfectly symmetrical equipotential connexions (Chapter XII, § 18).

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[1] For a case illustrating the effect of a want of magnetic symmetry, see J. Dixon, *El. World*, Vol 76, p. 1024

[2] But for more than $2p$ parallel branches per phase in turbo-alternators, with a slight phase difference between the branches, see E. Wilczek, *Engineering*, Vol. 118, p 206.

CHAPTER XXVI

THE LOOP E.M.F. OF THE TOOTHED ARMATURE

§ 1. The general case of a varying flux-density curve.—The flux-density curve in the air-gap as cut by the conductors on the surface of a smooth armature is stationary with respect to the poles and of constant shape, so that it may be expressed as a simple Fourier series, $B_x = \Sigma B_n \sin (na + \theta_n)$, where n is the order of the harmonic. Or if B_n is again split up into $A_n = B_n \cos \theta_n$ and $C_n = B_n \sin \theta_n$, it becomes

$$B_x = \Sigma A_n \sin na + \Sigma C_n \cos na$$

Bu actually with a toothed armature there is no one such fixed wave-shape of flux. It passes through a complete cycle of shapes while the poles move through a tooth-pitch relatively to a stationary armature or a rotating armature moves through a tooth-pitch relatively to stationary poles Fig. 487 illustrates *e.g.* the different shapes at each quarter of a tooth-pitch cycle.

In the general case of a flux wave-shape varying cyclically in time, let a number of instants of time within the cycle be taken, and let each of the instantaneous wave shapes be separately analysed into simple Fourier series. Taking all the values of A_1 so obtained, let them be plotted on a time basis of $\omega_e t$; similarly, let the values of $A_3, A_5 \ldots$ and $C_1, C_3, C_5 \ldots$ be plotted separately against time. In each case if a sufficient number of instants have been taken, a continuous periodic curve can be drawn which represents the instantaneous value of the coefficient in question. Each of these can in turn be resolved into a Fourier series. If the curve is not divided by the zero line into equal positive and negative areas, it is replaceable by a constant term and an alternating cycle superposed on it. In place, therefore, of a constant A_n or C_n, we have other series for these, and the actual flux-density becomes a series of products of sine and cosine terms. Lastly, each of these can be resolved into a sum or difference of sine or cosine terms, of constant amplitude representing a wave of fixed length, either travelling at a constant angular velocity with or against the movement of the rotating member, or stationary.

Any one of these component waves may be expressed[1] as

$$B_n \sin(na \pm k\omega_e t + \theta_{nk})$$

Here B_n is the amplitude of a sinusoidal wave of length $2\pi/n$, *i.e.* of space-order n, as if produced by pn pole-pairs (where p is the actual number). The angle a is measured from the interpolar line of symmetry and defines a certain point in the air-gap ahead of or behind this line. If k is zero, the sinusoidal wave of amplitude B_n is stationary with respect to the poles (assumed moving); if the wave moves in time relatively to the poles, and the sign before $k\omega_e t$ is positive, the direction of its movement is opposite to that of the actual poles, but if the sign before $k\omega_e t$ is negative, it is moving in the same direction as the poles; its electrical angular velocity relatively to the poles in the two cases is $\mp k\omega_e/n$, and the value of k is usually 0, 2, 4, 6, etc.

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[1] The writer in this and the following section has followed in principle the treatment in "Application of Harmonic Analysis to the Theory of Synchronous Machines," by Prof. Waldo V. Lyon (*Trans. Amer. I.E.E.*, Vol. 37, Part II, p. 1477), but with altered symbols to suit the usage of the present book.

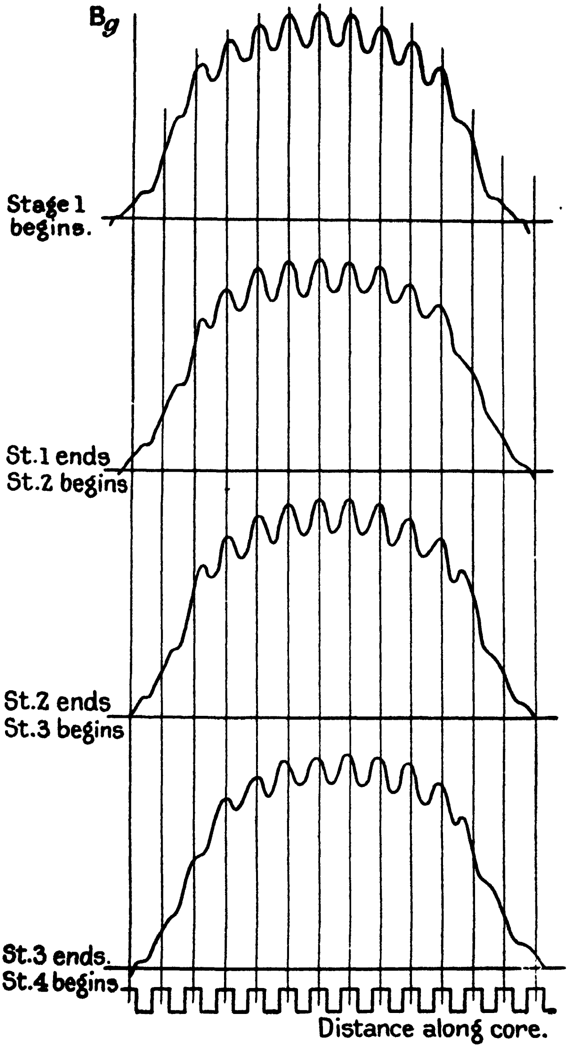

FIG. 487.—Flux waves for 4 moments in a tooth-cycle.

The angle θ_{nk} fixes the phase of the component flux-density whether this is stationary or moving relatively to the poles, *i.e.* when $\omega_e t$ is zero or any multiple of π, the value of the component flux-density on the interpolar line of symmetry is $\pm B_n \sin \theta_{nk}$.

So far as a component flux-wave travelling with respect to the poles penetrates within the closed field-circuit, it generates an alternating voltage and alternating current therein, the frequency of which is $\frac{k\omega_e}{n} \div \frac{2\pi}{n} = \frac{k\omega_e}{2\pi} = kf$, where f is the fundamental frequency. The same is also true within the solid iron of pole-pieces, and the alternating effect would be held in check by a low-resistance amortisseur winding embedded in the poles The angular velocity of the flux-wave relatively to the stationary armature is $\omega_e \mp k\omega_e/n$, and the frequency $(n \mp k)f$. This shows that in the case of a component harmonic flux-wave of space-order n travelling with respect to the poles, the frequency is not simply nf. By analysis on these lines, the two sides of a loop could be treated separately as E M F.-giving units, and their results added. It also illustrates why the pitch factor cannot in all cases be relied on to eliminate harmonics in the voltage of the loop; *e g* a steady 5th space-harmonic in the flux-curve moving with the poles, *i e* at the fundamental speed, or a 3rd space-harmonic moving at five-thirds of the fundamental speed, will equally produce a 5th harmonic in a conductor, but a loop designed to eliminate the former will not eliminate the latter

§ 2. The general case of the toothed armature.—Returning to the toothed armature, since the instantaneous flux-wave varies cyclically during the time of a tooth-cycle according to the position of the rotor poles relatively to the stator slots and teeth, the A coefficients must vary. Further, if the neutral line marking the change of direction of flux into and out of the stator teeth does not strictly follow the interpolar line of symmetry, but oscillates on either side of it, C coefficients must arise even at no load and will also vary cyclically with time.

In place, therefore, of simple values for A_n and C_n, we have in general

$$A_n = \Sigma_m a_n \cos 2gm\omega_e t + \Sigma_m c_n \sin 2gm\omega_e t$$

$$C_n = \Sigma_m c'_n \sin 2gm\omega_e t + \Sigma_m a'_n \cos 2gm\omega_e t$$

where g is the number of slots per pole, ${}_m a_n$ and ${}_m c_n$ are the coefficients of the mth harmonic in the Fourier series representing the varying amplitude of the nth harmonic sine term A_n, and ${}_m c'_n$ and ${}_m a'_n$ are the coefficients of the mth harmonic in the Fourier series representing the varying amplitude of the nth harmonic cosine term C_n.

The periodic time of a tooth-cycle is $1/2g$ of the time of a fundamental cycle, and its frequency is $2gf = 2g\omega_e/2\pi$. By giving m the values 0, 1, 2, 3 . . . the flux-distribution is resolved into waves stationary with respect to the poles ($m = 0$) and waves moving relatively to the poles ($m = 1, 2, 3 \ldots$). The stationary waves may vary in amplitude, and when $m = 0$, the value ${}_0 a_n$ is the average amplitude of the nth harmonic in the stationary wave. The fundamental slot harmonics are given when $m = 1$ and are of the orders $2g \pm 1$.

When the distribution of each field about the centre line of its pole is symmetrical, as on no load with a whole number of slots per pole at the beginning of stage 1 or 3 (Fig. 487), *i.e* for 0° and 180°, A_n has no sine terms, and on account of the symmetry C_n has no cosine terms. At the moments of transition from stage 1 to stage 2 and from stage 3 to stage 4, *i.e.* for 90° and 270°, C_n again has no cosine terms; further at these moments, the distribution of one field from a N. pole exactly repeats the distribution of the next field from a S pole, but in the reverse order, and in order that this may be the case there cannot be any sine terms in A_n But at all other times owing to the magnetic circuits not being symmetrical about the centres of the poles, each series must contain both sine and cosine terms, although the cosine terms are predominant in A_n and the sine terms in C_n.

With a fractional number of slots per pole, and the fraction other than $\frac{1}{2}$, one field at least must always be unsymmetrical about its centre line, and except at moments both are unsymmetrical.

§ 3. Procedure by average E.M.F.'s in quarter tooth-cycles.—The process of analysis is, however, too lengthy to be practicable, and a simpler investigation will suffice to indicate at least qualitatively the nature of the mechanism which causes tooth ripples to appear in the E.M.F. wave. Apart from the very small influence of hysteresis arising when one member rotates, and of eddy-currents in the teeth under rapidly changing flux, a static examination of the flux-distribution over a pole-pitch for different relative positions of teeth and poles brings to light all that, in the writer's opinion, is required to explain the phenomena.

Taking a single tooth-pitch from slot-centre to slot-centre, *i.e.* with a tooth at its centre as a unit, let the total flux entering it in some particular position relatively to the pole be calculated, and let the process be repeated for a sufficient number of positions, especially in the interpolar gap, to enable a continuous curve to be drawn, showing the flux of a tooth-pitch in any position (as the lower dotted curve of Fig. 491). Let the time of the passage of a fixed point on the pole past a tooth-pitch, *i.e.* a tooth-cycle, be divided into the four equal stages shown in Fig. 488, beginning with the moment when *IS*, the interpolar line of symmetry, coincides with the centre line of a slot. By use of the above curve the fluxes entering each of the tooth-pitches can be added up to give the total flux of a pole-pitch for the beginning of each of the 4 stages, after which they again repeat. Usually its amount will vary, unless the total permeance happens to remain constant.

Taking for simplicity a single loop whose sides are supposed to be situated on the centre-lines at the bottom of two slots, and a whole number of tooth-pitches in a pole-pitch, the flux of the tooth-pitches enclosed within it, *i.e.* its linked flux, can be calculated. Beginning with the moment of time when the fixed axis of the stationary loop is at 90 electrical degrees from the interpolar line of symmetry and its embraced flux is at its maximum (Fig. 488), let the linked flux be written down for the beginning of each of the four stages or quarter-cycles, and starting always from the leading edge of the moving field. When the fourth stage is passed, the next or fifth, so far as the linked flux is concerned, is given by deducting from the linked flux of the beginning of stage 1 *twice* the flux of the first tooth-pitch; for whatever the amount by which the positive flux is reduced at the one side of the loop, the same amount of negative flux is thrust into the loop at the other side. Similarly at the beginning of stages 9, 13, 17 . . . . twice the fluxes of the 2nd, 3rd, 4th . . . . tooth-pitches of stage 1 must be progressively deducted. The same process, but using twice the fluxes of the 1st, 2nd, . . . tooth-pitches of stage 3 will yield the linked flux at the beginning of stages 7, 11, 15 . . . ; for in both these cases, with a slot in the one case and a tooth in the other on the interpolar line of symmetry, the permeance and flux on the one side will be repeated on the other side of that line.

At the beginning of stages 2 and 4 the permeance in each half of the pole-pitch is not symmetrical, and the fluxes are not symmetrical. The total flux of a pole-pitch is given arithmetically by the addition of the fluxes for half the tooth-pitches on each side of the neutral line *NL*, where the flux changes direction, and its value is the same for both stage 2 and stage 4. As previously, for the linked flux at the beginning of stages 2, 6, 10, . . . twice the fluxes for the tooth-pitches of stage 2 must progressively be deducted,

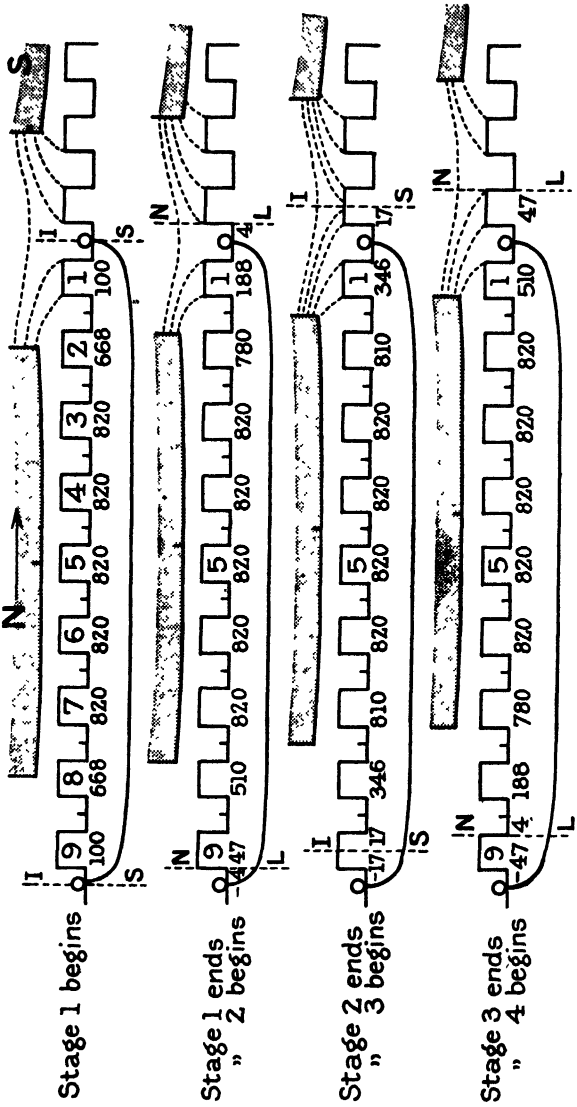

FIG. 488.—The four quarter stages of a tooth-cycle.

the reckoning having been begun at the leading edge of the field, and at the beginning of stages 4, 8, 12, . . . twice the fluxes for the tooth-pitches of stage 4 must progressively be deducted; the amount of negative flux thrust in on the trailing side is in each case equal to the amount of positive flux withdrawn at the leading edge. The whole process is illustrated in Tables XXVII and XXIX for the case of 3 slots per pole per phase or 9 tooth-pitches per pole in a 3-phase machine.

A further table can then be drawn up, showing the linked flux at the beginning of each stage, and the differences when the linked flux at any stage is deducted from that immediately preceding it (Table XXVIII or XXX). To these differences the average E.M.F. during the passage through each stage is proportional, since the time taken to pass each quarter of a tooth-pitch is the same. Plotting the differences or average E.M.F.'s as small rectangles on a time base (Figs. 489, 493, 495), a curve drawn through them to yield over each stage the same average E.M.F.'s will show the instantaneous E.M.F. induced in the loop at any moment of time.

§ 4. Elementary case with very small interpolar gap.—In order to understand clearly the physical origin of tooth-ripples, it is worth while to spend a little time in considering in detail a hypothetical case, leading up ultimately to a more practical case.

Taking the simple case when the pole-pitch contains an exact whole number (say, 9) of tooth-pitches, and a tooth-pitch electrically = 180/9 = 20°, let the flux-density under the pole with a smooth armature be supposed for further simplicity to be uniform, so that its curve is there quite flat and of value $B_{g\,max} = 10{,}000$. Then when set out as in Table XXVII, the fluxes of the tooth-pitches or portions of tooth-pitches into which the smooth surface can in imagination be divided, when summed up for a pole-pitch, must at the beginning of each of the 4 stages and at any time during them yield the same total flux. When the pole moves, whatever takes place at one side of the loop due to removal of positive flux out of the loop is exactly repeated at the other side by the insertion of negative flux, and the unlinking or linking of flux proceeds continuously after a law determined by the shape of the constant flux-density curve.

But let the surface be slotted with open slots 0·6 in. wide out of a tooth-pitch of 1·4 ins., and let the air-gap under the pole be uniformly ¼ in. The contraction coefficient is then

$$C = \frac{w_{t1} + w_s(1 - K_s)}{w_{t1} + w_s} = \frac{0{\cdot}8 + 0{\cdot}6 \times 0{\cdot}68}{1{\cdot}4} = 0{\cdot}863$$

the value of K_s for the ratio 0·6/0·25 = 2·4 being 0·32. Let the axial length of the air-gap after allowance for the air-vents in the surface of the core be 0·95 of its over-all length. Then the average density over a tooth-pitch under the pole will be 0·863 × 10,000 × 0·95 = 8200, and if the length of the core is made for simplicity 11·08 ins., so that the gross area of a tooth-pitch is 1·4 × 11·08 × 6·45 = 100 sq. cm., the total flux of a tooth-pitch under the pole is k = 820 kilolines. Over a quarter of a tooth-pitch on either side of a tooth-centre the density will be closely 9600, and over a quarter tooth-pitch containing a half slot,[1] also under the pole, will be 6800, these figures giving for a whole tooth-pitch $(9600 + 6800) \times 50$ sq. cm. $= 820$ kilolines, as above.

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[1] It must be noted that 6800 is not the average density *over a slot,* since the quarter tooth-pitch exceeds in width half the slot. It is the density averaged over the slot and a small piece of the adjacent tooth on either side.

Now let the poles be imagined to approach one another so closely that the dying away of the flux-density from its normal value down to zero on *IS* or *NL* is confined to a quarter tooth-pitch. The flux-density curve of the smooth core thus descending steeply between, say, 3° and 0° (Fig. 489), let the average density in the toothed case when half a tooth occupies the quarter tooth-pitch next to *IS* or *NL* be 6300, and when half a slot occupies the same tooth-pitch be 3500. We then have—

For the whole tooth-pitch at either end of the pole-pitch at the beginning of stage 1 when *IS* bisects a slot (*cp.* Fig. 488),

$$d = (6800 + 9600 + 9600 + 3500) \times 25 = 737{\cdot}5 \text{ kilolines.}$$

For the three-quarters tooth-pitch at the right-hand edge of the N pole-pitch at the beginning of an earlier stage 4, or at the left-hand edge at the beginning of stage 2, when in either case a tooth is next to *NL* and its centre-line is 5° therefrom,

$$c = (6800 + 9600 + 6300) \times 25 \qquad = 567{\cdot}5 \text{ kilolines.}$$

For the half tooth-pitch at either end of the pole-pitch at the beginning of stage 3, when *IS* bisects a tooth,

$$b = (6800 + 6300) \times 25 \qquad = 327{\cdot}5 \text{ kilolines.}$$

For the quarter tooth-pitch at the right-hand end of the N. pole-pitch at the beginning of stage 2 or at the left-hand end at the beginning of stage 4, when in either case a slot is next to *NL* and the centre-line of a tooth is $-5°$ therefrom,

$$a = 3500 \times 25 \qquad = 87{\cdot}5 \text{ kilolines.}$$

When arranged schematically as in Table XXVII, the fluxes are

	a	b	c
d	k	k	k
k	k	k	k
k	k	k	k
k	k	k	k
$\frac{1}{2}k$	—	—	—

Total flux at beginning of stage 1 . . . . . $2d + 7k$
„ „ „ 2 or 4 . . . $a + c + 8k$
„ „ „ 3 . . . . . $2b + 8k$

When the linked fluxes are tabulated by progressive deduction, it is found that after the first stage, their differences for every tooth-cycle are the same, viz.,

Stage 1. $k - (c - a) = 820 - 480 = 340$
„ 2. $c - a = 567{\cdot}5 - 87{\cdot}5 = 480$
„ 3. $c - a = 480$
„ 4. $k - (c - a) = 340$

as plotted in Fig. 489. There are, in fact, two different average rates of unlinking or of re-linking, one above and the other below the average rate over a tooth-cycle; the latter is simply twice the flux k of a tooth-pitch per tooth-cycle or $k/2 = 410$ kilolines per quarter tooth-cycle. As soon as the k values are reached in the table, the tooth ripple persists unchanged, the same values continually repeating. For the E.M.F. wave to become smooth in this particular case, $c - a$ must be $\frac{1}{2}k$; whether there is actually a crest or a hollow at its centre depends upon whether $c - a$ is greater than or less than the average rate $\frac{1}{2}k$.

The object of the above treatment of a simple, although not as yet a practical, case has been to emphasize the importance of the differences in the permeances at the two ends of the pole-pitch at the beginnings of stages 2 and 4. The two values of the slot and tooth densities next to the interpolar line of symmetry have been chosen to maintain the same ripple as would result if the poles be imagined to touch (leakage being presumed

TABLE XXVII

FLUXES CORRESPONDING TO FIG. 489

STAGE 1 BEGINS.		**STAGE 2 BEGINS.**		**STAGE 3 BEGINS.**		**STAGE 4 BEGINS.**	
Angle of centre-line of tooth from I.S.	Flux of tooth-pitch in kilolines.	Angle of c.l. of tooth from I.S.	Flux of tooth-pitch in kilolines.	Angle of c.l. of tooth from I.S.	Flux of tooth-pitch in kilolines.	Angle of c.l. of tooth from I.S.	Flux of tooth-pitch in kilolines.
		− 5°	(¼t.p.)87·5	0°	(½t.p.)327·5	5°	(¾t.p.)567·5
10°	737·5	15°	820	20°	820	25°	820
30°	820	35°	820	40°	820	45°	820
50°	820	55°	820	60°	820	65°	820
70°	820	75°	820	80°	820	85°	820
90°	(½t.p.)410						
			3367·5		3607·5		3847·5
	3607·5						
	× 2		3847·5		× 2		3367·5
Total flux of pole-pitch	7215		7215		7215		7215
			175		655		1135
Linked flux of column 3 of Tab. XXVIII	7215		7040		6560		6080
	1475		1640		1640		1640
of col. 5 .	5740		5400		4920		4440
	1640		1640		1640		1640
of col. 7 .	4100		3760		3280		2800
	1640		1640		1640		1640
of col. 9 .	2460		2120		1640		1160
	1640		1640		1640		1640
of col. 11 .	820		480		0		−480

TABLE XXVIII

LINKED FLUX DIFFERENCES PER STAGE FOR QUARTER-WAVE (FIG. 489)

	Total flux of pole-pitch.	Linked flux.	Diff.	Linked flux.	Diff.	Linked flux.	Diff.	Linked flux.	Diff.	Linked flux.	Diff.
	7215	7215		5740		4100		2460		820	
Stage 1.			*175*		*340*		*340*		*340*		*340*
	7215	7040		5400		3760		2120		480	
„ 2.			*480*		*480*		*480*		*480*		*480*
	7215	6560		4920		3280		1640		0	
„ 3.			*480*		*480*		*480*		*480*		
	7215	6080		4440		2800		1160			
„ 4.			*340*		*340*		*340*		*340*		
	7215	5740		4100		2460		820			

absent); for in this imaginary case, a' would be $6800 \times 25 = 170$ kilolines, and c' would be $(9600 + 9600 + 6800) \times 25 = 650$ kilolines, giving $c' - a' = 650 - 170 = 480$, as above. But now a higher value for the density of the slot quarter tooth-pitch as compared with the tooth value 6300 would directly increase a and d, leaving b and c unaltered. But d does not appear in the later stages, so that the difference $c - a$ would decrease and the amplitude of the tooth ripple would diminish. If the density of a slot quarter tooth-pitch under the pole stands to the density of a tooth quarter tooth-pitch as 6800 : 9600 or as 0·7 : 1, the ratio beyond the pole-edge where the effective air-gap length is greater will be higher, but, on the other hand, it must be borne in mind that in estimating the linked flux (although not the total flux of a pole-pitch), some allowance must be made for the fact that in a slot or tooth coinciding with IS or near thereto the flux crosses from side to side nearly transversely (*cp.* Fig. 249) and is not effectively cut by or linked with the loop; the radial component which is alone effective is therefore less than the flux entering the outer cylinder of the core surface.

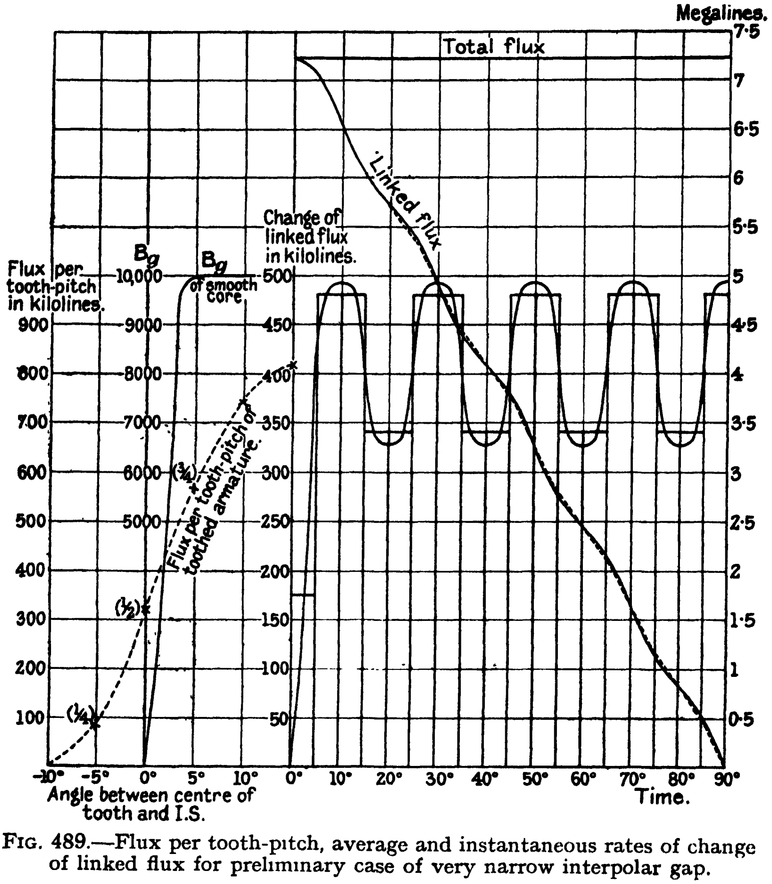

FIG. 489.—Flux per tooth-pitch, average and instantaneous rates of change of linked flux for preliminary case of very narrow interpolar gap.

§ 5. Effect of wider interpolar gap.—The chosen figures were not, therefore, unreasonable in the given case, but mark the greatest amplitude that the ripple can attain and serve as an upper limit. A closer approach to the usual cases of practice will be made by increasing the interpolar gap. A further object of the schematic treatment (p. 90) is that it renders it easy to follow the effect of cutting back the pole-edges. Let this be done until, say, the flux per tooth-pitch only reaches its normal value k under the pole for a tooth-pitch of which the tooth-centre stands at 25° behind the interpolar line of symmetry. We then have

$$\begin{array}{cccc} & a' & b' & c' \\ d' & e' & f' & g' \\ k & k & k & k \\ k & k & k & k \\ k & k & k & k \\ \frac{1}{2}k & — & — & — \end{array}$$

Then after one tooth-cycle, the differences of the linked fluxes are the same for each tooth-cycle, viz.,

Stage 1. $k + (c' - a') + (g' - e')$
,, 2. $(c' - a') + (g' - e')$
,, 3. $(c' - a') + (g' - e')$
,, 4. $k + (c' - a') + (g' - e')$

which again shows the vital importance of the interpolar gap.

At the end of every half tooth-cycle along the top of the E.M.F. wave, the change of linked flux is the flux of a tooth-pitch under a pole, and at the end of every complete tooth-cycle it is *twice* the flux of a tooth-pitch under a pole. But it is the fluxes or the permeance of the interpolar gap which fix the two average rates of change when the established values are reached. It is, therefore, futile to consider any case when the flux is supposed to be artificially confined within the limits of certain imaginary " boundary lines " reaching, say, to one tooth beyond the pole-edge. Only consideration of the pole-pitch as a whole yields valid results When the spacial flux-density of the smooth-core armature varies throughout owing to progressive increase of the air-gap up to the centre of the pole, the whole of the 2nd and 4th columns are involved at the centre of the half-wave of E.M.F.

§ 6. The instantaneous induced E.M.F.—Plotting each of the values determined for the linked flux on a horizontal line basis, and joining the points by dotted straight lines, the tangent of the angles which these straight lines make with the horizontal correspond strictly to the heights of the rectangles of average E.M.F., since they measure the *average* rate of change of the linked flux during each stage. When the loop-sides are well under the poles, the dotted straight lines joining the points of linked flux fall into a zigzag of steps, with different slopes described about a mean line of the average rate of change for one or more complete tooth cycles (410 kilolines per stage). A meeting at one point of two rates of change cannot hold for instantaneous values, and the true curve of linked flux which yields the instantaneous E.M.F. rounds off the corners at which the different average rates of change meet (480 and 340 kilolines per stage in Fig. 490). But the true curve of linked flux *must* pass through each of the determined points; it follows that the rounding off cannot be done on the inside of each corner. On the contrary the true curve passes smoothly through each of the determined points on the *outside* of the corner, and crosses the zigzag line backwards and forwards at the junctions of two similar stages. It thence follows, as will be seen from Fig. 490, which shows the latter part of Fig. 489 to a larger scale, that when passing from a crest to a hollow, the instantaneous rate of change or E.M.F. equals the average rate or average E.M.F. of the stage later than the middle of the rectangle forming part of the crest preceding the hollow, and earlier than the middle of the rectangle forming the first part of the hollow. But though the area of a crest above the mean E.M.F. corresponding to the mean rate of linking or unlinking must be equal to the area of a hollow below it, the shape of the superposed ripple need not be symmetrical above and below the mean.

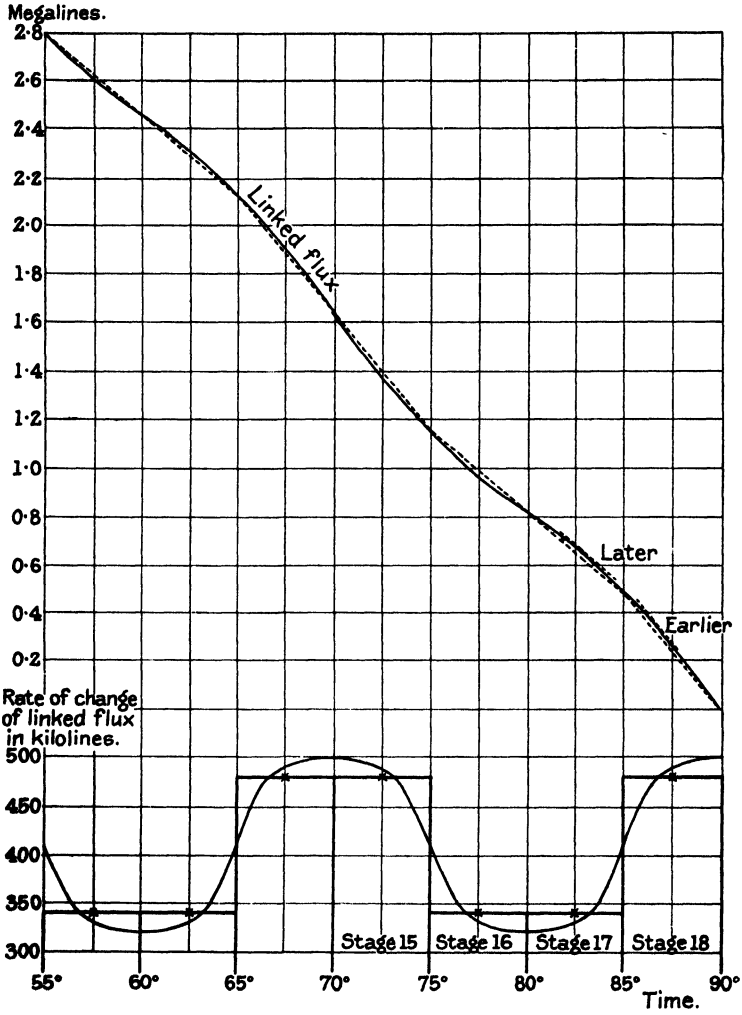

FIG. 490.—Portion of Fig. 489 to a larger scale.

§ 7. Flux-swing.—The origin of tooth ripples has often been assigned to "flux-swing," upon the supposition that the weakening of the fringe at one edge of a pole and its strengthening at the other edge in the unsymmetrical positions of teeth and pole causes a corresponding amount of flux to travel backwards and forwards across the slots under the pole, cutting the conductors therein and virtually either increasing or reducing the normal rate of cutting, such as would occur in an equivalent smooth armature with the same constant relative velocity of movement. It has, however, been argued in Chapter V that the growth and decay of the flux-density in the teeth is to be regarded rather as occurring *in situ*, *i.e.* in the path along the direction of the flux without passage of any lines from the teeth across the slots; the phenomenon would not, therefore, appear to be so correctly explained by ascribing it to flux oscillating backwards and forwards in the above manner.

A clear meaning can be given to the term, "flux-swing," if it be defined as the swinging of the neutral line about the geometrical line of symmetry between the poles. It will have been noticed that to render it possible to take into account any such action, the neutral line NL has alone been spoken of at the beginning of stages 2 and 4 in place of the interpolar line of symmetry IS, and has been shown in Figs. 488 and 492. The neutral line may, in fact, first hang back behind and then advance in front of IS in each tooth-cycle, recovering the symmetrical position at the beginning of stages 1 and 3. The effect therefrom is, however, demonstrably very small—the more so, owing to the flux in the tooth and slot nearest to IS being so largely tangential and not radial. No refinement of distinction between NL and IS appears, therefore, to be necessary in practice, as shown by the figures deduced above, which are not largely altered by assigning different proportions of fluxes in the first tooth-pitches of stages 2 and 4, so long as these proportions are rational and consistent. It is true that a neutral search coil in the interpolar geometrical plane close to the stator core shows an E.M.F. due to variation of the flux within its embrace and that this flux variation is almost entirely confined to the immediate neighbourhood of the stator core.[1] But this is immediately explained by the alteration from instant to instant in the configuration of the leakage lines passing from pole to pole as an unsymmetrical succeeds to a symmetrical position or *vice versa*; it does not necessarily imply any appreciable divergence between NL and IS, although doubtless to a small extent this may be present.

§ 8. The flux per tooth-pitch.—Proceeding now to more real cases, but still assuming the flux-density curve of the smooth core to be flat under the poles, let the ratio of pole-arc to pole-pitch be 0·69, so that $c/l_g = 1{\cdot}95/0{\cdot}25 = 7{\cdot}8$. For the calculation of

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[1] G. W. Worrall, "Magnetic Oscillations in Alternators," *Journ. I.E.E.*, Vol. 39, p. 213.

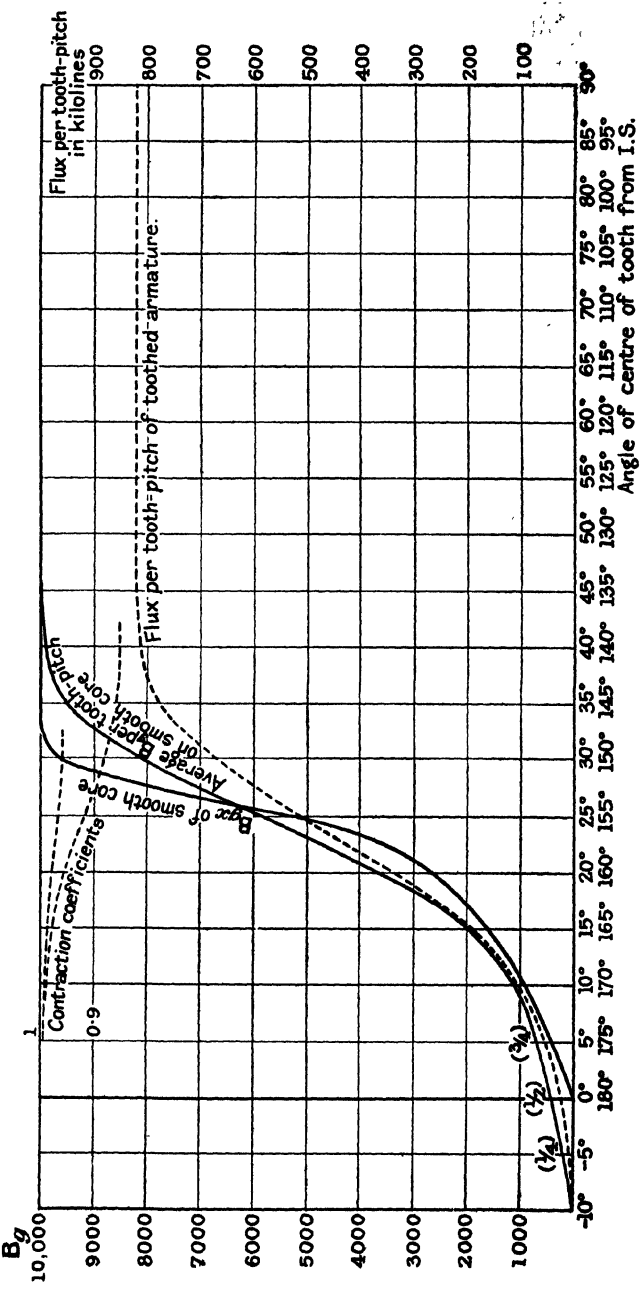
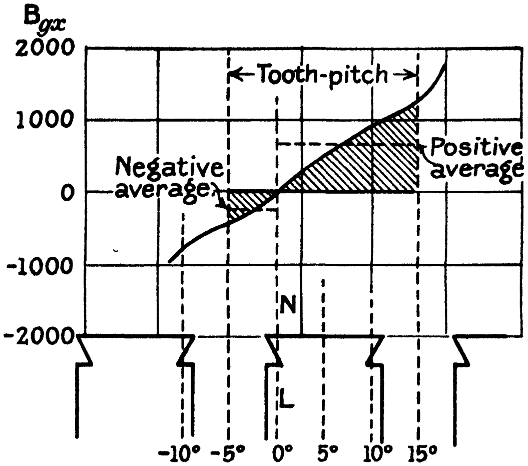
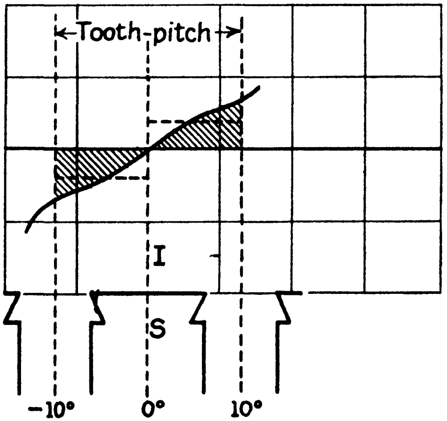


FIG. 491.—Calculation of flux per tooth-pitch for any position of the tooth relatively to the pole.

the initial curve for the flux in a tooth-pitch at any position relatively to the pole, it is best, as in Chapter XVIII, § 3, to take the difference of magnetic potential acting between the bored face of the pole and the cylindrical surface of the stator core at the level of the bottoms of the slots, *i.e.* $1{\cdot}257\ (AT_g + AT_t) = 1{\cdot}257\ (AT_p - AT_{cx})$ on the analogy of equations (134) and (135). In



(i) Beginning of Stages 2 and 4



(ii) Beginning of Stage 3.

FIG. 492.—Tooth-pitch containing *NL* or *IS*.

the interpolar gap $\mathfrak{R}_{tx}$ has little influence, and taking $\xi x + l_g$ as the length of path, its division into the M.M.F. yields as quotient B_{gx} and an average over a tooth-pitch on a supposed smooth core as shown by the full lines of Fig. 491. On the toothed core, taking $\xi x + l_g$ as the length of path to the *centre* of a tooth, the same density represents the maximum value $\mathbf{B}_{gx}$ (Chapter XVI, § 7), and the average over the tooth-pitch is $C\mathbf{B}_{gx}$, where C is the contraction coefficient. The flux for each tooth-pitch of width $(w_{t1} + w_3)$ and extending across the axial length of the stator face is then

$$\{w_{t1} + w_3\ (1 - K_3)\}\ \mathbf{B}_{gx}\ (L_f + K_2\, l_g - K_3\, w_d\, n_d)$$

as shown dotted in Fig. 491. As $w_3/(\xi x + l_g)$ diminishes, $1 - K_3$ approaches unity, so that, proceeding from under the pole, the dotted curve of flux per tooth-pitch on the toothed armature would gradually draw towards a similar curve for the smooth armature. The B_g curve for the smooth armature, unslotted and without ducts, is merely used as a ready means for finding $\mathbf{B}_{gx}$ to which the contraction coefficients are to be applied.

A further consequence of passing to the more real case with wider interpolar gap is that the effective axial length of the armature must be progressively increased when we have emerged from under the pole and approach the interpolar line of symmetry. As $\xi_x + l_g$ lengthens, the contraction due to the air-ducts diminishes, until the effective axial length of the core becomes nearly equal to its gross length on *IS*.

§ 9. The treatment of tooth-pitches within which *NL* or *IS* falls.—When the centre line of a tooth-pitch stands at less than the electrical angle of a tooth-pitch away from *NL* or *IS*, *i.e* in the first tooth-pitches of stages 2, 3, and 4 of Tables XXVII and XXIX, the + and − fluxes which combine to give the average value must be reckoned out separately and the proper portion taken into account. Thus, as shown in Fig 492 (ii) for a particular case, at the beginning of stage 3, when the interpolar line of symmetry coincides with the centre-line of a tooth, the average density on either side of the line is estimated as ± 420 ; at the beginning of stages 2 and 4 (Fig. 492 (i)), with an average density of 465 over the whole tooth-pitch, the average densities on the two sides of the neutral line are estimated as + 700 and − 198 After this manner it is estimated below that at the beginning of stage 3, 420 × 50 = 21,000 lines enter on one side and leave on the other side of the interpolar line, and at the beginning of stages 2 and 4 that 198 × 25 = 4950 lines enter on one side and 700 × 75 = 52,500 lines leave on the other side of the neutral line There is also within the same region some deduction to be made in the linked flux to allow for the fact that the actual flux is not truly radial, but contains a tangential component.

§ 10. Whole number of slots per pole and pole-pieces not chamfered at edges.—The approximate data for the calculations of Table XXIX are then as follows—

Centre-line of tooth . .	− 5°	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
Slot contraction coefficient .	0·995	0·994	0·992	0·988	0·978	0·95	0·91	0·866	0·863	0·863	0·863
Length of core contrac. coeff.	0·995	0·994	0·992	0·99	0·985	0·978	0·965	0·953	0·95	0·95	0·95
Product . .	0·99	0·988	0·985	0·979	0·964	0·94	0·878	0·825	0·82	0·82	0·82
Average B_g per tooth-pitch on smooth-core .	200	425	710	1050	1960	3690	5800	8100	9500	9870	10,000
Average B_g per tooth-pitch on toothed core .	198	420	700	1028	1890	3465	5100	6680	7795	8100	8200
Rad. percentage.	0·71	0·81	0·9	0·97	0·995	1·00	—	—	—	—	—

TABLE XXIX

FLUXES CORRESPONDING TO FIG. 491 (REPEATED AS CURVE *aa* IN FIG. 494); SEE ALSO FIG. 488.

STAGE 1 BEGINS.		STAGE 2 BEGINS.		STAGE 3 BEGINS.		STAGE 4 BEGINS.	
Angle of centre-line of tooth from I.S.	Flux of tooth-pitch in kilolines.	Angle of c.l. of tooth from I.S.	Flux of tooth-pitch in kilolines.	Angle of c.l. of tooth from I.S.	Flux of tooth-pitch in kilolines.	Angle of c.l. of tooth from I.S.	Flux of tooth-pitch in kilolines.
		–5°	(¼ t.p.) 3·5	0°	(½ t.p.)17	5°	(¾ t.p.)47
10°	100	15°	188·5	20°	346·5	25°	510
30°	668	35°	780	40°	810	45°	820
50°	820	55°	820	60°	820	65°	820
70°	820	75°	820	80°	820	85°	820
90° (½)	410						
			2612		2813·5		3017
	2818						
	× 2		3017		× 2		2612
Total flux of pole-pitch	5636		5629		5627		5629
			7		34		94
Linked flux of col. 3 of Tab XXX	5636		5622		5593		5535
	200		377		693		1020
of col. 5.	5436		5245		4900		4515
	1336		1560		1620		1640
of col. 7.	4100		3685		3280		2875
	1640		1640		1640		1640
of col. 9.	2460		2045		1640		1235
	1640		1640		1640		1640
of col. 11.	820		405		0		–405

TABLE XXX

LINKED FLUX DIFFERENCES PER STAGE FOR QUARTER-WAVE (FIG. 493)

	Total flux.	Linked flux.	Diff	Linked flux.	Diff.	Linked flux.	Diff.	Linked flux.	Diff.	Linked flux.	Diff.
	5642	5636		5436		4100		2460		820	
Stage 1			*14*		*191*		*415*		*415*		*415*
	5637	5622		5245		3685		2045		405	
,, 2			*29*		*345*		*405*		*405*		*405*
	5635	5593		4900		3280		1640		0	
,, 3			*58*		*385*		*405*		*405*		
	5637	5535		4515		2875		1235			
,, 4			*99*		*415*		*415*		*415*		
	5642	5436		4100		2460		820			

The reckoning of the assumed radial percentages makes but little difference in the final result, but it is taken into account for the sake of completeness in Table XXIX. It will now be found that in the given case $(c + g + k) - (a + e + i)$ is less than $\frac{1}{2}k$. Consequently, as shown by Fig. 493 in which the results of Fig. 491 are plotted, a *hollow* occurs at the centre of the half wave of E.M.F., but the resulting ripple is of no great amplitude.

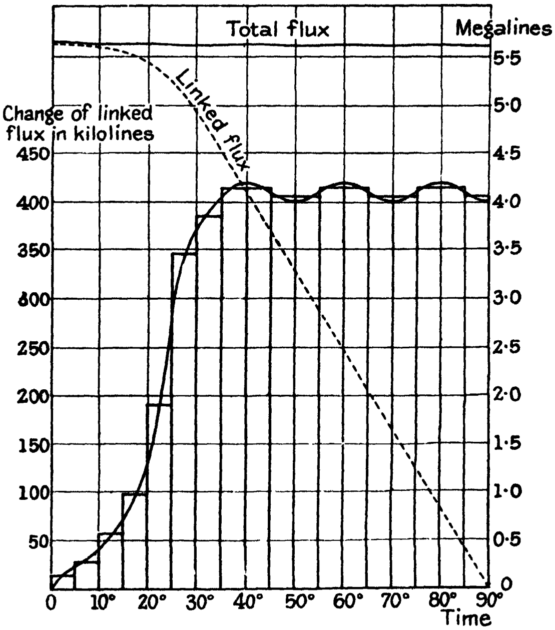


FIG. 493.—The tooth-ripples of a quarter-wave resulting from Fig. 491.

Now between the two shapes of Fig. 489 and Fig. 493, the one strongly marked with a crest and the other with a hollow at its centre, any intermediate figure is possible. But it must not be supposed that the one will necessarily pass gradually into the other by gradual shortening of the polar width, although it is the case that as the curve of the flux per tooth-pitch becomes more rounded or sinusoidal, the less the amplitude of the tooth-ripple. The exact shape is dependent on *all* the circumstances, including the maximum density and reluctance of the teeth under the pole as compared with their almost negligible reluctance in the interpolar gap. In fact, no general rules or precise conditions for the pole-width to contain a whole, even or uneven, number or a fractional number of tooth-pitches can be laid down as obviating tooth-ripples.

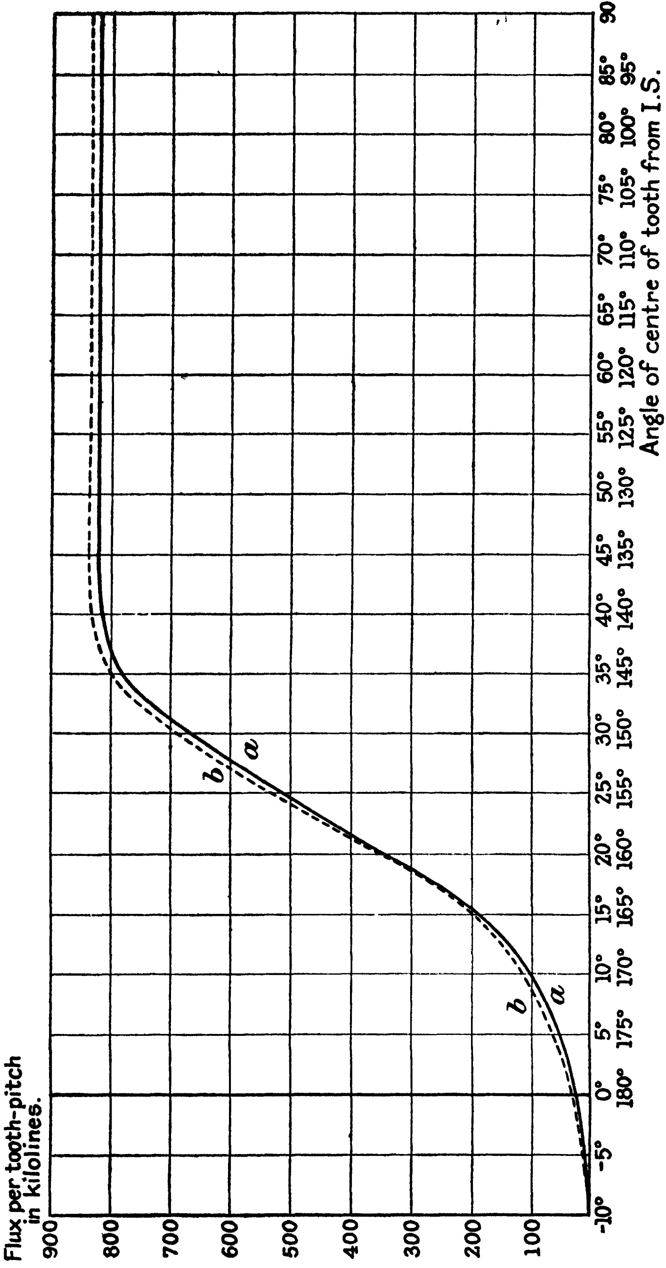


FIG. 494.—Altered flux per tooth-pitch with curve *aa* repeated from Fig. 491 for comparison.

§ 11. Effect of alteration of conditions in the interpolar gap.—In proof of this, with the same polar-width, let the air-gap be slightly reduced and the reluctance of the teeth under the pole be increased, so that the flux per tooth-pitch under the pole rises from 820 to 836 kilolines, but only when the applied ampere-turns are increased; in consequence the flux near the interpolar line of symmetry as well as under the pole will rise slightly, but at some intermediate point it will be the same as before. The very slight difference between the two curves of Fig. 494 in which *aa* repeats the curve of Fig. 491, now leads to the entirely different result of Tables XXXI and XXXII and Fig. 495 .

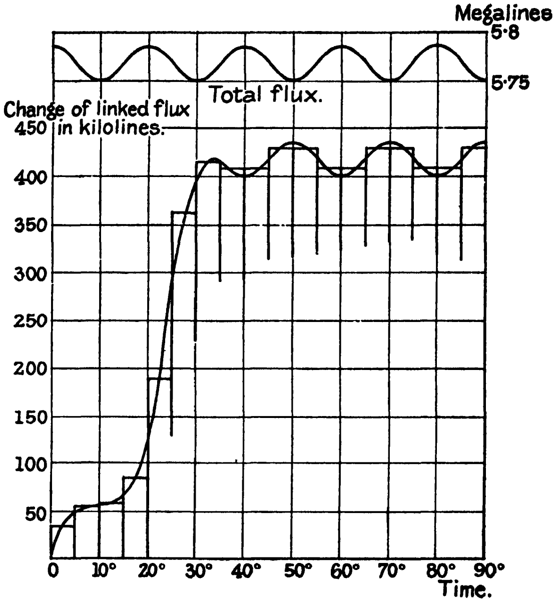


FIG. 495.—Tooth-ripple resulting from *bb* curve of Fig. 494.

§ 12. Pulsation of total flux of the pole-pitch.—But a further difference is that the total flux or permeance of the pole-pitch now pulsates appreciably, *i.e.* passes through a cycle of values from a maximum to a minimum and back again during a tooth-cycle. The amplitude of the pulsation, even when undamped, will be found in all practical cases to be very small. That the number of tooth-pitches covered by the pole-face, the degree of chamfering of their edges and the reluctance of the teeth should be so nicely related that the total permeance of the air-gap and teeth remains strictly constant must be recognized as exceptional. Yet it does not appear to be physically impossible, and, if realized, it does not secure the elimination of tooth-ripples. In the true case of the

TABLE XXXI

FLUXES CORRESPONDING TO DOTTED CURVE *bb* OF FIG. 494

STAGE 1 BEGINS.		STAGE 2 BEGINS.		STAGE 3 BEGINS.		STAGE 4 BEGINS.	
Angle of centre-line of tooth from I.S.	Flux of tooth-pitch in kilolines.	Angle of c.l. of tooth from I.S.	Flux of tooth-pitch in kilolines.	Angle of c.l. of tooth from I.S.	Flux of tooth-pitch in kilolines.	Angle of c.l. of tooth from I.S.	Flux of tooth-pitch in kilolines.
		-5°	(¼ t.p.)10	0°	(½ t.p.)27	5°	(¾ t.p.)67
10°	117	15°	193	20°	345	25°	524
30°	686	35°	796	40°	830	45°	836
50°	836	55°	836	60°	836	65°	836
70°	836	75°	836	80°	836	85°	836
90° (½)	418						
			2671		2874		3099
	2893						
	× 2		3099		× 2		2671
Total flux of pole-pitch	5786		5770		5748		5770
Linked flux of col. 3 of Tab.XXXII			20		54		134
	5786		5750		5694		5636
	234		386		690		1048
of col 5 .	5552		5364		5004		4588
	1372		1592		1660		1672
of col. 7 .	4180		3772		3344		2916
	1672		1672		1672		1672
of col. 9 .	2508		2100		1672		1244
	1672		1672		1672		1672
of col. 11 .	836		428		0		-428

TABLE XXXII

FIRST APPROXIMATION TO QUARTER-WAVE (FIG. 495) WITHOUT DAMPING

	Total flux of pole-pitch.	Linked flux.	Diff.	Linked flux.	Diff.	Linked flux.	Diff.	Linked flux.	Diff.	Linked flux.	Diff.
	5786	5786		5552		4180		2508		836	
Stage 1			*36*		*188*		*408*		*408*		*408*
	5770	5750		5364		3772		2100		428	
,, 2			*56*		*360*		*428*		*428*		*428*
	5748	5694		5004		3344		1672		0	
,, 3			*58*		*416*		*428*		*428*		
	5770	5636		4588		2916		1244			
,, 4			*84*		*408*		*408*		*408*		
	5786	5552		4180		2508		836			

constant permeance of the smooth armature, the total flux of columns 4 and 8 of the flux Tables, at the beginning of stages 2 and 4, when divided about the centre of the pole, would yield two equal halves. But in the toothed armature even if the sum of the two columns is the same as the total at the beginning of stage 1 or 3, the division about the centre of the pole is unequal.

In cases in which the pulsation is large, marked rises and falls are shown on the sloping sides of the E.M.F. wave, which is obtained as a first approximation. It remains, therefore, to consider the effect of the damping to which the flux pulsation must be subjected, and thence to obtain a second approximation, which, although it cannot be accurate, may at least indicate qualitatively what may be expected.

In reality there is *no* portion of the actual E.M.F. wave which does not contain within itself the combined effect of flux-pulsation, if present, and of what may be regarded as the true unlinking or linking of flux by relative movement between pole and loop. This is true, as will be shown by a later case, even when the resulting E.M.F. wave is a pure sine curve. It can, however, be said that the nearer the practical case approaches to the case of flux limited to the polar face or a little beyond it, the nearer we approach to a pure pulsation effect about the zero of the E.M.F. wave, lasting until the pole-edges meet the sides of the loop, when the two effects, from the flux-pulsation and from such withdrawal or addition of linked flux as would occur with a constant flux, become blended.

§ 13. Damping of pulsations of the total flux.—Many causes combine to produce damping, chief among which are the action of the field-magnet circuit as a secondary, and the similar effect due to the presence of solid iron in the field system. Due to these, secondary ampere-turns arise which lower the flux when it would otherwise be at its maximum, and raise it when it would otherwise be at its minimum But any such action has for its concomitant a shifting of the phase of the flux variation relatively to the instantaneous positions of poles and teeth. Thus, say that the maximum variation is reduced to one-half; then the time and position of the poles when the flux passes through its maximum or minimum value[1] will not correspond to the beginnings of stages 1 and 3 (*cp.* Fig. 496), but in each case will be delayed The maximum flux will, therefore, occur at some relative position of poles and teeth intermediate between the beginnings of stages 1 and 2, as shown in Fig. 496, and so on. Since the damping can never be perfect, the delay must be less than one stage or one-quarter of a tooth-pitch, *i.e.* less than 90° of a tooth-cycle, and its uncertain amount introduces difficulty into the calculation of the E.M.F.

In order to attempt an approximate estimate of the effect of partial or imperfect damping, some reduced curve of flux pulsation must be assumed, with a proportionate degree of shift in the phase, as shown dotted in Fig. 496. It is here, too, assumed that the shift of the phase is 45° or an eighth of a tooth-pitch, so that the maximum flux from curve *bb* of Fig. 494 occurs halfway through stage 1; if the pulsation were further reduced and the shift increased to 67·5°, then the maximum would occur three-quarters of the way through stage 1.

---

[1] The average value of the flux does not necessarily occur, *e.g.* when undamped at the beginning of stage 2, although so shown in Fig. 496.

The relative permeances of the tooth-pitches for the new intermediate stages must then be calculated, and a measure for these is at once found by calculating the total and linked fluxes for them on the assumption that the full ampere-turns still act without any allowance for the secondary ampere-turns. The proportions of linked to total flux are thus found for each intermediate stage throughout a quarter wave, and it will only be necessary to increase or reduce them in proportion to the assumed actual value of the damped flux at the beginning of each new stage. It is thus assumed that the increased or reduced flux is distributed among the tooth-pitches in the same proportions as the undamped flux for the same relative position of teeth and poles.

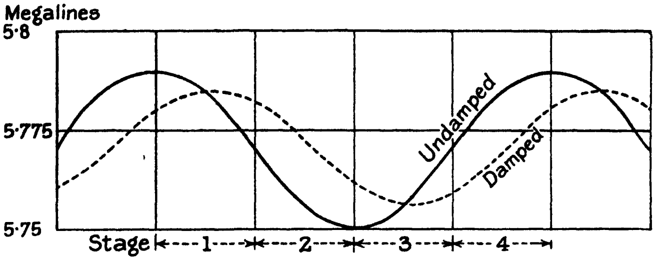


FIG. 496.—Undamped and damped total fluxes for curve *bb* of Fig. 494.

In Tables XXXIII and XXXIV the total and linked undamped fluxes are calculated from curve *bb* of Fig. 494 for the middle of the 4 stages previously treated. Each of the rows of Table XXXIV has then to be raised or lowered in the proportion

$$\frac{\text{assumed partially damped total flux}}{\text{undamped total flux calculated for the intermediate stages}}$$

before insertion in Table XXXV. But only the 2nd and 4th rows have been so treated in Table XXXV, the 2nd having been multiplied by 5769/5758 = 1·002, and the 4th by 5769/5781 = 0·998; the 1st and 3rd rows are repeated in Table XXXV unaltered, since the curve of damped flux in Fig 496 has been assumed to cut the undamped curve in the middle of stages 1 and 3, so that the partially damped and undamped intermediate fluxes are the same.

§ 14. Dissymmetry produced by damping.—A high degree of accuracy is not to be expected, but if the suggested treatment is approximately correct, the action is clear from Figs. 497 and 498 The curve of average or instantaneous E.M.F.'s at no-load no longer remains strictly symmetrical about the centre, so that an entire half-wave must be plotted. Then, as might be expected, the initial ripple of Fig. 495 about the zero point is appreciably reduced. Lastly, the top of the wave becomes tilted,[1] the amplitude of the true tooth ripples being a maximum at one corner, and with 45° of shift the other corner is also beginning to rise With a higher degree of damping the latter corner rises, until finally in the imaginary case of perfect damping the curve would again become symmetrical about the centre, each corner having risen by the same amount.

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[1] Confirmation may perhaps be found in the oscillogram of the coil pressure, Fig. 12 of Dr. S. P. Smith's paper on " The Shape of the Pressure Wave in Electrical Machinery " (*Journ. I.E.E.*, Vol. 53, p. 217); in Fig. 330*a* of Prof. Miles Walker's *Specification and Design of Dynamo-Electric Machinery* (p. 315); and in Fig. 4 of Mr. Worrall's paper on " Magnetic Oscillations in Alternators " (*Journ. I.E.E.*, Vol. 40, p. 418).

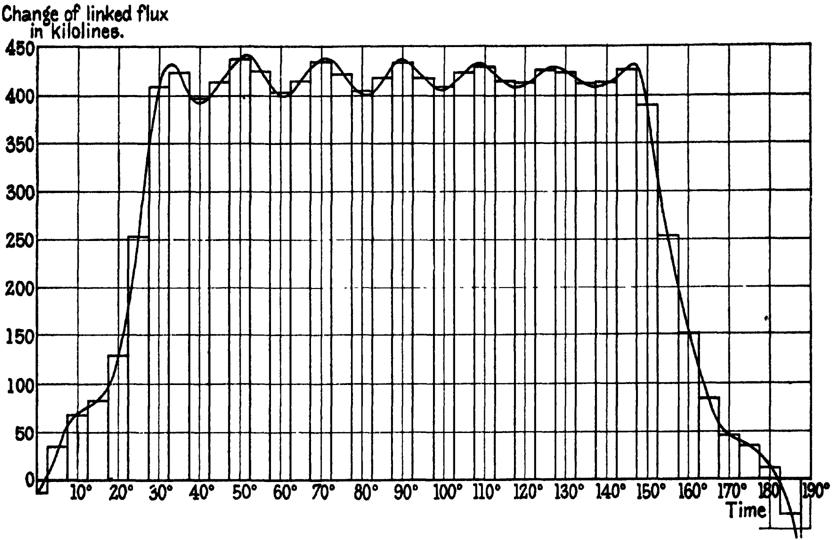

FIG 497.—Half-wave of E.M.F. with partially damped flux (Fig. 495 modified).

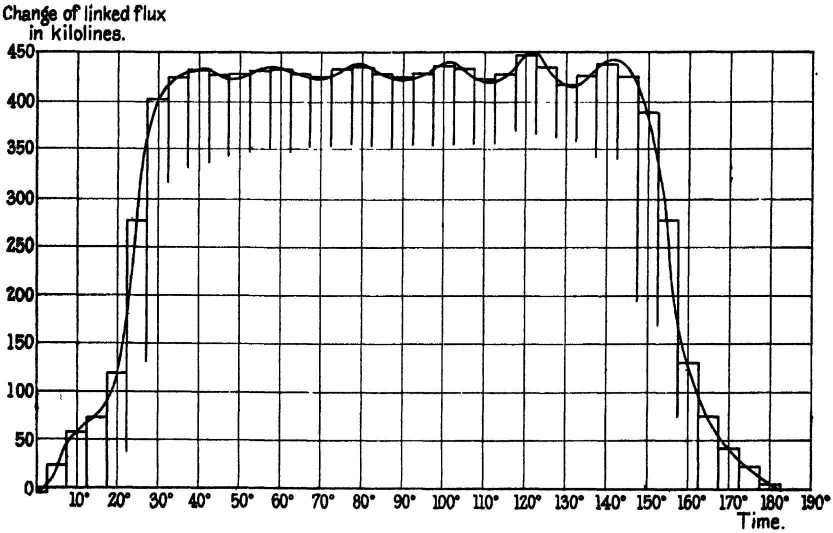

FIG. 498.—Half-wave of E.M.F. with partially damped flux.

TABLE XXXIII

UNDAMPED FLUXES FROM CURVE *bb* OF FIG. 494 FOR INTERMEDIATE STAGES

Angle of centre-line of tooth from I.S.	Flux of tooth-pitch in kilo-lines.	Angle of c.l. of tooth from I.S.	Flux of tooth-pitch in kilo-lines.	Angle of c.l. of tooth from I.S.	Flux of tooth-pitch in kilo-lines.	Angle of c.l. of tooth from I.S.	Flux of tooth-pitch in kilo-lines.
-7·5° (part)	3¾	-2·5°(pt.)	16	2·5°(pt.)	45	7·5°(pt.)	91¾
12·5°	158	17·5°	265·5	22·5°	435·5	27·5°	608
32·5°	740	37·5°	818	42·5°	834	47·5°	835
52·5°	836	57·5°	836	62·5°	836	67·5°	836
72·5°	836	77·5°	836	82·5°	836	87·5°	836
	2573¾ 3206¾		2771·5 2986·5		2986·5 2771·5		3206¾ 2573¾
Total flux of pole-pitch	5780·5 7·5		5758 32		5758 90		5780·5 183·5
Linked flux of cols. 2 & 10 of Tab. XXXIV	5773 316		5726 531		5668 871		5597 1216
of cols. 3 & 9	5457 1480		5195 1636		4797 1668		4381 1670
of cols. 4 & 8	3977 1672		3559 1672		3129 1672		2711 1672
of cols. 5 & 7	2305 1672		1887 1672		1457 1672		1039 1672
of col. 6 . .	633		215		-215		-633

TABLE XXXIV

LINKED FLUXES PER STAGE FOR HALF-WAVE.

Middle of Stage	1	5773	5457	3977	2305	633	- 1039	- 2711	- 4381	- 5597
	2	5726	5195	3559	1887	215	- 1457	- 3129	- 4797	- 5668
	3	5668	4797	3129	1457	- 215	- 1887	- 3559	- 5195	- 5726
	4	5597	4381	2711	1039	- 633	- 2305	- 3977	- 5457	- 5773

The same procedure applied to another case gave the curve shown in Fig. 498.

The direction of the tilt in relation to time depends on the particular circumstances of the case as affecting the permeances for the intermediate stages. The linked fluxes of Table XXXIV are, of course, symmetrical about the centre, but if *e.g.* the multipliers of two consecutive quantities b and c in the second and third rows, but in the same column, are x and y, the differences for the same stages reappear in Table XXXV as $(xb - yc)$ and $(yb - xc)$, which renders the values different towards the two ends of the half-wave.

TABLE XXXV

LINKED FLUX DIFFERENCES PER QUARTER TOOTH-CYCLE FOR A HALF-WAVE WITH PARTIAL DAMPING (*Angle of shift* 45°)

FIG. 497

	Total flux.	Linked flux.	Diff.	Linked flux	Diff.	Linked flux	Diff.	Linked flux.	Diff.	Linked flux	Diff.	Linked flux	Diff.	Linked flux.	Diff.	Linked flux.	Diff.	Linked flux.	Diff.
Middle of Stage 1	5780·5	5773		5457		3977		2305		633		– 1039		– 2711		– 4381		– 5596	
			35·6		*251·7*		*410·9*		*414·2*		*417·6*		*420·9*		*424·2*		*425·6*		*83·3*
Middle of Stage 2	5769	5737·4		5205·3		3566·1		1890·8		215·4		– 1459·9		– 3135·2		– 4806·6		– 5679·3	
			69·4		*408·3*		*437·1*		*433·8*		*430·4*		*427·1*		*423·8*		*388·4*		*46·7*
Middle of Stage 3	5758	5668		4797		3129		1457		– 215		– 1887		– 3559		– 5195		– 5726	
			82		*424·8*		*423·4*		*420*		*416·8*		*413·4*		*410*		*251*		*35·5*
Middle of Stage 4	5769	5586		4372·2		2705·6		1037		– 631·8		– 2300·4		– 3969		– 5446		– 5761·5	
			129		*395·2*		*400·6*		*404*		*407·2*		*410·6*		*412*		*150*		*11·5*
Middle of Stage 5	5780·5	5457		3977		2305		633		– 1039		– 2711		– 4381		– 5596		– 5773	

§ 15. Fractional number of slots per pole.—If the number of slots per pole is fractional, and the coil-span in consequence is less than the full pole-pitch, the one side does not repeat the action of the other side. Let there be, for example, 9½ slot-pitches in the pole-pitch, and let the span of the loop be 9 slot-pitches (Fig. 499). Now the beginning of stage 3 represents the flux of the pole-pitch successive in space on the machine to the pole-pitch at the beginning of stage 1 and simultaneous in time. The two fluxes calculated separately would vary, there being 9 teeth in the pole-pitch in the first case and 10 in the second. But the fluxes must of necessity be equal, and the equalization is obtained in Nature by an unequal division of the ampere-turns acting on the gap and teeth forming the two reluctances in series, and also as a corollary by a slight expansion of the polar field at the beginning of either stage 1 or 3, as shown by the dotted lines *NL* on the outside of *IS*, and a corresponding reduction of the polar field at the beginning of either stage 3 or 1 (Fig. 499). There is not, therefore, any swing of the flux first in one and then in the other direction, but an alternate expansion and contraction of the width of the field beyond or less than the pole-pitch. At the beginning of the intermediate stages 2 and 4, there are 9½ teeth in the pole-pitch in either case, and the fields recover their equal width.

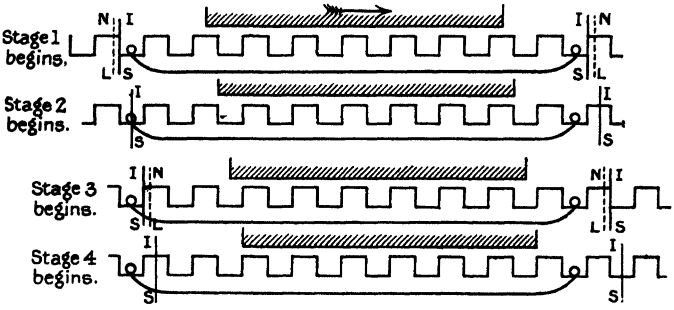

FIG. 499.—Number of slots per pole = a whole number + ½.

In a given case if the fluxes of a pole-pitch between the *IS* lines calculated separately at the beginning of stages 1 and 3 were 6076 and 6170 kilolines, the addition of 3 kilolines at each end of the pole-pitch at the beginning of stage 1 and its subtraction at the beginning of stage 3 more nearly equalizes them to an average of 6123. The normal ampere-turns expended over a tooth and single air-gap and correspondingly the fluxes originally calculated per tooth-pitch must then be increased in the proportion 6123/6082 in the first case and reduced in the proportion 6123/6164 in the second case, when equality between them results. Whether the polar field at the beginning of stage 1 must be expanded (as shown in Fig. 499) or contracted, depends on the shape of the flux curve; the presence of a slot at the pole centre in stage 3 may easily reduce the flux more than the addition of a half tooth at each end will increase it, and in this event stage 3 will show the expansion and stage 1 the contraction.

The pulsating flux of a polar field now passes through a cycle of values in the time of the passage of a pole-edge past half a tooth-pitch instead of a whole tooth-pitch as in Fig. 495.

At the beginning of stage 1 the loop fails to embrace some of the total flux at both edges; the first result of movement of the pole is then to add flux at the left side of the loop (Fig. 499), although at the other side the neutral line is drawing away from the loop side. With the beginning of stage 5, the flux within the left edge of the loop reproduces that at the head of stage 3. When the tooth-pitch fluxes are tabulated as before, the successive deductions to find the linked fluxes are not, therefore, so simple as in previous cases, since at the beginning of stages the sums of different rows in pairs of columns have to be subtracted; the unlinking process is best followed by moving a pair of lines marking the polar field in steps of tooth-pitches past the teeth.

The results fully serve to show that even with damping the tooth-ripple is not eliminated or reduced, as it would be if the coil-span differential factor of the smooth core with constant flux-wave strictly applied.

§ 16. Sinusoidal basis flux-wave.—When the flux pulsates in value, it is not necessarily the case that there must be tooth-ripples. The rising and falling of the total flux *may* so work in with the true linking and unlinking rates as to produce a pure sine wave of E M F. Damping of the pulsation will then introduce small ripples.

Thus let the variation of the flux-density on an imaginary flat core follow a sine law with a crest value of $\hat{B}_t$ over the centre of a tooth and of $\hat{B}_s$ at the centre of a slot (Fig 500 (i)); the spacial flux-density at any point reckoned from the centre of a slot is then

$$B = \tfrac{1}{2}(\hat{B}_t + \hat{B}_s) - \tfrac{1}{2}(\hat{B}_t - \hat{B}_s)\cos 2g\theta$$

where g is the number of teeth per pole and θ is the electrical angle in radians reckoned from one side of the full-pitch loop. If it then further be assumed that the basis flux curve is not a rectangle, but is sinusoidal, and that each ordinate of the above curve for a flat core may be multiplied by $\sin\theta$ when the pole-centre coincides with the centre of the tooth in the middle of the loop (Fig 500 (ii)), the spacial flux-density at that instant becomes $B\sin\theta$, and when the pole has moved through an angle $\omega_e t$ from its symmetrical position (Fig. 500 (iii)), it becomes $B\sin(\theta - \omega_e t)$. The present case then becomes amenable to simple mathematical treatment, which yields a continuous result for each and every instant as contrasted with the previous treatment by quarter tooth-pitch stages

When expanded, the spacial flux-density is

$$\begin{aligned} B = {} & \tfrac{1}{2}(\hat{B}_t + \hat{B}_s)\sin(\theta - \omega_e t) \\ & - \tfrac{1}{2}(\hat{B}_t - \hat{B}_s)\{\cos\omega_e t \,.\, \sin\theta \,.\, \cos 2g\,\theta - \sin\omega_e t \,.\, \cos\theta \,.\, \cos 2g\,\theta\}. \end{aligned}$$

At any instant, the total flux of a pole-pitch per cm. of core length is

$$\Phi = \frac{R}{p}\int_{\omega_e t}^{\pi + \omega_e t} B\,d\theta$$

where R is the radius of the stator face.

At the initial moment when $\omega_e t = 0$,

$$\Phi = \frac{R}{p}\int_0^{\pi} B\,d\theta \text{ becomes } \frac{R}{p}\left[\left(\hat{B}_t + \hat{B}_s\right) + \frac{1}{2}\left(\frac{1}{4g^2 - 1}\right)\left(\hat{B}_t - \hat{B}_s\right)\right],$$

since, the loop being of full pitch, $2g$ must be even.

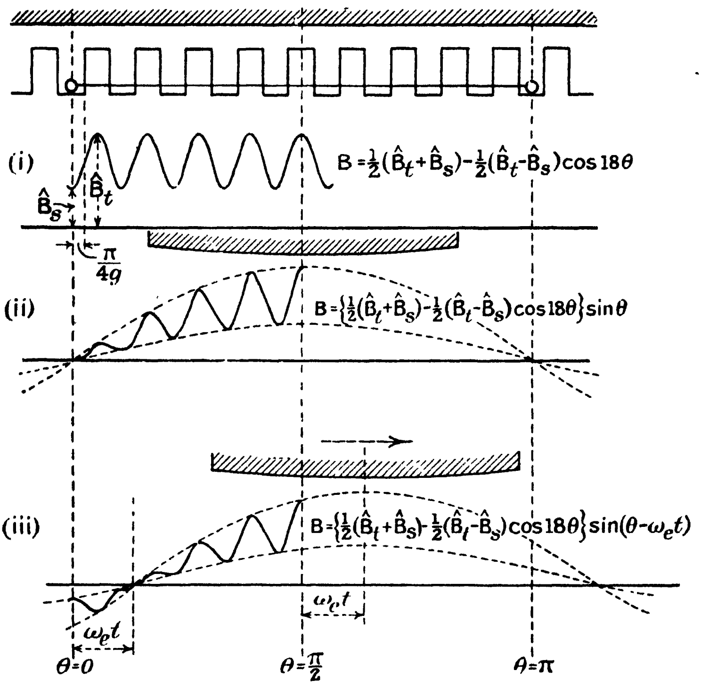
(i)
$B = \frac{1}{2}(\hat{B}_t + \hat{B}_s) - \frac{1}{2}(\hat{B}_t - \hat{B}_s)\cos 18\theta$
$\hat{B}_s$
$\hat{B}_t$
$\frac{\pi}{4g}$
(ii)
$B = \{\frac{1}{2}(\hat{B}_t + \hat{B}_s) - \frac{1}{2}(\hat{B}_t - \hat{B}_s)\cos 18\theta\}\sin\theta$
(iii)
$B = \{\frac{1}{2}(\hat{B}_t + \hat{B}_s) - \frac{1}{2}(\hat{B}_t - \hat{B}_s)\cos 18\theta\}\sin(\theta - \omega_e t)$
$\omega_e t$
$\omega_e t$
$\theta = 0$
$\theta = \frac{\pi}{2}$
$\theta = \pi$

FIG. 500.—Sinusoidal basis curve of flux-density.

At the beginning of stage 2 or a quarter tooth-cycle later, when $\omega_e t = \pi/4g$, the total flux is

$$\Phi = \frac{R}{p}\int_{\frac{\pi}{4g}}^{\pi\left(1 + \frac{1}{4g}\right)} B\,d\theta$$

The integral is then, owing to the cancelling out of a second and third term, simply $\Phi = \frac{R}{p}(\hat{B}_t + \hat{B}_s)$. The amount of the pulsating flux is, therefore, $\frac{R}{2p}\left(\frac{1}{4g^2 - 1}\right)(\hat{B}_t - \hat{B}_s)$, and its instantaneous value

$$\frac{R}{2p}\left(\frac{1}{4g^2 - 1}\right)(\hat{B}_t - \hat{B}_s)\cos 2g\omega_e t.$$

The flux embraced by the stationary loop is throughout $\Phi_l = \frac{R}{p}\int_0^{\pi} B\,d\theta$, which with $2g$ even is at any instant

$$\Phi_l = \frac{R}{p}\left[\frac{1}{2}(\hat{B}_t + \hat{B}_s)\{\cos(0^\circ - \omega_e t) - \cos(\pi - \omega_e t)\} - \frac{1}{2}(\hat{B}_t - \hat{B}_s)\frac{1}{1 - 4g^2}\cos\omega_e t + 0\right]$$

$$= \frac{R}{p}\left[(\hat{B}_t + \hat{B}_s) + \frac{1}{2}\left(\frac{1}{4g^2 - 1}\right)(\hat{B}_t - \hat{B}_s)\right]\cos\omega_e t$$

It is thus a pure cosine curve, and the E.M.F. induced in the loop at any instant per cm. length of core

$$-\frac{d\Phi_l}{dt} = R\left[(\hat{B}_t + \hat{B}_s) + \frac{1}{2}\ (\hat{B}_t - \hat{B}_s)\frac{1}{4g^2 - 1}\right]\omega \sin \omega_e t$$

is a pure sine curve. Thus in the above case, although there is pulsation in the value of the total flux, and although the harmonics in the instantaneous air-gap flux-density curve of the orders $(2g + 1)$ and $(2g - 1)$ may be great, there are no harmonics in the E M.F. induced in a full-pitch loop.[1]

§ 17. Oscillograms of the loop E.M.F. wave.—The previous detailed calculations from assumed data have only been directed to show the main factors entering into the production of tooth-ripples. The practical difficulty in laying down the curve of flux per tooth-pitch for each and every position relatively to the pole, with sufficient accuracy to enable the ripple in a given machine to be predicted with certainty, is fully admitted. The actual operation in Nature, apart from other secondary effects further complicating the case, is one of great delicacy, and small changes suffice to produce large differences in these results. Thus the more rounded B_g curve of Fig. 501 due to the air-gap being gradually increased from 0·25 in. at the centre of the pole to 0·35 in. at the edge, with its consequent gradual rise in the flux per tooth-pitch up to a maximum at the centre, yields a practically constant flux and an entirely negligible tooth-ripple, which would confirm the extent of the benefit that might be gained by a very careful adjustment of combined air-gap length and tooth-reluctance.

Yet actual oscillograms of the E.M.F. wave of a simple loop or concentrated coil bear out the main results at least with salient poles. Thus most show a crest at the centre (Fig. 502), but some a hollow (Fig. 503). Any initial ripples are usually damped out. Fig. 504 shows a rise in the crests at one end and a smaller rise at the other end with a slight dip in the centre. They are, in fact, seldom entirely symmetrical, and the pitch of the ripples is seldom perfectly uniform. Deductions from oscillograms, even when their complete accuracy is ensured, are therefore beset with difficulty on these accounts.

The doubly-slotted structure of the turbo-alternator with non-salient poles shows greater ripples on the sloping sides (Fig. 505), and for the treatment of such cases it would be necessary to consider the cycles in which the stator slots reach the same position relatively to the rotor slots, which introduces still greater complexity. The effect of damper bars in the shoes of salient poles, and even of clamping bolts in laminated poles, may often be traced as contributory causes of ripples.

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[1] The result of the above section has, since it was written, been confirmed by Prof. Waldo V. Lyon in the paper (p. 1494) quoted previously in § 1. He has further pointed out that it presupposes the flux entering a tooth on the right of a slot and on the left to divide exactly on the centre line of the slot, which is not strictly true. The integrated linked flux, therefore, contains some small error of indeterminate amount.

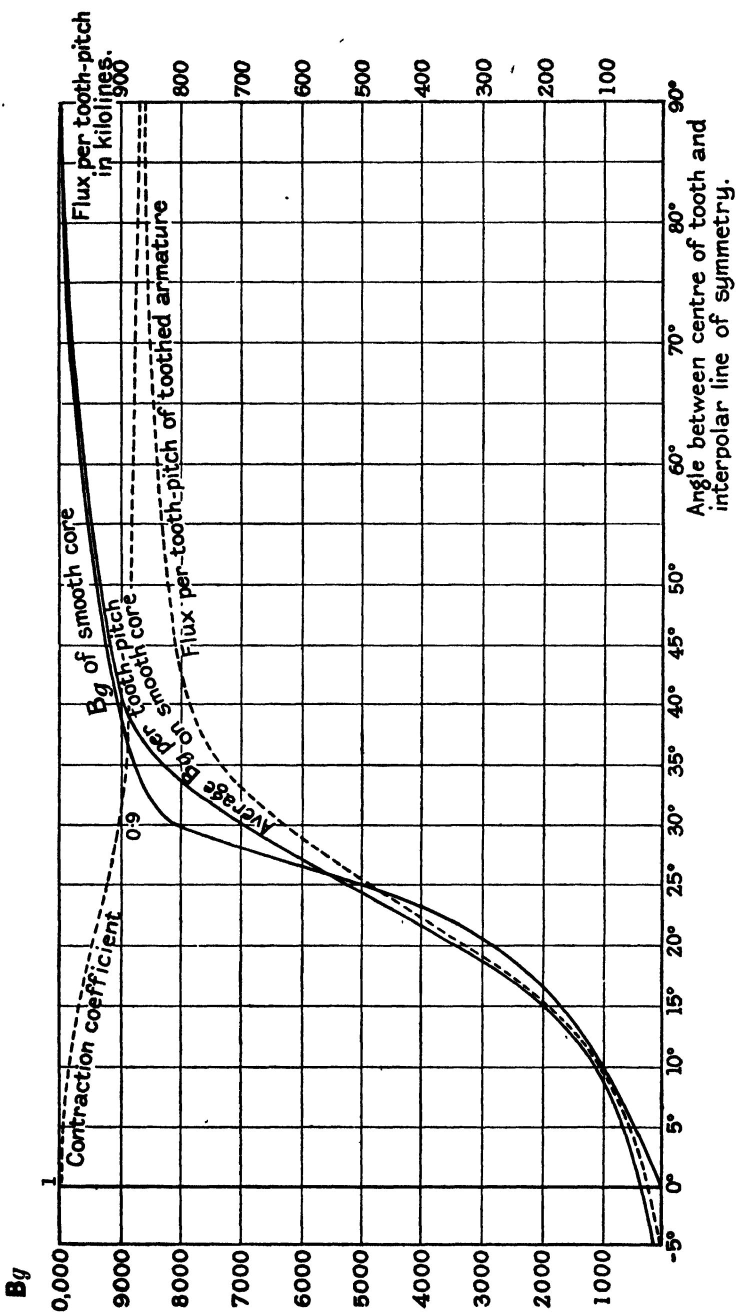

FIG. 501.—Flux per tooth-pitch with bevelled pole-pieces.

§ 18. The elimination of tooth-ripples from the loop E.M.F. wave or their reduction in magnitude.—If either the armature slots or the edges of the pole-shoes are skewed along their axial length through an amount equal to a tooth-pitch, the permeances at either end of a pole-pitch at the beginning of either stage 2 or 4 are rendered similar. A return is thus made to the condition of a smooth core; when the loop is multiplied, the actual value of the number of slots g' or g or S becomes immaterial, and the reduction factor of uniform distribution is obtained, when $\sigma\pi$ is made equal to the whole of the arc within the outer slots of the coil-side, over which the conductors are in effect spread. To skew the armature slots is, however, a troublesome operation in the manufacturing shop, and laminated pole-shoes can only be skewed by arranging packets in a few steps to avoid numerous dies for making the stampings.

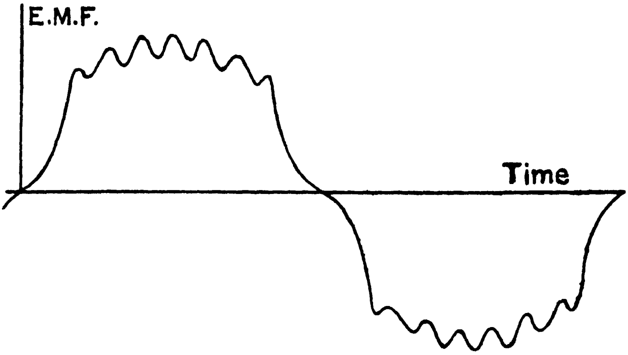

FIG. 502.—Loop E.M.F. wave on open circuit.
Salient poles, 9 slots per pole, 0·63 in opening; air-gap, 0·216 in.

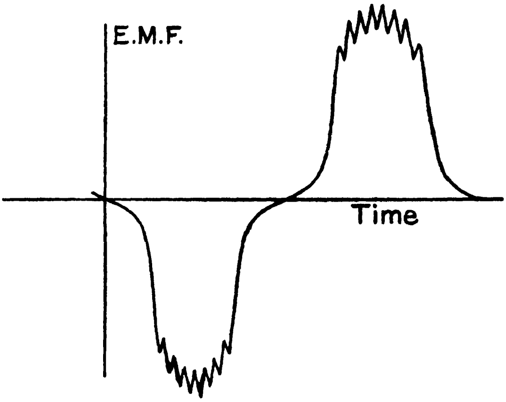

FIG. 503—Loop E.M.F. wave on open circuit.
Salient poles, 15 slots per pole; air-gap, 0·158 in.

A simpler and almost equally effective method is to move all the poles of one sign circumferentially in the same direction through half a tooth-pitch. This has the effect of making alternate interpolar gaps shorter by one tooth-pitch than the remainder of the interpolar gaps. The curve of flux-density as it dies away to zero on the interpolar line of symmetry is thus rendered different in the two sets of gaps. It then becomes possible to make the difference $(c+g+\ldots)-(a+e+\ldots)$ equal or nearly equal to $\frac{1}{2}k$, and the tooth-ripple disappears or is reduced to an inappreciable amount.

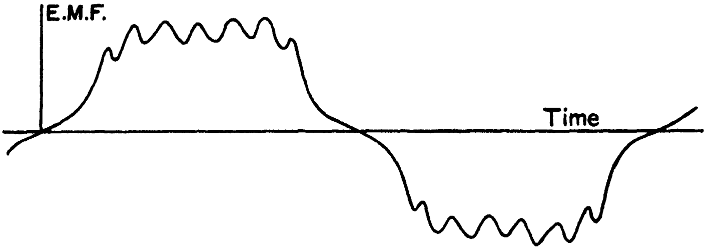

FIG. 504.—Loop E.M.F. wave on open circuit.
Salient poles, 9 slots per pole, 0·63 in opening; air-gap, concentric, 0·2 in.

§ 19. The tooth-ripple under analysis into harmonics.—When tooth-ripples are present in the loop E.M.F. wave, they often do not persist throughout the entire half wave, but may rise and die away again on the top or on the sloping sides, more usually on the former. With a uniform pitch such that, if repeated over the whole pole-pitch, their frequency due to g slots per pole-pitch would be $2g$ times the frequency of the machine, they cannot when the E.M.F. wave is analysed give rise immediately to a corresponding harmonic; for this would be an even multiple of the machine frequency, and only harmonics whose frequencies are odd multiples can appear in the analysed curve. But if the E.M.F. tooth-ripple of even

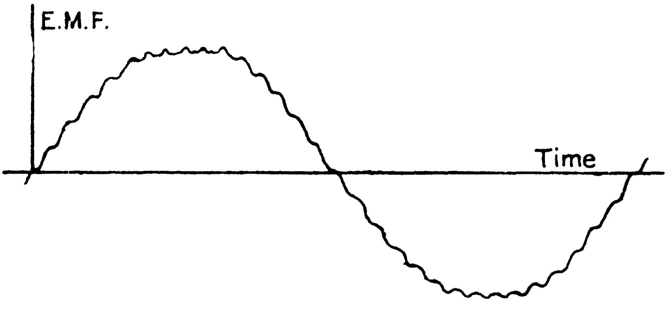

FIG. 505.—Loop E.M.F. wave on open circuit.
Two-pole non-salient-pole turbo-alternator; 30 stator slots per pole; 0·236 in. opening; 16 rotor slots per pole, 0·4 in. opening; air-gap, 0·472 in.

frequency *itself* varied sinusoidally over the whole pole-pitch with an amplitude $\mathbf{e}_g$ (*cp.* Fig. 500), so that it is expressible at any time t in the wave as

$$\mathbf{e}_g \sin\omega_e t \cos 2g\omega_e t,$$

it may be completely replaced by

$$\tfrac{1}{2}\,\mathbf{e}_g \sin\,(2g+1)\,\omega_e t - \tfrac{1}{2}\,\mathbf{e}_g \sin\,(2g-1)\,\omega_e t$$

since by trigonometry

$$\sin ba \,.\, \cos aa = \tfrac{1}{2}\sin\,(a+b)\,a - \tfrac{1}{2}\sin\,(a-b)\,a$$

It is thus replaceable by two odd harmonics of equal amplitude, which would appear when the E.M.F. wave was analysed; at the zero points of the E.M.F. wave they are in opposition, but they gradually draw together in phase and coincide at the centre of the wave (Fig. 506). Now the condition presupposed, implying a maximum tooth-ripple at the crest of the E.M.F. wave of the loop, is often fairly well reproduced in actual cases, so that in general it may be said that harmonics of the orders $2g \pm 1$ may be expected to be pronounced in the toothed machine. Hence, *e.g.* the E.M.F. wave of a loop on a three-phase toothed machine with $g = 3$, usually gives harmonics of quintuple and septuple frequency, but in addition a triple harmonic is usually present, due to the field itself if the slots were absent being a complex harmonic function.

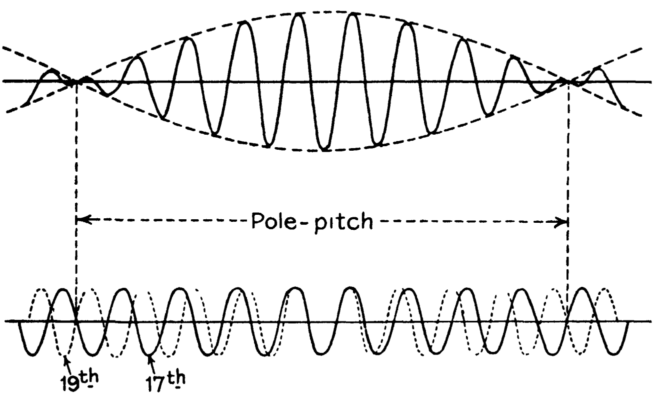

FIG. 506.—Uneven harmonics arising from a tooth-ripple of $2gf$ frequency, the amplitude of which varies sinusoidally.

By turning the diagram (Fig. 506) upside down, it will be seen that the same two uneven harmonics will represent either a crest or a hollow at the centre of the wave, according to whether the lower or higher harmonic begins by being positive.

§ 20. Summary of means for further reduction or elimination of harmonics in the phase or terminal E.M.F.—The E.M.F. wave of a loop[1] once determined by calculation or trial for a given machine, it can be resolved into a simple Fourier series, and the processes described in the preceding chapter become immediately applicable. In the presence of higher harmonics in the loop E.M.F. wave or waves, the first step towards reduction of them in the E.M.F. of the complete coil or of the phase as a whole will, therefore, be to make the slot-positions as numerous as possible relatively to a standard reference field of double pole-pitch, and thus to work through the belt and group differential factors. Taking the methods of Chapter XXV in reverse order, the use of a *fractional number of slots per pole-pair* here takes precedence, and is obtained either (i) by the addition to a single layer winding of $xN_{ph}f$ extra slots which are left empty, or (ii) by the employment of a continuous-current wave-winding in two layers, which, although essentially a closed winding, may be used either closed or opened.

In a single-layer winding with N_{ph} uneven, if N_{ph} *empty slots* are added within each pole-pair to an original whole number g_o, so that $g = g_o + N_{ph}/2 =$ an integer $+ \frac{1}{2}$, then as explained in Chapter XXV, § 10, 1 (*b*), the mean span is made to diverge from a full pitch which may be advantageous, but the positions corresponding to one pole-pair are exactly repeated under every other pole-pair, so that only the belt differential factor remains further available.

With no empty slots and $g = 3g'$ where g' is > 1, the belt differential and reduction factors for the phase E.M.F. of a 3-phase alternator by eq. (231) are respectively

$$k_{sn}' = \frac{\sin n\,30^\circ}{g' \sin n\,\dfrac{30^\circ}{g'}} \text{ and } k_{rn}' = \frac{\sin n\,30^\circ \times \sin \dfrac{30^\circ}{g'}}{\dfrac{1}{2} \sin n\,\dfrac{30^\circ}{g'}}$$

as shown for each value of g' in Tables XXIII and XXIV.

With $g' = 2$ or a multiple of 2, and divided coils, it becomes possible even with coils in a single layer to suppress harmonics with frequencies which are multiples of 3, by placing in series pairs of divided coils which are electrically 60° apart, instead of pairs of divided coils, the sides of which are adjacent and form a single belt. In relation to the third harmonic, the vectors of their E.M.F.'s will be at 3 × 60° = 180° (electrical), and therefore, opposed to each other, so that no third harmonic can appear in the joint E.M.F. of a pair of divided coils; and consequently a third harmonic cannot appear in the phase or line E.M.F.

Since the number of slots per pole must remain finite even with a large number per pole per phase so long as g' is an integer, a single-layer winding can never, as previously mentioned, do more than approach the case of uniform distribution so far as the earlier, but more important, harmonics are concerned. In this respect the single-layer windings of classes D and E (Chapter XXIV, § 11), although wound on the wave principle (*cp.* Fig. 96), are in no better case, since they are based on a whole number of slots per pole; they may, therefore, show pronounced tooth-ripples of the orders $2g \pm 1$.

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[1] Or of specimen loops when two or more loops of different span are required to represent the actual coils of the machine.

Lastly, the effect of the *interlinking of phases* affects only the terminal voltage between the lines with star-connected armatures, but not the phase E.M.F. Thus, in a 3-phase alternator, when the phase E.M.F.'s are at 120°, the differential factor in the interlinked pressure for the nth harmonic present in the phase E.M.F. is $\cos n\pi/6$, which becomes zero when $n = 3, 9, 15$, etc. The third harmonic and all uneven harmonics whose frequencies relatively to the machine frequency are multiples of 3 are, therefore, eliminated from the voltage between the lines, although still present in the phase voltage and capable of acting round the delta-connected mesh or between the lines and the centre-point of a star-connected alternator, as shown in the following section.

§ 21. E.M.F. harmonics of triple frequency.—When a 3-phase alternator is giving equal induced E.M.F.'s in each of its 3 phases, if these phase E.M.F.'s are resolved into Fourier series, they are respectively

$$\begin{aligned}
e_{\text{I}} &= \mathbf{E}_1 \sin \omega t + \mathbf{E}_3 \sin (3\omega t + \theta_3) + \mathbf{E}_5 \sin (5\omega t + \theta_5) + \dots \\
e_{\text{II}} &= \mathbf{E}_1 \sin (\omega t - 120^\circ) + \mathbf{E}_3 \sin \{3(\omega t - 120^\circ) + \theta_2\} \\
&\qquad\qquad + \mathbf{E}_5 \sin \{5(\omega t - 120^\circ) + \theta_5\} + \dots \\
&= \mathbf{E}_1 \sin (\omega t - 120^\circ) + \mathbf{E}_3 \sin (3\omega t + \theta_3) + \mathbf{E}_5 \sin (5\omega t - 240^\circ + \theta_5) + \dots \\
e_{\text{III}} &= \mathbf{E}_1 \sin (\omega t - 240^\circ) + \mathbf{E}_3 \sin \{3(\omega t - 240^\circ) + \theta_3\} \\
&\qquad\qquad + \mathbf{E}_5 \sin \{5(\omega t - 240^\circ) + \theta_5\} + \dots \\
&= \mathbf{E}_1 \sin (\omega t - 240^\circ) + \mathbf{E}_3 \sin (3\omega t + \theta_3) + \mathbf{E}_5 \sin (5\omega t - 120^\circ + \theta_5) + \dots
\end{aligned}$$

The peculiarity of the harmonics of third order—and also of ninth, fifteenth, etc., order—is seen to be that at any instant they have the same sign and value. This means in a mesh-connected armature that all harmonics, the frequencies of which are multiples of 3 times the machine frequency, act in the same direction round the mesh; each order will, therefore, cause its own local current to circulate round the mesh—a point on which, as will shortly be seen, the mesh-connexion is inferior to the star or Y-connexion.

In a star-connected armature the similarity of sign and value of the harmonics of triple frequency means that they are all simultaneously directed either outwards from the centre of the star or inwards to the centre. The harmonics of the 3rd, 9th, 15th, etc., order must, therefore, disappear in the curve of the interlinked pressure on no-load or on any balanced load (*i.e.* so long as the three phase E.M.F.'s remain similar); for the potential difference between any pair of lines is such as would correspond to an instantaneous induced E.M.F. between phases of value $e_{\text{I}} - e_{\text{II}}$, just as if no harmonics of triple frequency existed. They cannot, therefore, show on a voltmeter connected between any two lines. Yet a voltmeter between the centre points of the stars of the alternator armature and load would show the voltage due to the 3rd, etc., harmonics. $\mathbf{E}_1$, $\mathbf{E}_3$, $\mathbf{E}_5$, etc., being the amplitudes of the components of the phase pressure, its virtual value is $\sqrt{\frac{1}{2}(\mathbf{E}_1^2 + \mathbf{E}_3^2 + \mathbf{E}_5^2 + \ldots)}$; the virtual pressure between any pair of lines is then $\frac{\sqrt{3}}{\sqrt{2}}\sqrt{\mathbf{E}_1^2 + \mathbf{E}_5^2 + \mathbf{E}_7^2 + \mathbf{E}_{11}^2 \ldots}$, yet between two neutral points which are earthed even in a balanced system $\frac{1}{\sqrt{2}}\sqrt{\mathbf{E}_3^2 + \mathbf{E}_9^2 + \mathbf{E}_{15}^2 + \ldots}$ is present, but can produce no current so long as the load is balanced for want of a connexion between the neutral points of the armature and load.

But now let these neutral points be connected or both grounded. Then at once the above triple-frequency voltage will cause equal currents to flow through each of the three lines and to unite to return by the additional connexion between the centres of the stars. These currents in the three lines and phase windings will be in phase with each other, and the current in the neutral connexion will be 3 times that in each line, its total value being limited by the parallel impedance of the 3 circuits to triple harmonics.

Next let another alternator be substituted for the above load. Then so long as the two machines are exactly similar, equally excited and are running in perfect parallel without any variations of angular velocity, so that they divide the load equally and give equal waves of E.M.F., no current will flow even if their neutrals are connected. But so soon as these conditions are departed from, any *difference* between the values of their third-harmonic E.M.F.'s will cause a current to flow exactly analogous to the case of the balanced load described above, with a short-circuit neutral connexion. Such triple-frequency currents between star-wound alternators running in parallel with their centres grounded, may give rise to such fluctuations as to render their working unsafe. If their origin cannot be eliminated, impedances must be inserted in the neutral connexions in order to minimize their amount.[1]

Even with a system grounded at one neutral point only, a third harmonic E.M.F. causes a charging current to oscillate backwards and forwards between the earth connexion and the three lines in parallel, which may seriously interfere with the working of neighbouring telegraphs.[2]

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[1] *Cp.* J. H. Rider, *Journ. I.E.E.*, Vol. 43, p. 260, and E. P. Hollis, p. 341.

[2] See S. C. Bartholomew, *Journ. I.E.E.*, Vol. 62, p. 826.

§ 22. The effect of capacitance.—Resonance with the higher harmonics of the E.M.F. wave when the cables have capacitance may seriously strain the insulation of the system and even cause its breakdown;[1] since it occurs when $2\pi f_n \mathscr{L} = 1/(2\pi f_n C)$ where C is the capacitance in series with inductance $\mathscr{L}$ and f_n is the frequency of the harmonic, it is evident that the higher the order of the harmonic, the smaller the capacitance that will cause resonance.[2]

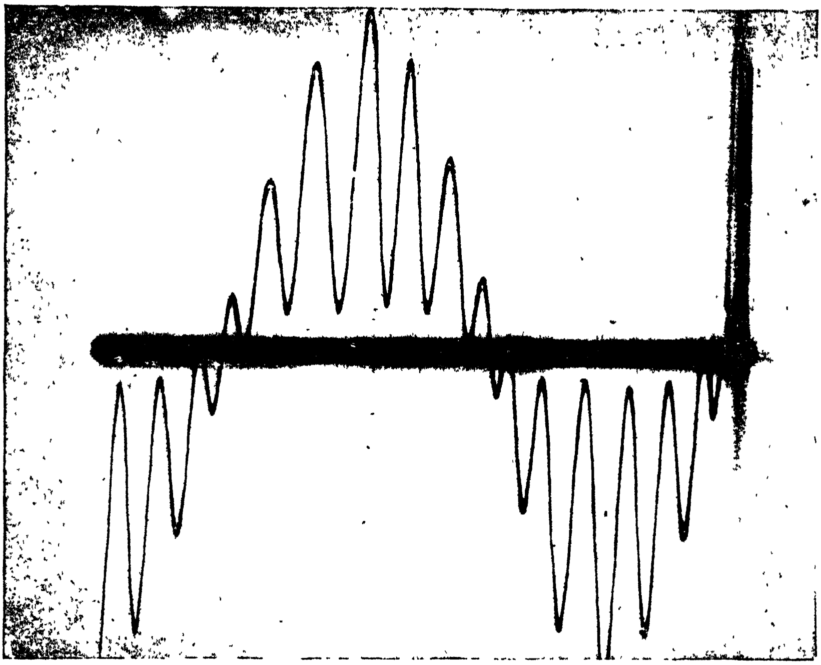

Fig. 507.—P.D. curve of alternator on unloaded cables, harmonic of 13 times machine frequency prominent.

The effect of capacitance as magnifying the amplitude of ripples is shown in the oscillogram (Fig. 507) of an alternator running on unloaded cables; the harmonic of 13 times the machine frequency is prominent. A sinusoidal E.M.F. without harmonics is, therefore, always to be aimed at,[1] and the desirability of avoiding all higher harmonics when the external lines possess capacitance affords a powerful argument for the superiority of semi-closed as compared with open slots.

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[1] *Journ. I. E. E.*, Vol. 32, p. 734, "Resonance in Electric Circuits," by M. B. Field; also p. 707, "Distribution Losses in Electric Supply System," by A. D. Constable and E. Fawssett, and various speakers in the discussion of both papers.

[2] For experimental methods of resolving the E.M.F. wave into its harmonics in order to determine their amplitude, see Dr A. Russell, *Alternating Currents*, Vol. 2, pp. 144–152 (2nd edit.); A. F. T. Atchison, *Journ. I.E.E.*, Vol. 33, p. 1109; P. G. Agnew, *Electr. World*, Vol. 54, p. 142; and J. D. Cockcroft, R. T. Coe, J. A. Tyacke, and Miles Walker, *Journ. I.E.E.*, Vol. 63, p. 69 with bibliography.

Lastly, a practical conclusion is that, when starting up an alternator connected to a cable system, the full speed should be obtained before the excitation is slowly brought up to the normal, and *vice versa,* on shutting down, the excitation should be withdrawn before the speed is reduced; otherwise the critical speed at which resonance with some higher harmonic occurs may be passed through.[2]

§ 23. The E.M.F. wave under load.—Although with a winding distributed among 3 or 4 slots per pole per phase a near approach to a sine wave of E.M.F. naturally results at no load, even with a comparatively flat-topped flux-curve, or may be obtained by special shaping of the pole-shoes, it is practically impossible to retain this shape so long as the pole-faces are salient and entirely separate from one another. Further, the shape will vary widely according to the nature of the external load; *e.g.* if the alternator be working on a non-inductive load such as a water-resistance or fully loaded transformers, as contrasted with the same alternator working on an inductive load such as lightly loaded transformers or motors. The reason is to be found in the distortion due to the cross-magnetizing turns of the armature current; as will be more fully explained in Chapter XXVII, the cross flux which is virtually superposed upon the main flux shows with salient poles a marked depression in the gap between the pole-tips (*cp.* Fig. 317), and in fact almost falls to zero, yet it is just here that it should be a maximum if the cross E.M.F. is also to be sinusoidal, and this latter is the necessary condition for the resultant curve of E.M.F. under load to remain sinusoidal. This difficulty decreases as the inductiveness of the load is increased, and in consequence the cross ampere-turns of the armature are decreased, but under any fairly non-inductive load it can never be entirely removed. Even when the field coils instead of being wound on projecting poles are embedded in the slotted surface of a smooth iron rotor as in turbo-alternators, the pole-centre is often unslotted; there must then under load be some loss of any exact sinusoidal shape of the E.M.F. which might be secured on open circuit.[3]

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[1] For the permissible deviation from a sine wave, and measurement of the deviation factor and telephone interference factor, see Standards of the American Institute of Electrical Engineers (*Trans*, Vol. 40, p. 1619, 1921), and British Standardisation Rules for Electrical Machinery; also Dr. A. Russell, *Journ. I.E.E.*, Vol. 62, p. 13 ff. For sine-wave testing sets, see E. J. Burnham, *Gen. El. Rev.*, Vol. 23, p. 177, and B. Hague, *Journ. I E E.*, Vol. 62, p. 921.

[2] For even harmonics in alternating-current circuits, see C. P. Steinmetz, *Transient Electrical Phenomena and Oscillations*; also *Electr. World*, Vol. 53, p. 734; and J. B. Taylor, *Trans. Amer. I.E.E.*, Vol. 28, Part I, p. 725.

[3] For full-load wave, see Chap. XXX and Part II of the writer's article in *Papers on the Design of Alternating-Current Machinery* (Pitman), pp. 202-239.

CHAPTER XXVII

ARMATURE REACTION IN ALTERNATORS

§ 1. **Regulation of alternators.**—In order to judge of the practical merits of an alternator, its behaviour under various loads with different amounts of inductance must be examined. A constant terminal voltage for all loads within the capacity of the machine being assumed as desirable, the degree by which it falls short of this ideal depends on the drop in volts, *i.e.* on the difference in the terminal voltages (with constant excitation and speed) at no-load and under a certain definite load formed by an external current I_e with an angle of lag ϕ_e such that $V_e I_e \cos \phi_e =$ the full rated output of the machine per phase. If the fall of the voltage when load is thrown on or the rise when load is thrown off exceeds a certain permissible amount, the load has passed the limit set by armature reaction, even though the heating of the armature winding might be within the permissible limit. Thus the output of the alternator is fixed essentially by the two considerations of heating and armature reaction, just as is that of the continuous-current dynamo. To secure a constant terminal voltage, it is evident that the excitation must be adjusted by hand or automatically to the amount required by varying loads. The inherent *regulation* of the alternator, as it is termed, or the percentage variation of its voltage under given conditions is, therefore, a matter of practical importance (*cp.* § 40).

§ 2. **Value of power-factor of external circuit.**—For lighting distribution with transformers fully loaded, ϕ_e is about 10° and the *power-factor* may be taken as nearly equal to 1. For power distribution with fully loaded asynchronous motors, ϕ_e on the average is about 37° lagging ; $\cos \phi_e$ is then 0·8, and this value of the power-factor is commonly adopted in specifications for alternators. In ordinary use, however, the motors will seldom all be fully loaded, and the average value of $\cos \phi_e$ may for long periods not be more than 0·6 to 0·7. Next, when a large motor is switched on, its starting current is high and lags behind the terminal voltage by a considerable angle ; if the effect of this is to cause a large drop in the volts of the generator, it will seriously interfere with the steadiness of light in a mixed system of combined incandescent lamps and motors, or with motors only may cause these to stop, since their torque is proportional to the square of the terminal pressure. It is not, however, only in motor or combined light and power systems that it may be necessary to consider the behaviour of the alternator under low power-factors. With a number of small transformers, each lightly loaded as may be the case during the daytime, the power-factor of a lighting installation may fall as low as 0·3 or even to 0·2. The full voltage and current of an alternator which is run to supply the network in the daytime, might then be taken up in magnetizing transformers, although its true power in watts would be quite small. An analogous but opposite case is that of a long transmission line for high voltage, which may take the whole apparent power of one machine to supply a leading current to charge it when open.[1]

§ 3. **The inductance of an alternator armature.**—The cause of the reduction in the terminal voltage of an alternator under a given load of defined character as compared with the open-circuit voltage for the same excitation and speed is partly (*a*) ohmic, due to the loss of volts over the ohmic resistance of the winding, but chiefly (*b*) magnetic or inductive, due to the effect of the alternating current as linked with a magnetic field. The " inductance " of the armature is often used as a general term to cover all effects connected with the flux in the magnetic circuit or circuits upon which the alternating current acts, and it may then be subdivided into three principal heads, according as it is related to (1) a real self-induced flux arising in local circuits independent of the main magnetic circuit, (2) an alteration of the *distribution* of the resultant main field over the pole-pitch, or (3) an actual alteration in the *total number* of resultant lines flowing through the main magnetic circuit for the same excitation. If with a constant field excitation and with the rotor held fast, the armature currents in all phases were gradually brought up by means of regulators to their normal simultaneous values under full load with some given angle of lag, and if the lines of flux were visible under the process there would be seen a progressive diminution in the number of lines that actually penetrate to the roots of the teeth of the stator core, and also a progressive alteration in their distribution ; the lines in the air-gap would become less radial, and more slanting in direction, a transversal component having arisen, shunting them as it were peripherally round the air-gap and, although lessened in total number, a greater proportion would leak across between salient poles.

Although each of the items (1), (2), and (3) forms one component of the total inductance of the armature, in the sense that the withdrawal of the armature current would cause a change of the linked flux and a consequent self-induced E.M.F., yet from their different origin they bear different characters and require to be clearly distinguished. The two latter may together be termed the " armature

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[1] Thus to charge 240-mile, 150,000-volt transmission lines of the Southern California Edison Co. when open, about 24,000 kVA are required, which is a considerable overload for one hydro-electric 17,500 kVA generator. And with only one generator the line-charging current self-excites the generator without any field-current up to a voltage above the normal. (*Trans. Amer. I.E.E.*, Vol. 39, Part II, pp. 1635 and 1637.)

reaction " in the proper sense of the words, yet part of their action is often expressed through the medium of a supposed self-induced flux which brings it into line with (1) and renders an analogous treatment of it correct at least as a first approximation to the real truth.

The complete vector diagram for the E.M.F. of an alternator when giving current has now to be built up out of the several component items, and the effect of the " armature reaction " will be found to take two somewhat different forms in the non-salient-pole machine and the machine with definite salient poles.

§ 4. The vector diagram of the internal or armature E.M.F.—The first steps in the process are, however, common to every type of alternator whether its rotor has salient poles, is semi-salient with unslotted pole-centre, or is truly non-salient with a uniformly slotted rotor and an air-gap of uniform length and reluctance all round it. The active external voltage is itself the sum of the volts lost over the ohmic resistance of the external circuit and of any E.M.F. which is absorbed in doing useful work against a back E.M.F., as *e.g.* in the primary coils of a transformer or the armature of an alternating-current motor. The vector sum of this active external voltage with the voltage expended in overcoming the reactive E.M.F. due to the reactance of the external circuit, the two being in quadrature, yields the terminal voltage with its phase angle ϕ_e relatively to the current vector. Next, the internal E.M.F. of an armature phase must contain an item corresponding to the loss of volts over its own ohmic resistance, and the vector of this item will be in phase with the current I.

In regard to the ohmic loss, a further effect due to non-uniform distribution of the current, or, as it is also expressed, to " eddy-currents " in the armature conductors, must here be mentioned, and, as will be shown in Chapter XXVIII, § 6, this effect may become of considerable magnitude in the case of solid bars of great depth and thickness. It may be taken into account by assuming the apparent resistance r_a' of the armature to be higher than its true ohmic resistance to a steady current. Thus if r_a be the real resistance of one phase of the armature winding, r_a' may be estimated by calculation to be, say, $1\frac{1}{2}$ times r_a. The armature current will further set up eddies in the armature discs and their cast-iron case, and, so far as these eddies are in phase with the armature current, they add to the apparent resistance and consume energy; they are, however, very largely wattless, and their actual effect is rather to reduce the net magnetizing force of the armature turns, for they are almost exactly opposed to the armature ampere-turns and, therefore, act like the closed secondary of a transformer or an amortisseur winding.[1]

Next, when a current is flowing through the coils of the armature there arises a certain flux linked with the ends of the coils, but not passing through the main magnetic circuit; the lines of this flux are confined to local circuits almost entirely outside the active length of the core. Being magnetically independent of the main circuit, their distinguishing feature is that they are independent of the position of the coils relatively to the poles. As the current alternates, these lines set up a self-induced E.M.F. e_{sa} proportional to the inductance $\mathcal{L}_a$ of the end-windings of a phase. Assuming $\mathcal{L}_a$ to be constant, which is permissible since the local paths if not

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[1] *Cp.* A. Potier, *L'Eclairage Electrique*, Vol. 24, p. 138.

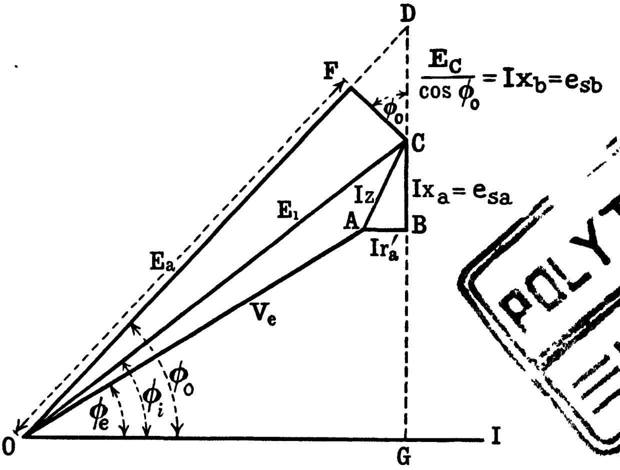

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FIG. 508.—Vector diagram of E M F.'s of alternator per phase.

entirely in air are in iron which is far from any saturation, we thus have with a sinusoidal rate of change of current an end-connexion reactance per phase $x_a = 2\pi f \mathcal{L}_a$, and corresponding thereto a reactance voltage of virtual value Ix_a, lagging in time-phase 90° behind the current in the armature, which must be counter-balanced by an equal voltage e'_{sa} impressed by the main field. This latter E.M.F. or the *voltage expended in overcoming the end-connexion reactance*, is thus numerically equal to e_{sa} or Ix_a, but *preceding* the phase of the current by 90°, and to produce it an equivalent number of lines of the main field will be called for.

Given the virtual value of the terminal voltage V_e per phase as OA inclined to the current vector OI at the angle ϕ_e (Fig. 508), from A draw AB parallel to OI, and of length $= Ir_a'$, and at right angles to AB draw the perpendicular $BC = Ix_a$. The resultant of AB and BC, or AC, is the vector of the E.M.F. consumed by the true armature impedance, or Iz_a, and the resultant of OA and AC, or OC, is the virtual value of the E.M.F. E_i actually induced in the armature winding per phase in magnitude and phase. Its phase angle relatively to the vector of current is ϕ_i. Since this

E.M.F. is actually produced internally in the active lengths of the armature conductors, the real flux in its resultant state of distribution in the teeth and slots where it is linked with the armature conductors must be proportional thereto, due regard being paid to the form-factor of the E.M.F. Strictly speaking, the representation of the complete E.M.F. by a single rotating vector of constant length implies that there are no higher harmonics in the flux curve or loop E.M.F.; either, therefore, the fundamental of the flux curve and of the E.M.F. must be isolated for sole consideration, and the harmonics be afterwards considered separately, or the actual E.M.F. with higher harmonics must be replaced by an equivalent sine wave. But in the practical treatment of the problem it has been shown (Chapter IX, § 14), that the harmonics have no appreciable effect on the virtual E.M.F., although in other ways important.

When the flux curve is distorted so that it becomes more peaked under the action of the current, the form-factor of the E.M.F. is increased. There thus arises a distinction between the internal E.M.F. E_i and the E.M.F. E_a that would be due to the same number of lines Φ_a through the armature if undistorted and distributed similarly in the two halves of each field. It will be seen later that this distinction between E_i and E_a, although clearly present in the salient-pole alternator, does not apply to nearly the same degree in the non-salient-pole alternator so that E_a and E_i may in the latter often be treated as identical, especially when the rotor is not highly saturated.

§ 5. The representation of flux, E.M.F. and current by vectors.—In either case, however, there is a certain real flux to which the sinusoidal E_i or E_a is due, and it suffices here to state that this sinusoidal flux Φ_1, or more strictly its maximum density B_{g1}, may be represented by a vector 90° ahead of the E.M.F. to which it gives rise (cp. F_g preceding E_i in Fig. 519). The radius vector representing the sinusoidal E.M.F. is best conceived as coinciding spacially with the *axis* of a drum coil when its two sides are together cutting the greatest number of lines, so that it lags 90° behind the radius vector of the sinusoidal field to which it is due. In the case of long coils the vector of an E.M.F. may equally well be made to coincide with the position of a *coil-side* when cutting the maximum flux-density B_{g1}, but this ceases to be true in the more general case of coils of fractional pitch. It is, therefore, best to adopt the former convention, which agrees with the definition of E as $= -\frac{dN}{dt}$, where N is the number of linkages of lines with the coil. Correspondingly the vector of a current I must spacially be made to coincide with the position of the *axis* of a coil when carrying maximum current.

§ 6. The slot inductance.—Putting on one side the true end-connexion inductance, the problem of the value and distribution of the main resultant field can, strictly speaking, only be solved by writing down the M.M.F. of the armature ampere-turns acting within each stator slot and the M.M.F. of the rotor field-winding, in their respective positions at any moment of time, and thence by a process of trial and error finding the corresponding flux when due account is taken of the resulting reluctivity of each portion of the iron circuit. In the non-salient-pole machine with cylindrical rotor carrying exciting coils embedded in slots, the M.M.F. curve of the field-winding will be the stepped trapezium of Fig. 304 or 521. An additional datum will be that in the non-salient-pole machine the surface of the unwound pole-centre, and in the salient-pole machine the face of the salient pole, must be in each case at a uniform magnetic potential.

The difficulty then arises as to the exact location of the armature ampere-conductors or of the axis of a coil when carrying the maximum current **I** relatively to the pole-centre for any given value of ϕ_e. This relative position, as will be seen later, is vital to the problem. There is, however, one further portion of the armature effect which may be treated as independent of the position of the armature coils relatively to the poles, and this is the effect corresponding to what would be the *slot inductance* of the armature winding in the absence of any rotor or rotor field-winding.

At any particular instant of time, let AT_s be the ampere-conductors within a stator slot bounded by two teeth over which the ampere-turns AT_x' and AT_x are respectively expended, while AT_c are the ampere-turns expended over the tooth-pitch at the bottom of the slot in the stator core. Then on the analogy of the similar case of the rotor slot (Chapter XVIII, § 7), the difference of magnetic potential acting across the stator slot at any level x is in ampere-turns

$$AT_s \frac{x}{h_s} + AT_c \frac{w_s}{t_s} + (AT_x' - AT_x) \quad . \quad . \quad . \quad (233)$$

The only new feature is that AT_s now varies in slots carrying currents of different phases. Calculating the values of this expression and multiplying by the permeance of a slot and wedge, a series of transverse component fluxes are obtained. The differences between the transverse components of adjacent slots escape radially up or down the intervening stator tooth, increasing or reducing the main flux, but on the whole always reducing it, and reducing the total number of linkages, since the flux passing the roots of the stator teeth is less than that which has entered at the stator face. The change in the peripheral direction of the stator transverse slot flux occurs at right angles to the coil-sides carrying the maximum current, *i.e.* along the axis of this coil or coils and coinciding with the position of the radius vector I.

In the 3-phase alternator the two crucial moments of time, which have been emphasized in Chapter XXIV, § 3, (1) when the current in one phase, say, A, is at its maximum and in the two other phases has half this value, and (2) when the current in one phase is zero and in the two other phases is $\frac{\sqrt{3}}{2}\mathbf{I}$ might be taken, and two pictures would thus be obtained, differing slightly. But practically it suffices to consider only the fundamental of the joint M.M.F. of the armature ampere-conductors of all phases as housed in small groups within the stator slots, and the determination of this fundamental forms the subject of the next sections.

Further, the influence of the core and tooth-reluctance AT of the stator is but small, so that the result becomes *independent of the relative position of the poles.* We thus return to an ideal system of transverse slot fluxes with differences passing up or down the stator teeth proportional to $AT_s' - AT_s$ in any tooth. These differences yield a true sine curve of radial flux and an E.M.F. in strict quadrature with the current.

Lastly, calculation shows that the result so obtained corresponds with but small divergencies to that which is obtained by calculating the slot inductance and the reactance due thereto for a single phase alone as giving rise to an independent self-induced slot flux.[1] It is, therefore, justifiable to treat the slot effect by an addition of slot inductance to the true inductance of the end-connexions, and to embrace both under a joint term $\mathcal{L}_a$ giving reactance $2\pi f \mathcal{L}_a$ and the reactive balancing voltage $Ix_a = e_{sa}$, shown in its correct position in the vector diagram of the internal E.M.F. (Fig. 508), and this course is henceforth assumed.

§ 7. The varying amplitude of the fundamental sine-wave of armature M.M.F. from a single phase.—Before proceeding to the consideration of the influence of the relative position of coils and poles as modifying the remaining inductive effects of the true "armature reaction," the M.M.F. system of the armature ampere-conductors must first be considered *independently of the poles.* The next step must, therefore, be to obtain a clear understanding of the spacial and temporal distribution of the magnetic potential round the bore of the stator due to the armature ampere-turns for any given winding placed in slots or distributed over its surface, when the alternating current which these carry is assumed to be sinusoidal.[2]

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[1] The two methods have been compared by the writer in his paper on "The Flux-wave of the Turbo-Alternator" in *Papers on the Design of Alternating-current Machinery* (pp 219 238) by Hawkins, Smith and Neville, where the subject is more fully treated.

[2] *Cp.* J. B. Henderson and J. S. Nicholson, "Armature Reaction in Alternators," *Journ. I.E.E.*, Vol. 34, p. 465 ; and also especially B. Hague, *Journ. I.E.E.*, Vol. 55, p. 489, and Dr. A. E. Clayton, *Journ. I.E.E.*, Vol. 61, p. 749, where the subject is fully discussed.

Consider in the first place long coils with a mean span equal to the pole-pitch, arranged on the internal surface of an armature, concentric with which is an iron cylinder representing the field-magnet system. The exact nature of the latter need not at present be further defined, but will be again referred to later. The coils in Figs. 509–511 are shown undivided, but may equally well be divided. Describing closed paths round the coil-sides as shown by the dotted lines of Fig. 509, and assuming the iron of both stator and rotor to have negligible reluctance, it is seen that from considerations of symmetry with the rotor surface assumed to be at zero potential, half of the total M.M.F. of the coil at any moment may be assigned to the air-path within and half to the air-path outside the span of a coil, and that within each of these two regions the magnetic potential along the face of the bore will be uniform, the sign changing when a slot is passed.

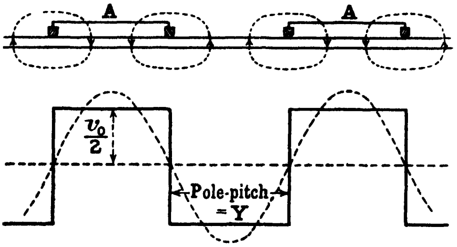


FIG. 509.—M.M.F. of coil with one slot per side.

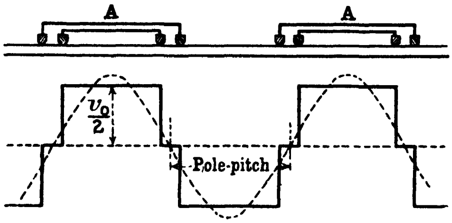


FIG. 510.—M.M.F. of coil with two slots per side.

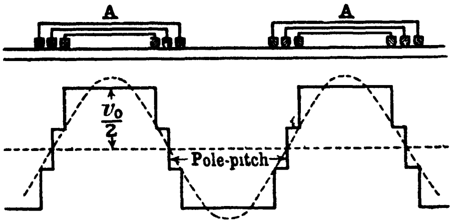


FIG. 511.—M.M.F. of coil with three slots per side.

(a) *The joint fundamental of M.M.F. from the coils of a single phase with a constant current of one ampere.* In the first place, let a single-slot coil of span equal to the pole-pitch be taken, and let it contain t/g' turns, where t is the number of conductors in the belt per pole of a single phase, and there are g' slots per pole per phase. Let it carry the constant current of one ampere. Then if the spacial distribution of the magnetic potential in direction and magnitude along the face of the armature bore is plotted on a straight line base, a series of rectangles of width equal to the pole-pitch and of height proportional to half the ampere-turns of the coil (*i.e.* in this case $t/2g'$) is obtained (Fig. 509). As each slot is passed, the magnetic potential at the air-gap undergoes an increase from a negative to an equal positive value, or a similar decrease, the change in either case being equal to 1·257 times the ampere-conductors within the slot, and this increase or decrease is so rapid that it may practically be identified with a vertical rise or fall coinciding with the centre line through the slot.

As is well known, the amplitude of the fundamental sine curve of a rectangle is $\frac{4}{\pi}$ times the height of the rectangle; hence the amplitude of the fundamental in the present case is $\frac{4}{\pi} \times \frac{t}{2g'} = \frac{2}{\pi} \frac{t}{g'}$ ampere-turns.

When there are two or more slots in the coil-side, the sides of the joint curve of M.M.F. for the assumed constant current become stepped as in Figs. 510, 511. Now without alteration of their magnetic effect, the end-connexions of the entire belt of g' slots, if wound concentrically, can be re-arranged (Fig. 472) so that they are coupled up into g' single-slot coils, each of full pitch and each containing $\frac{t}{g'}$ turns; each, therefore, will have its own rectangle of height $t/2g'$, and its own fundamental of amplitude $\frac{2}{\pi} \times \frac{t}{g'}$. These component fundamentals, being sinusoidal in shape and exactly similar, can be added as vectors mutually displaced spacially by angles of γ_e radians, and by the rules for the addition of g' vectors uniformly displaced (Chapter IX, § 12), the amplitude of the joint fundamental is equal to g' times the amplitude of a component multiplied by k_s'. Therefore for the belt as a whole when carrying one ampere the amplitude of the joint fundamental is

$$t \times c_A = t \times \frac{2}{\pi} k_{s1}'$$

(b) *The joint fundamental of M.M.F. from the coils of a single phase with an alternating current.* Next, instead of a direct current, let a sinusoidal alternating current of virtual value I/q flow through the coils.[1] Then the vertical heights of the original rectangles will vary periodically, and in synchronism temporally with the current, between equal positive and negative maxima values, *i.e.* the curves will shrink or grow vertically up or down in proportion to the current. To avoid the necessity of continually repeating the combination of the quantities giving the maximum ampere-turns of the whole belt, let $t\sqrt{2}I/q = v_o$; then the maximum height of each component rectangle measured in ampere-turns is $\frac{v_o}{2g'}$ and of the stepped curve for the magnetic potential from the joint coil is $v_o/2$, one-half of the M.M.F. being expended over the air-gap of each pole-pitch. It follows that the maximum amplitude of the fundamental of the magnetic potential at the moment of maximum current will be

$$\mathbf{a} = c_A\, v_o = \frac{2}{\pi} k_{s1}'\, v_o$$

and the instantaneous amplitude will be

$$a = \mathbf{a} \sin \frac{2\pi t}{T}$$

where T is the periodic time of the current, and t is the time reckoned from the moment when the current is passing through zero from a negative to a positive value. Thus, on the one hand the amplitude of the instantaneous fundamental for the sinusoidally-varying current of a single-phase is itself a periodic function of the *time*, and on the other hand the magnetic potential along the face of the armature for any height of the fundamental is a periodic function of *space*, the space-period being equal to twice the pole-pitch or $2Y$. In Figs. 510–511 the fundamentals of the joint magnetic potential for the stepped rectangle of height $= v_o/2$ are shown by dotted curves.

Let x be the distance from a coil-side of any particular point P on the armature, reckoned positively in the direction of rotation of the rotor or negatively if against it, and let a current directed away from the observer be positive, so that zero time corresponds to the moment when the current changes its direction from towards the observer to away from him. Then the full expression for the magnetic potential $v_{x,t}$ (expressed in ampere-turns) for the point P on the armature at time t is

$$v_{x,t} = c_A \sin \frac{2\pi x}{2Y} \cdot v_o \sin \frac{2\pi t}{T}$$
$$= \mathbf{a} \sin \frac{2\pi t}{T} \cdot \sin \frac{2\pi x}{2Y}$$

---

[1] The divisor q takes into account the case of a phase divided between q parallel paths covering p/q pole-pairs.

which will be found to yield positive values when the M.M.F. acts from stator to rotor, or the magnetic potential of the stator is above the assumed zero potential of the rotor.

Taking any particular value of t and keeping it constant, while x is varied, the fundamental sine wave of the instantaneous spacial distribution of the magnetic potential round the armature at that moment is obtained. Taking any fixed point P on the stator bore and varying t, we obtain the temporal variation of the potential at that spot, which when plotted on a time base is also sinusoidal.

Starting with any point x on the stator bore at time t, let the point P on the rotor opposite to x at time t be marked. Now, the current and voltage pass through a periodic cycle in time T when the rotor moves over a space $2Y$; if, therefore, another point be taken on the stator at the farther distance $x + x'$ and at a later time $t + t'$, the additions bearing respectively the same proportion to the double pole-pitch and the periodic time, *i.e.* so that $t'/T = x'/2Y$, the new point on the stator will again be exactly opposite to the marked point P on the rotor. If α be the electrical angle through which the rotor has moved between the two stages, $t' = T\frac{\alpha}{2\pi}$ and $x' = 2Y\frac{\alpha}{2\pi}$ The magnetic potential of the armature in relation to any point marked on the rotor and moving with it is then

$$v' = \mathbf{a}\Big\{\sin 2\pi\frac{t}{T} \,.\, \sin 2\pi\frac{x}{2Y} \,.\, \cos^2\alpha + \Big(\sin 2\pi\frac{t}{T} \,.\, \cos 2\pi\frac{x}{2Y}$$

$$+ \cos 2\pi\frac{t}{T} \,.\, \sin 2\pi\frac{x}{2Y}\Big)\sin\alpha \,.\, \cos\alpha + \cos 2\pi\frac{t}{T} \,.\, \cos 2\pi\frac{x}{2Y} \,.\, \sin^2\alpha\Big\}$$

If now we start with the marked point on the rotor opposite to the coil-side carrying zero current, $t = 0$ and $x = 0$. Therefore $v' = \mathbf{a}\sin^2\alpha$, *i.e.* above a marked point on the rotor which is opposite to a coil-side when its current changes from towards to away from an observer, the magnetic potential of the armature is proportional to the square of the sine of the angle moved through and is always positive. The average value of $\sin^2\alpha$ being $\frac{1}{2}$, its average value is $\bar{a} = \frac{1}{2}\,\mathbf{a}$.

§ 8. The constant amplitude of the fundamental sine wave of armature M.M.F. in the polyphase machine.—In the polyphase machine the other phases have also to be taken into account in their relative phases of time and space. Thus, in a quarter-phase machine if t and x refer to phase A

$$v_{A1} = \mathbf{a}\sin\frac{2\pi t}{T} \,.\, \sin 2\pi\frac{x}{2Y}$$

$$v_{B1} = \mathbf{a}\sin 2\pi\left(\frac{t}{T} - \frac{1}{4}\right) . \sin 2\pi\left(\frac{x}{2Y} - \frac{1}{4}\right)$$

and adding the two together

$$v = v_{A1} + v_{B1} = \mathbf{a} \,.\, \cos 2\pi\left(\frac{t}{T} - \frac{x}{2Y}\right)$$

Similarly in a 3-phase alternator

$$v_{A1} = \mathbf{a} \sin \frac{2\pi t}{T} . \sin 2\pi \frac{x}{2Y}$$

$$v_{B1} = \mathbf{a} \sin 2\pi \left(\frac{t}{T} - \frac{1}{3}\right) . \sin 2\pi \left(\frac{x}{2Y} - \frac{1}{3}\right)$$

$$v_{C1} = \mathbf{a} \sin 2\pi \left(\frac{t}{T} - \frac{2}{3}\right) \sin 2\pi \left(\frac{x}{2Y} - \frac{2}{3}\right)$$

and adding the three together

$$v = v_{A1} + v_{B1} + v_{C1} = 1{\cdot}5\, \mathbf{a} \cos 2\pi \left(\frac{t}{T} - \frac{x}{2Y}\right)$$

or in general for N_{ph} phases symmetrically loaded,

$$v = \frac{N_{ph}}{2} \mathbf{a} \cos 2\pi \left(\frac{t}{T} - \frac{x}{2Y}\right) = A \cos 2\pi \left(\frac{t}{T} - \frac{x}{2Y}\right)$$

Taking any particular value of t reckoned from the time when the current in phase A is zero and in a certain coil-side of that phase is passing from the negative to the positive direction, if t is kept constant while x is varied, the spacial distribution of the magnetic potential round the armature is found to remain sinusoidal as before, so far as the joint fundamental is concerned. Similarly the temporal variation for any fixed point on the stator at distance x from the coil-side remains sinusoidal. But whatever time t and distance x is taken, the value of v falls on a cosine curve of *constant* amplitude $A = \frac{N_{ph}}{2} \mathbf{a}$ in contrast to the previous case of a single phase.

§ 9. The movement of the fundamental sine wave of armature M.M.F. in the polyphase machine.—Applying to the polyphase machine the same procedure as was before applied to the single phase, and starting with

$$v = A \cos 2\pi \left(\frac{t}{T} - \frac{x}{2Y}\right)$$

as the magnetic potential opposite to some point P marked on the rotor and at some time t, at any later time $t + t'$ the potential opposite the same point on the rotor is

$$v = A \cos 2\pi \left(\frac{t}{T} + \frac{t'}{T} - \frac{x}{2Y} - \frac{x'}{2Y}\right)$$

But let $\frac{t'}{T} = \frac{x'}{2Y}$; then

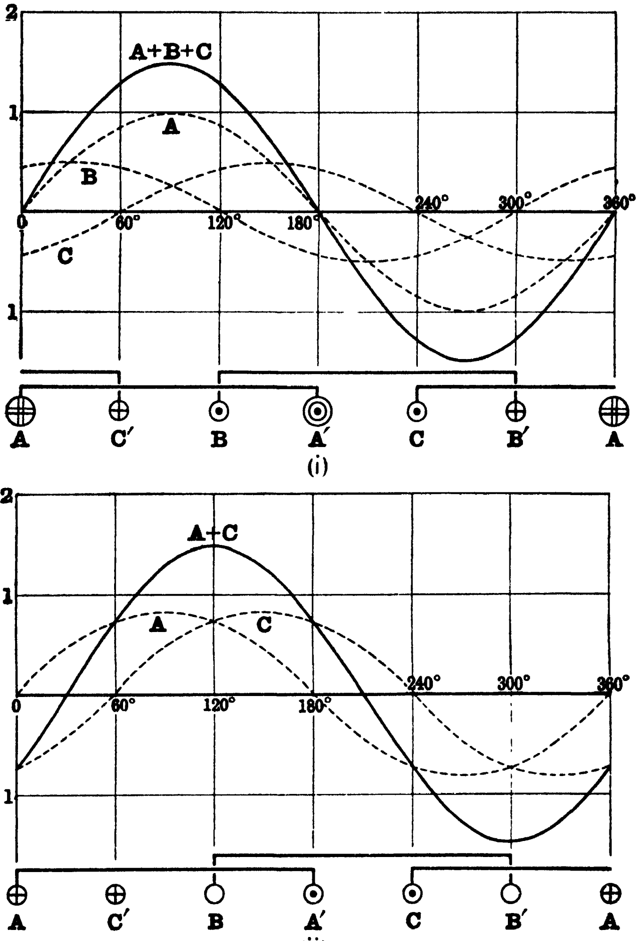
$$v = A \cos 2\pi \left(\frac{t}{T} - \frac{x}{2Y}\right)$$

i.e. exactly the same value as for point P. Hence, above a marked spot on the rotor the joint fundamental always has the same value, which shows that the joint fundamental sine-wave travels round the armature in the direction of rotation and at exactly the same constant speed as the poles. It is, therefore, stationary in relation to the rotor poles, although moving relatively to the armature or stator.

§ 10. Illustration of the uniform speed of movement of the fundamental sine wave of M.M.F.—Thus the joint fundamental wave, F_{AR}, from the several phases in combination does *not* shrink and grow vertically, like its components, but *travels round the armature* in space and time, with a constant amplitude which is numerically $A = \frac{N_{ph}}{2}\mathbf{a}$. This important result is simply illustrated by considering a 3-phase armature at the two crucial moments of time described in Chapter XXIV, § 3. Figs. 512 (i) and (ii) give the component fundamentals of the magnetic potential at these two moments. In the first case, $\frac{t}{T}$ for, say, coil-side A is $\frac{1}{4}$, the double crosses and circles indicating the maximum current in its conductors; at the same time the phase of the current in B and C is given by $2\pi\left(\frac{1}{4}-\frac{1}{3}\right)$ and $2\pi\left(\frac{1}{4}-\frac{2}{3}\right)$ respectively, so that the amplitudes of their component fundamentals are each one-half of that due to A. Their spacial values are $-\frac{1}{2}\mathbf{a}\sin 2\pi\left(\frac{x}{2Y}-\frac{1}{3}\right)$ and $-\frac{1}{2}\mathbf{a}\sin 2\pi\left(\frac{x}{2Y}-\frac{2}{3}\right)$, and their addition to $\mathbf{a}\sin 2\pi\frac{x}{2Y}$ from phase A yields the joint fundamental of amplitude $\frac{3}{2}\mathbf{a} = 1{\cdot}5\,\mathbf{a} = A$ for the 3-phase case.

In the second case, $\frac{1}{12}$th of a period later, $t = \frac{1}{3}T$, so that the amplitudes of A and C are each $0{\cdot}866\,\mathbf{a}$, the former having shrunk and the latter risen, while the component due to B has sunk to zero. Again their addition gives a joint fundamental of amplitude $1{\cdot}5\,\mathbf{a}$, and it is seen that this occurs at 120° instead of at 90°, so that it has moved forward $\frac{1}{12}$th of a space-period.

§ 11. The phase of the vector of the fundamental sine-wave of armature M.M.F. relatively to the current.—The ordinate v has its maximum positive value when $t/T = x/2Y$, *i.e.* for a point on the armature which is distant from a coil-side A by the same fraction of the double pole-pitch that the time is of the period. When $t/T = \frac{1}{4}$, the positive amplitude occurs at $x/2Y = \frac{1}{4}$, *i.e.* 90 electrical degrees from the coil-side for which $t/T = \frac{1}{4}$, and in which, therefore, the current is a maximum in the positive direction. In other words, the positive amplitude of the fundamental sine-wave is found to stand centrally over the axis of a coil at the instant when the current in it is a maximum. The preceding proof has for simplicity assumed long coils, so that the centre of a coil-side occurs at 90 electrical degrees from the axis of the coil, but the last statement is true even if the coils are shortened and their pitch is fractional. If the position of the radius vector of current *I* is made to coincide with the position of the *axis* of a coil when carrying maximum current, as recommended in § 5, it follows that the vector F_{AR}, representing the constant amplitude *A* of the joint fundamental sine wave of the armature M.M.F., coincides with the vector of current in a phase (Fig. 513).



(i)

(ii)

FIG. 512.—The component and joint fundamentals of M.M.F. in a 3-phase alternator
(i) at the moment when *A* carries maximum current and
(ii) one-twelfth of a period later.

If with long coils the vector of an E.M.F. is made to coincide with the position of a *coil-side* when cutting the maximum flux-density which corresponds to the E.M F., the current vector will then coincide with the position of a coil-side when cutting the maximum resultant B_{g1}, and it results that the vector of F_{AR} lags 90° behind the vector of the current. This method in the case of long coils has the practical advantage that it shows more quickly the angle of lag of the maximum current in a coil-side behind the centre of a pole. But, as already shown, in order to meet the more general case of coils of fractional pitch, it is best to adhere to the first-mentioned method, which is the more strictly correct in theory.

Thus if ϕ_0 is the angle by which the vector of the current lags behind or leads in front of the interpolar line of symmetry, reckoned negative in the former case and positive in the latter case, the maximum ordinate of the resultant fundamental sine curve is displaced behind or before the centre of a pole by the angle $(90° - \phi_0)$. Its exact position relatively to the field poles will be shown later to depend on the power-factor of the entire circuit. If the power-factor of the external circuit were unity, and there were no armature inductance so that $\phi_0 = 0°$, the maximum ordinate would coincide with the dividing line between the poles; if the power-factor of external circuit and armature were zero, so that $\phi_0 = -90°$, the maximum ordinate would stand over the centre of a pole.

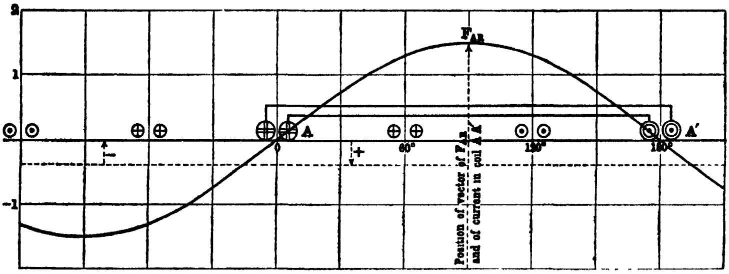


Fig. 513.—Position of vector of F_{AR} relatively to the phase carrying maximum current.

§ 12. The special case of uniform distribution in the three-phase alternator.—For uniform distribution of each coil-side in a 3-phase machine over 60 electrical degrees, $k_{s1}' = 3/\pi$ and A assumes the special value $\frac{3}{2} \times \frac{2\sqrt{2}}{\pi} \times \frac{3}{\pi} Jt = \frac{9}{\pi^2} \sqrt{2} Jt$. It will be of interest to check the above result by consideration from first principles of the special case of such a 3-phase machine. It is in practice nearly reproduced when there are several slots per pole per phase as in the stator of a turbo-alternator, and its simplicity enables the mathematical result to be easily followed. Let the curve of the spacial distribution of the amperes or ampere-conductors round the bore be first plotted on a straight-line base representing one double pole-pitch for the same two particular moments of time as in § 10. The first moment yields the stepped curve of Fig. 514 (i). The second selected moment yields two rectangles, each extending over 120 electrical degrees, separated by gaps of 60 electrical

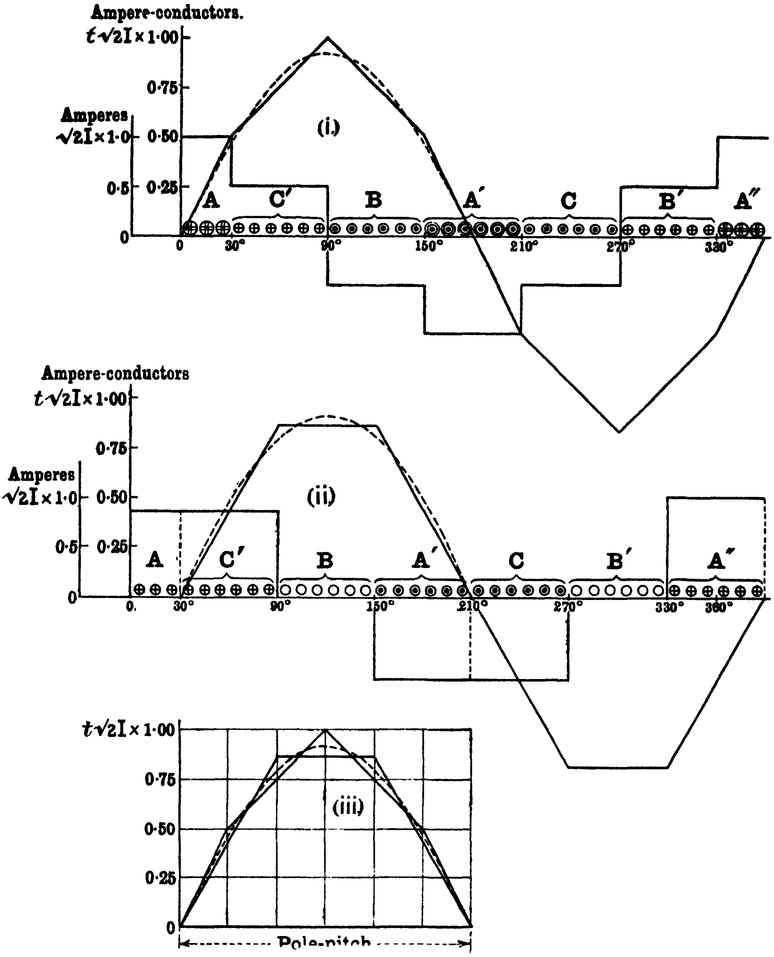


FIG. 514 —The varying shape and constant fundamental of the M.M.F. of a 3-phase alternator, with uniformly distributed conductors, at two moments of time in a period.

degrees Fig. 514 (ii). Next, in each case let the ampere-conductors be summed up, and let their sum to which the M.M.F. at any point is proportional be plotted as ordinates on the same base-line. In the first case for the starting-point of the summation is taken the middle of a coil-side belonging to phase *A*, in which the current is at its maximum as shown by the double crosses and circles; the ampere-conductors rise to $\frac{t}{2}\mathbf{I}$ in 30°, and then more gradually to a peak of $t\mathbf{I}$ after a further 60°. In the second case, where the start is made 30° later or from the edge of coil-side C', *i.e.* from the middle of a rectangle where coil-sides A and C' adjoin, the ampere-conductors rise uniformly from zero to $t \times \frac{\sqrt{3}}{2}\mathbf{I}$ at 60°, remain constant at this value for 60°, and then sink again to zero at the end of the pole-pitch, so forming a trapezium.

The simple mathematical shape of the curves enables them to be easily resolved into Fourier series, and if as before,

$$\frac{v_o}{2} = \frac{t}{2}\frac{\mathbf{I}}{q} = \frac{t\sqrt{2}\,I/q}{2}$$

for the pointed curve

$$v = \frac{18}{\pi^2}\cdot\frac{v_o}{2}\left(\sin\theta + \frac{1}{25}\sin 5\theta - \frac{1}{49}\sin 7\theta + \ldots\right) \quad .$$

for the flat-topped curve or trapezium

$$v = \frac{18}{\pi^2}\cdot\frac{v_o}{2}\left(\sin\theta - \frac{1}{25}\sin 5\theta + \frac{1}{49}\sin 7\theta - \ldots\right)$$

The fundamental of the M.M.F. curve is, therefore, precisely the same in each,[1] and is shown in Fig. 514 (iii); its amplitude measured in ampere-turns is

$$\frac{18}{\pi^2}\cdot\frac{v_o}{2} = \frac{9}{\pi^2}t\sqrt{2}I/q = 0{\cdot}911t\sqrt{2}I/q \quad . \quad . \quad . \quad (234)$$

Since $t = \frac{Z}{2pN_{ph}} = \frac{Z}{6p}$ in the 3-phase case, this expression which gives the effective armature ampere-turns as related to a half magnetic circuit or per pole may also be expressed as

$$0{\cdot}86IZ/4pq = 0{\cdot}86JZ/4p \quad . \quad . \quad . \quad . \quad (234a)$$

The factors which enter into the coefficient $\frac{9}{\pi^2}$ will be recognized from what has preceded as being $\frac{1}{2}\left(\frac{4}{\pi}\times\frac{3}{2}\times\frac{3}{\pi}\right)$, the last factor being that for uniform distribution of the coil-sides over one-third of the pole-pitch.

The M.M.F. pulsates between the two limiting cases (Fig. 514 (iii)) at a frequency 6 times that of the current, and it is evident that it can at any moment be replaced with close accuracy by the fundamental. The variation of the wave-shape of the armature M.M.F. will cause a corresponding small variation of sextuple frequency in the flux curve, but its amount is greatly restricted by the damping action of eddy currents, especially with solid pole-shoes.

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[1] Dr. S. P. Smith and W. H. Barling, *Electr.*, Vol. 74, p. 42.

§ 13. The amplitudes and speeds of the harmonics of the armature M.M.F. in polyphase alternators.—So far only the combined fundamentals of the armature M.M.F. have been considered. So long as the mean span of the coils is equal to the pole-pitch, the two halves of the wave for each component are symmetrical and contain no harmonics whose frequency is an even multiple of that of the fundamental. If, then, there is no dissymmetry in the currents of the two or more phases, there are no even harmonics in the joint result from N_{ph} phases. As is well known, the complete expression for a simple rectangle of height $v_o/2$ resolved into a Fourier series is

$$v = \frac{4}{\pi} \cdot \frac{v_o}{2}\left(\sin\theta + \frac{1}{3}\sin 3\theta + \frac{1}{5}\sin 5\theta + \ldots\right)$$

The amplitudes of the harmonics of the M.M.F. due to a single phase with long coils divided among two or more slots are, therefore,

$$\mathbf{a}_n = \frac{1}{n} \times \frac{2}{\pi} v_o \times k_{sn}' \text{ where } n = 3, 5, 7, \text{ etc.}$$

The time-period T of the current remains unaltered, but the space-period of the moving wave[1] of space order n is $2Y/n$. Hence in a three-phase alternator the component moving waves of the magnetic potential, of length $1/n$ of the pole-pitch, are

$$v_{An} = \mathbf{a}_n \sin\frac{2\pi t}{T} \cdot \sin 2\pi n\frac{x}{2Y}$$

$$v_{Bn} = \mathbf{a}_n \sin 2\pi\left(\frac{t}{T}-\frac{1}{3}\right) \cdot \sin 2\pi n\left(\frac{x}{2Y}-\frac{1}{3}\right)$$

$$v_{Cn} = \mathbf{a}_n \sin 2\pi\left(\frac{t}{T}-\frac{2}{3}\right) \cdot \sin 2\pi n\left(\frac{x}{2Y}-\frac{2}{3}\right)$$

Making $n = 3, 9, 15$ or any other uneven multiple of 3, and adding the three components together, it will be found that they cancel out, so that there are no waves of space order 3, 9, 15 . . . in the joint armature M.M.F. from the fundamental currents of the three sets of coils. But the waves of length 1/5th of the pole-pitch combine to give a resultant

$$v_5 = \frac{N_{ph}}{2}\mathbf{a}_5 \cos 2\pi\left(\frac{t}{T}+\frac{5x}{2Y}\right)$$

Similarly the joint wave of space order 11, time order 1, is

$$v_{11} = \frac{N_{ph}}{2}\mathbf{a}_{11} \cos 2\pi\left(\frac{t}{T}+\frac{11x}{2Y}\right)$$

---

[1] A moving wave which is a function of *two* variables, space and time, is not properly expressible in terms of " harmonics " ; a component due to a current of *m* times fundamental frequency and having $1/n$th of the pole-pitch for its wave-length is a " constituent moving wave, of time order *m*, space order *n*, and speed m/n of synchronism." Thus *e.g.* from a 5th time-harmonic in the current will arise a M.M.F. wave moving at 5 times synchronous speed, and a sinusoidal constituent of this of wave-length 1/5th of the pole-pitch, *i.e.* time order 5, space order 5, moves at synchronous speed.

On the other hand, those of space order 7 or 13, time order 1, are respectively

$$v_7 = \frac{N_{ph}}{2}\,\mathbf{a}_7 \cos 2\pi\left(\frac{t}{T}-\frac{7x}{2Y}\right)$$

$$v_{13} = \frac{N_{ph}}{2}\,\mathbf{a}_{13} \cos 2\pi\left(\frac{t}{T}-\frac{13x}{2Y}\right)$$

The similarity of the expressions for the 7th and 13th to that for the fundamental shows that these waves, like the fundamental, rotate in the same direction as the rotor, but at the corresponding fractions of the synchronous speed.

But from the expressions for the 5th and 11th it will be found that they rotate in the opposite direction to the rotor at one-fifth and one-eleventh respectively of its speed. If in relation, say, to the 5th moving wave at some time t and distance x, the magnetic potential of the stator bore opposite a marked point P on the rotor is

$$\frac{N_{ph}}{2}\,\mathbf{a}_5 \cos 2\pi\left(\frac{t}{T}+\frac{5x}{2Y}\right)$$

the potential opposite the same marked point on the rotor at any later time will be

$$\frac{N_{ph}}{2}\,\mathbf{a}_5 \cos 2\pi\left(\frac{t+t'}{T}+\frac{5(x+x')}{2Y}\right)$$

If $\frac{t'}{T} = -\frac{5x'}{2Y}$, this yields the same value as initially, showing that the constituent of space order 5, time order 1, rotates backwards. The waves are, therefore, divisible into two groups according to their direction of movement.

$$\begin{array}{ccc} & & \text{Movement of rotor.} \\ \longleftarrow\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!- & & -\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!\longrightarrow \\ \text{etc., } 29,\ 23,\ 17,\ 11,\ 5 & & 1,\ 7,\ 13,\ 19,\ 25, \text{ etc.} \end{array}$$

Since all the joint M.M.F. constituent waves (except the fundamental) of the polyphase alternator move relatively to the rotor, they will induce in the field-coils and pole-faces damping currents, and the magnetizing effect that they would otherwise have is thereby neutralized to so great an extent that for most purposes we are justified in dismissing them from consideration and concentrating attention solely on the fundamental.

§ 14. The general case with coils of mean pitch differing from the pole-pitch.—If the original coil with which we started is shortened until its pitch y' is a fraction of the pole-pitch Y, then the curve of its magnetic potential for a constant current of one ampere when plotted on a straight line base, which forms the starting-point, instead of being a pair of equal rectangles as in Fig. 509, is a pair of rectangles of unequal height and width. The total M.M.F. of the single coil will not be divided equally over the two air-gaps, but will be so much greater over the shorter span that the area of the two rectangles may be equal; if actually called into existence, the same flux due to the coil M.M.F. must pass through the two areas, and the greater flux-density in the smaller area will call for a proportionately greater share of the total M.M.F. The two portions of the M.M.F. in ampere-turns are, therefore, $\frac{t}{2g'}\cdot\frac{y'}{Y}$ over a width $2Y-y'$, and $\frac{t}{2g'}\cdot\frac{2Y-y'}{Y}$ over a width y' (Fig. 515). Even

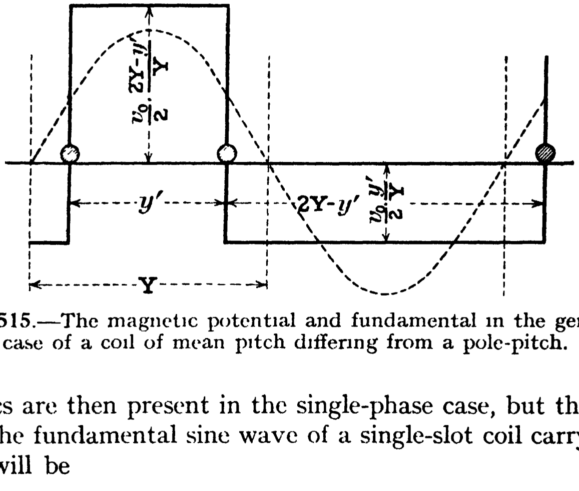

FIG. 515.—The magnetic potential and fundamental in the general case of a coil of mean pitch differing from a pole-pitch.

harmonics are then present in the single-phase case, but the amplitude of the fundamental sine wave of a single-slot coil carrying one ampere will be

$$\frac{1}{\pi}\left[\int_{0}^{\frac{\pi}{2}\cdot\frac{Y-y'}{Y}} -\frac{t}{2g'Y}y'\sin a\,.\,da+\int_{\frac{\pi}{2}\cdot\frac{Y-y'}{Y}}^{\frac{\pi}{2}\cdot\frac{Y+y'}{Y}} \frac{t}{2g'Y}(2Y-y')\sin a\,.\,da\right.$$

$$\left.+\int_{\frac{\pi}{2}\cdot\frac{Y+y'}{Y}}^{2\pi} -\frac{t}{2g'Y}y'\sin a\,.\,da\right]$$

$$=\frac{2}{\pi}\frac{t}{g'}\sin\left(\frac{\pi}{2}\cdot\frac{y'}{Y}\right)$$

Now y' has above denoted the pitch between the centre-lines of two slots of a single component coil. For the total number of g' slots in a belt per pole, whether the coils of the phase are concentric or lap-wound and whether g' be even or uneven, it is only necessary to interpret y' as the *mean pitch between the two belts,* and to multiply the expression for the component fundamental by $g'\ k_{s1}'$ to find the amplitude of the joint fundamental, in accordance with the rules for the addition of vectors displaced by equal angles from each other. The general expression for the joint fundamental of M.M.F. for a continuous current of one ampere in the turns of the belt corresponding to a pole is thus

$$t \times c_{\text{A}} = t \times \frac{2}{\pi}\, k_{s1}'\, k_{s1}'' \quad . \quad . \quad . \quad . \quad . \quad . \quad (235)$$

and for a sinusoidal alternating current is

$$c_{\text{A}}\, v_o = \frac{2}{\pi}\, k_{s1}'\, k_{s1}''\, t\, \sqrt{2}\, J \quad . \quad . \quad . \quad . \quad . \quad (236)$$

for $\sin\left(\frac{\pi}{2}\cdot\frac{y'}{Y}\right)$, when y' is the *mean* pitch between the belts, is

$$\sin\left(\frac{\pi}{2}\lambda_c\right) = \sin\left\{\frac{\pi}{2} - \frac{\pi}{2}\,(1 - \lambda_c)\right\} = \cos\frac{\varepsilon}{2} = k_{s1}'' \text{ as in eq. (33).}$$

§ 15. The general expression for the armature reacting ampere-turns.—Recapitulating the results reached so far, we have found the amplitude of the joint fundamental of the armature M.M.F. in ampere-turns as acting over a pole-pitch or on one-half of a magnetic circuit and as due to a single phase at the moment of maximum current to be

$$\mathbf{a} = c_{\text{A}}\, v_o = \frac{2}{\pi} k_{s1}'\, k_{s1}''\, t\, \sqrt{2}\, J \quad . \quad . \quad . \quad . \quad . \quad (236a)$$

The *average* value for a single phase is

$$\bar{a} = \tfrac{1}{2}\,\mathbf{a} = \frac{1}{\pi}\, k_{s1}'\, k_{s1}''\, t\, \sqrt{2}\, J \quad . \quad . \quad . \quad . \quad . \quad (237)$$

The *constant* value of the joint fundamental amplitude for all phases of a polyphase machine symmetrically loaded is

$$A = N_{ph}\frac{\mathbf{a}}{2} = \frac{N_{ph}}{\pi} k_{s1}'\, k_{s1}''\, t\sqrt{2}\, J$$

or when it is expressed as

$$F_{\text{AR}} = k_{\text{A}}\frac{IZ}{4pq} = k_{\text{A}}\frac{JZ}{4p} \quad . \quad . \quad . \quad . \quad . \quad . \quad (238)$$

$$k_{\text{A}} = \frac{2\sqrt{2}}{\pi}\, k_{s1}'\, k_{s1}'' = \frac{k_{s1}'\, k_{s1}''}{1{\cdot}11} \quad . \quad . \quad . \quad . \quad . \quad (239)$$

Just as in the case of the E.M.F. (eq. 228), with g' slots per pole per phase equally spaced over a width $\frac{1}{N_{ph}}$ th. of the pole-pitch and long coils, $\gamma_e = \pi / N_{ph}\, g'$ radians, and

$$k_{s1}' = \sin g'\frac{\gamma_e}{2} \Big/ g' \sin\frac{\gamma_e}{2} \text{ becomes } \sin\frac{\pi}{2N_{ph}} \Big/ g' \sin\frac{\pi}{N_{ph}g'}$$

Hence in a 3-phase alternator with long coils of mean span equal to the pole-pitch ($k_{s1}''= 1$), we have

with 2 slots per pole per phase $k_A = \frac{2\sqrt{2}}{\pi} \times 0{\cdot}9659 = 0{\cdot}87$

" 3 " " " $= \frac{2\sqrt{2}}{\pi} \times 0{\cdot}96 = 0{\cdot}865$

and with uniform distribution over 60°, $= \frac{2\sqrt{2}}{\pi} \times \frac{3}{\pi} = 0{\cdot}86$

Thus, as the number of slots per pole per phase is increased, the amplitude of the fundamental of the M.M.F. in ampere-turns decreases just as the fundamental of the E.M.F.

Other special cases such as are given in Chapter XXV also follow the same law, and need not be recapitulated.

§ 16. The contrast between the non-salient-pole and salient-pole alternators.—Having thus found the maximum amplitude A or the value to be given to the vector F_{AR} of the fundamental sine-wave of M.M.F. due to the combined result from the armature ampere-turns of two or more phases, its effect on the actual magnetic circuit will take two more or less distinct forms, according to whether (1) the rotor is truly *non-salient, i.e.* is a cylinder carrying exciting coils embedded in slots in its surface that are distributed *uniformly over the whole* of its periphery and are all wound, or (2) the rotor has *salient* poles. A third intermediate case is that of the modern turbo rotor in which the *pole-centre is unslotted* or is differently slotted and in which, therefore, the air-gap is not throughout of uniform reluctance : it may be called the *semi-salient* case.

The special feature which differentiates the first case is that the air-gap is of uniform length and uniform as regards slotting all round the bore. The air-gap reluctance of a pole-pitch is, therefore, very nearly constant wherever the pole-pitch is taken, *i.e.* whatever position the flux occupies. But in the alternator with salient poles if the flux centre shifts sensibly away from the centre of the pole, the air-gap reluctance alters decisively, and by reason of this any displacement of the flux relatively to the centre of the poles implies *distortion* of its distribution.

THE NON-SALIENT-POLE ALTERNATOR

§ 17. The ideal non-salient-pole case.—Although the definite relation that the M.M.F. wave bears to the position of the armature coils carrying at any moment the maximum current has been determined, the question still remains as to where the fundamental wave is to be located in relation to the pole-centre, for any given value of ϕ_e, the external angle of lag.

In the first case of a non-salient-pole machine with rotor uniformly slotted throughout and with all of its numerous slots assumed for the moment to be filled with conductors carrying currents increasing sinusoidally from pole-centre to centre of exciting coil, or with conductors carrying the same exciting current but increasing in number per slot sinusoidally, the situation would be that a sinusoidal wave of M.M.F. of amplitude, say, F_E on the rotor would confront another sinusoidal wave of M.M.F. of amplitude F_{AR} on the other side of the air-gap due to the armature ampere-turns, both waves in the polyphase machine revolving at the speed of the poles and being, therefore, relatively stationary to one another and to the poles. Assuming the rotor core and teeth and also the stator core and teeth to be of negligible reluctance, the resultant magnetic potential F_g acting across the air-gap would be the algebraic sum of the two,[1] and the vector of its amplitude the vectorial sum of F_E and F_{AR}; the reluctance of the air-gap being perfectly uniform at every point, the flux would be sinusoidally distributed, and the vector representing it would coincide in phase with F_g (Fig. 516). To this would correspond E_t, the E.M.F. actually induced in the armature winding, since there would be *no distortion* of the resultant field which would remain sinusoidal whatever the value and

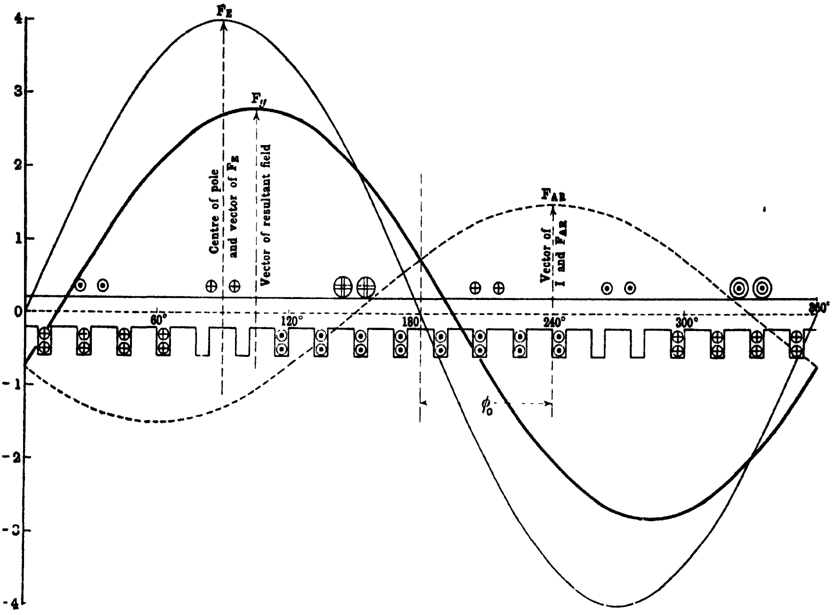

FIG. 516.—The ideal non-salient-pole case, with sinusoidal M.M.F. waves from both rotor and stator ampere-turns.

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[1] For an analogous process applied under short-circuit conditions, *cp.* Dr. A. E. Clayton, *Journ. I.E.E.*, Vol. 54, p. 94.

position of F_{AR} relatively to the given sinusoidal F_E. The vector F_E always marks the centre of a pole, and at right angles to it is always IS, the interpolar line of symmetry.

§ 18. **The displacement of the resultant field under load.**—The consequences of the assumed case can now be traced. At no load, the vector of the induced E.M.F. will upon the convention adopted in §§ 5 and 11 coincide with the line IS drawn 90° behind the pole-centre or behind F_E (Fig. 517). But as soon as any current flows and upon the supposition that the circuit possesses only ohmic resistance, the vector of F_{AR}, coinciding with the vectors of both I and E_i, must be compounded with F_E to yield the resultant M.M.F. or resultant field F_g : hence F_g must fall back behind F_E, the field, although not distorted, is displaced backwards behind the centre of the pole, and correspondingly the vector of $\mathbf{E}_i$ which is at right angles to F_g must fall back behind IS (Fig. 518). There thus arises a negative angle of lag ϕ_0 of the current vector $\mathbf{I}$ behind IS, and to maintain F_g the value of F_E must be increased as shown by the hypotenuse of the right-angled triangle (Fig. 518).

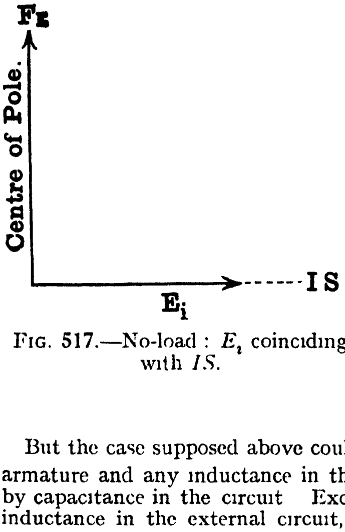

FIG. 517.—No-load : E_i coinciding with IS.

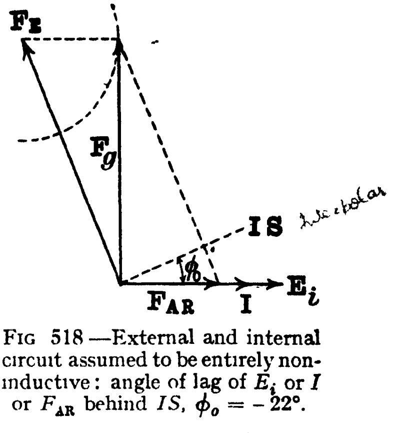

FIG 518—External and internal circuit assumed to be entirely non-inductive: angle of lag of E_i or I or F_{AR} behind IS, $\phi_0 = -22°$.

But the case supposed above could only occur if the inductance l_a of the armature and any inductance in the external circuit were exactly balanced by capacitance in the circuit Except, therefore, in such a rare case, any inductance in the external circuit, assisted by the true inductance of the armature end-connexions and stator slots, will cause I or F_{AR} to lag behind E_i, and will increase the angle of lag ϕ_0 (Fig. 519) ; the triangle then no longer remains right-angled, and the additional effect from the inductance adds greatly to the value of F_E that will be required to maintain F_g constant. The instantaneous position of the vectors shown in Fig. 519 with $\phi_0 = -60°$ corresponds to 105° in Fig. 516, where F_g has its maximum positive value. A difference also now arises between the angle of lag of $\mathbf{E}_i$ and the angle ϕ_0 by which the current vector lags behind IS, the latter being the greater. As the inductance of the load is increased, ϕ_0 increases negatively, until in the extreme ideal case of pure inductance this becomes 90°, when the vector F_{AR} is in line with F_E but diametrically opposed. The resultant M.M.F. F_g is then simply the arithmetical difference between F_E and F_{AR}, *i.e.* the reduced sinusoidal flux still remains with its crest over the centre of the pole as at no load (Fig 520). The vector of $\mathbf{I}$ when in line with F_E implies that the axis of a coil when carrying maximum current coincides with the centre of a pole, and, therefore, its coil-sides embrace the pole. It has its maximum direct effect on the rotor field which in this case is de-magnetizing. The reduction of the E.M.F. is, therefore, due to its de-magnetizing or back M.M.F. increased by a similar but smaller effect from the other phases.

Thus, broadly speaking, the active component of the current in phase with E_i displaces the field, and the reactive component more directly affects its magnitude.

Starting from the condition of pure inductance, as the effective resistance is increased, the resultant flux first falls back behind the centre of the pole. There is, however, a limit to its displacement backwards and to the lag of the E.M.F. behind *IS* for a given current, and this is reached when the phases of E_i and I coincide. Not only is the load then non-inductive, but there must also be just enough capacitance to balance the inductance of the armature winding. This situation has been illustrated in Fig. 518, where $\phi_0 = -22°$, and this is also the angle between F_E and F_g; it is easily seen by the construction that with given values of F_E and F_{AR}, F_g must be the tangent to the circle struck from the end of the vector F_E with radius F_{AR}.

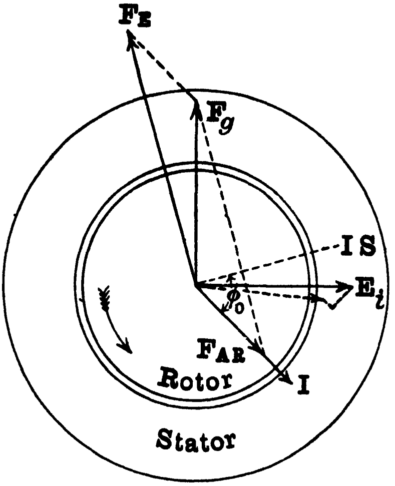

FIG 519—Inductive load: angle of lag of I or F_{AR} behind *IS*, $\phi_0 = -60°$.

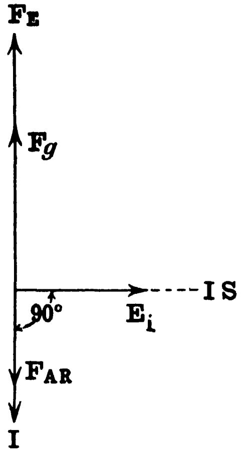

FIG. 520—Load and armature circuit assumed to be purely inductive; I or F_{AR} lagging behind *IS* by $\phi_0 = -90°$, and E_i again coinciding with *IS*.

Meantime the flux has steadily increased, and if the capacitance of the load is increased, so that I precedes E_i, it will increase still further, but now its displacement is reduced. With sufficient capacitance, the resultant flux may not only equal that due to F_E but will exceed it as soon as I precedes *IS*, *i.e.* when the angle ϕ_0 is positive, and the effect of F_{AR} is to assist the magnetization.

The angle ϕ_0 is with long coils the same as the angle between the centre of a *coil-side* when carrying maximum current and the centre of a pole, and it is, therefore, often spoken of as the angle of lag or of lead of the current behind or before the centre of a pole.

§ 19. The fundamental sine wave of M.M.F. in the non-salient-pole machine.—The determination of the fundamental of the M.M.F. of the exciting coils on the non-salient rotor is analogous to the similar determination in the case of the armature ampere-turns, save that now the ampere-turns are disposed on the external convex surface of the rotor, and there is no time-variation of the current to be considered.

If the exciting winding were concentrated within one slot per pole, the magnetic potential curve when plotted on a straight-line base on the same assumption as in § 7 (*a*) would yield a pair of rectangles in a two-pole machine (*cp*. Fig. 509), of height $I_f T_m/2$ in ampere-turns, where I_f = the exciting current and T_m is the number of conductors per pole. The amplitude of the fundamental is then

$$F_E = \frac{4}{\pi} \cdot \frac{I_f T_m}{2} = \frac{2}{\pi} I_f T_m$$

If the winding is perfectly uniformly distributed over the pole-pitch

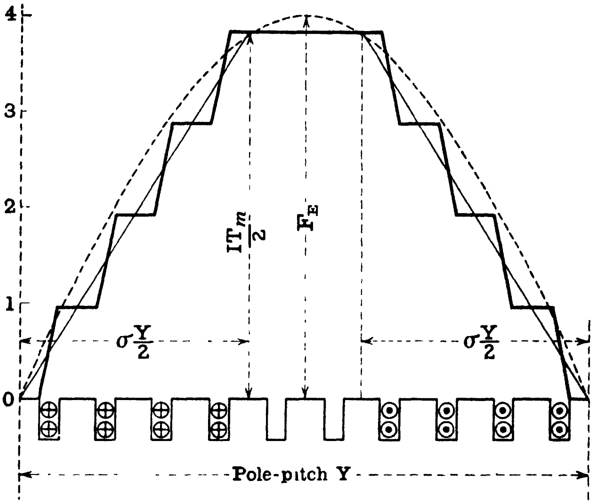

FIG. 521.—The trapezium of M.M.F. in a slotted rotor, and its fundamental.

or $\sigma = 1$, the M.M.F. curve is a pair of triangles in a two-pole machine, each of height $I_f T_m/2$, and the amplitude of the fundamental is

$$F_E = \frac{4}{\pi} \cdot \frac{I_f T_m}{2} \cdot \frac{\sin \pi/2}{\pi/2} = \frac{4}{\pi^2} \cdot I_f T_m$$

Between these two limiting cases lie all the intermediate cases of practice. When located in two or more slots, the sides of the curve for one pole-pitch are stepped, and when only a fraction σ of the pole-pitch Y is wound, the curve has a flat top extending over the fraction $1 - \sigma$ of the pole-pitch. For the stepped sides may then without much error be substituted continuously sloping lines, and a regular trapezium is obtained (Fig. 521) for which the amplitude of the fundamental is

$$F_E = \frac{4}{\pi} \cdot \frac{I_f T_m}{2} \frac{\sin \sigma \frac{\pi}{2}}{\sigma \frac{\pi}{2}} = \frac{8}{\sigma \pi^2} \frac{I_f T_m}{2} \sin \sigma \frac{\pi}{2} \quad . \quad (240)$$

The values for the amplitudes of the fundamental relatively to $I_f T_m/2$ and of the harmonics relatively to the fundamental for various ratios σ have been tabulated by Prof. S. P. Smith,[1] who has shown that throughout the practical range of $\sigma = 0{\cdot}6$ to $\sigma = 0{\cdot}8$, the amplitude of the 3rd and 5th harmonics of M.M.F. will never exceed 6·9 and 4·9 per cent. respectively of that of the fundamental, and higher harmonics never exceed 1 or 2 per cent., so that it is legitimate to deal solely with the fundamental sine wave. The amplitude of this within the same range falls from $1{\cdot}095\, I_f T_m/2$ to $0{\cdot}96\, I_f T_m/2$, *i.e.* from $0{\cdot}5475\, I_f T_m$ to $0{\cdot}48\, I_f T_m$.

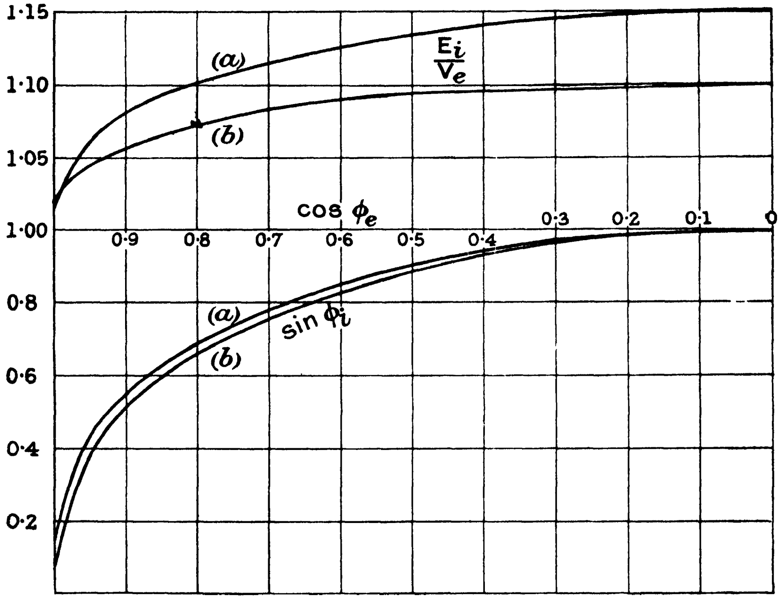

FIG. 522 —The ratio of induced to terminal voltage, and value of sin ϕ_i when (a) $Ir_a' = 0{\cdot}005\ V_e$ and $Ix_a = 0{\cdot}15\ V_e$, (b) $Ir_a' = 0{\cdot}012\ V_e$ and $Ix_a = 0{\cdot}1\ V_e$.

§ 20. **The approximate calculation of the rotor ampere-turns.**—The rotor teeth and core have above been assumed to be of zero reluctance, but actually have a marked influence on the portion of the exciting M.M.F. available over the air-gap and stator core. The theory is, therefore, brought more closely into relation with facts, if F_E is interpreted as the M.M.F. acting over the air-gap only. The process for finding the necessary field excitation will then be as follows. From the given ϕ_e, V_e and calculated or assumed values of Ir_a' and Ix_a, the value of the vector of E_i and its phase-angle ϕ_i relatively to the current vector I or to the fundamental of the armature reaction F_{AR} are found as in Fig. 508. In Fig. 522 the values of the ratio E_i/V_e and of $\sin \phi_i$ are plotted upon two assumptions (*a*) that Ir_a' is $\frac{1}{2}$ per cent. of V_e and Ix_a 15 per cent. of V_e, (*b*) that Ir_a' is 1·2 per cent. and Ix_a 10 per cent. of V_e, such values and other such combinations being found in practice. It will be seen that the ohmic resistance has but little effect, and that the upper curves rise up towards a maximum which is practically 1·15 V_e or 1·1 V_e at zero power-factor, while the sine of the internal angle of lag ϕ_i, slightly exceeding ϕ_e, does not differ widely in the two cases. The E.M.F. E_i will necessitate a sinusoidal flux of maximum density B_{g1}, and corresponding thereto a certain M.M.F. of amplitude $F_g = 0{\cdot}8\, B_{g1}\, K l_g$ in ampere-turns. The position of F_g at right angles to E_i determines its position relatively to F_{AR}, viz., leading it by the angle $(90° + \phi_i)$. By the parallelogram of Fig. 519 F_E is found as $= \sqrt{F_g^2 + F_{AR}^2 + 2\, F_g\, F_{AR} \sin \phi_i}$; thence a certain value of the equivalent ampere-turns for a single air-gap is found from equation (240), when AT_E is substituted for $I_f T_m/2$, viz.,

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[1] " The Non-salient Pole Turbo-alternator and its Characteristics," *Journ. I.E.E.*, Vol. 47, p. 562 ff., to which paper the reader is especially referred.

$$AT_E = F_E \, \frac{\sigma \pi^2}{8 \sin \sigma \dfrac{\pi}{2}} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (241)$$

Assuming different ratios, say, of F_g to F_{AR}, the ratio of the third side of the triangle F_E to F_{AR} can be found by the ordinary process for the solution of a triangle, and curves of the latter ratio in relation to the former can be plotted for any given value of $\cos \phi_e$. But such curves, when F_E is expressed in terms of AT_E, would only hold for a given value of σ.

Adding $AT_{t1} + AT_{c1}$ for the stator teeth and core, we reach AT_P as acting at the rotor pole-face. A further *scalar* addition of the ampere-turns $AT_{t2} + AT_{c2}$ for the rotor teeth and core must then be made in order to find the total AT_f per pole. It will be seen from Fig. 519 that the angle ϕ_0 between IS and I, although determined by the construction, is not necessary to the calculation; actually when F_{AR} is resolved into two components $F_{AR} \sin \phi_0$ and $F_{AR} \cos \phi_0$ (*cp.* Fig. 530), the former, in phase with or opposed to the field vector F_E, represents *direct-magnetizing ampere-turns* that affect the magnitude of F_E for a given required flux or the total value of the flux for a given F_E ; the latter, in quadrature with the field excitation, represents *cross-magnetizing ampere-turns* which displace F_g from F_E and the flux from the centre of the pole.

§ **21. The semi-salient-pole case of practice**—The basis of the above has been the assumption that the field is displaced unchanged in shape by the armature reaction. This really amounts to assuming that the reluctance of a pole-pitch as acted on by $F_{AR} \cos \phi_0$ is the same as the air-gap and tooth reluctance of a pole-pitch as acted on by $F_E - F_{AR} \sin \phi_0$.

But in the actual machine of modern practice the conditions are by no means so simple, when the pole-centre is unslotted or slotted differently from the sides. The pole-centre not being wound, the field M.M.F. wave is in reality a trapezium, and a curve of magnetic potential at the rotor surface such as AT_g in Fig. 308, with flat top and bowed sides, is more nearly what is presented to the M.M.F. wave of the stator. The machine thus becomes semi-salient, and starting with the two waves in some relative position, the flux-densities would have to be calculated tooth by tooth, adjustment being made in the shape of the curve of AT_g to suit the variation of tooth reluctance until all the magnetic potentials are satisfied.[1]

It is only, therefore, with an appreciable loss of accuracy that the simple construction of Fig. 519 is in practice used (*cp.* Fig. 641), the defect being that no attempt is made to calculate the reluctance of the cross circuit acted on by the cross-magnetizing ampere-turns, and the true value of the displacing and distorting effect which determines the real ϕ_0. As bearing also upon the intermediate semi-salient case, it is now proposed to consider the true salient-pole case wherein the effect of the relative position of F_{AR} and of poles is clearly marked and can be more definitely treated mathematically. The physical nature of the case will first be shown.

THE SALIENT-POLE ALTERNATOR

§ 22. The cross- and direct-magnetizing effects of the armature ampere-turns of the salient-pole alternator.—When we pass to the salient-pole machine, the flux is found to be not only displaced but also distorted by the armature reaction, owing to the fact that the air-gap reluctance per sq. cm. of path is not uniform round the bore but periodically increases very greatly in the gaps between the salient poles.

Taking one phase of a salient-pole alternator by itself, if the paths of the magnetic circuit presented at any moment to the armature coils of the phase are traced, they will be found to be different at different positions, and the reluctance presented to the armature M.M.F. is not constant but varies. When the axis of a coil falls on the interpolar line of symmetry (Fig. 523 (i)), its ampere-turns act round cross-circuits passing athwart the pole, and give a cross M.M.F. distorting the resultant field by increasing the density on one side of a pole and weakening it on the other side, so that its virtual centre is shifted towards the former side. When the same group of conductors reaches a similar position relatively to the next pole its current will in an alternator be exactly reversed, and so also will be the direction of the field; its M.M.F. will then again have the same effect, namely, entirely cross-magnetizing. But when the axis of the coil is moved away from the interpolar line, Fig. 523 (ii), two magnetic paths are presented to it, one of these being directly *through* a pair of poles round the main circuit of the field instead of *across* the pole. The effect of the ampere-turns is now

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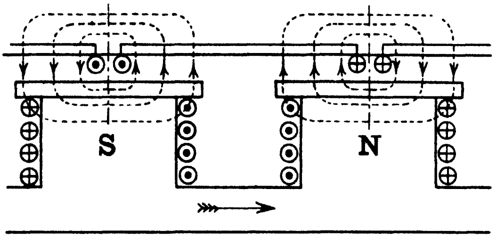
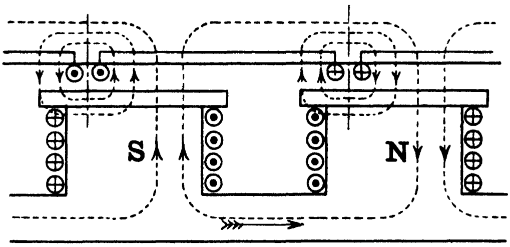
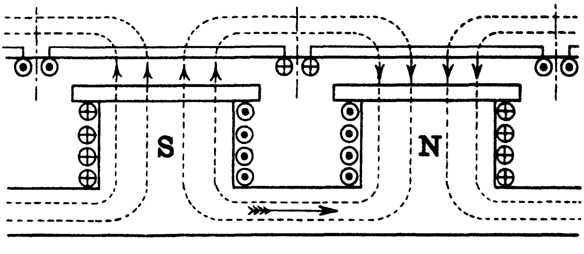
[1] Cp. *Papers on the Design of Alternating-Current Machinery*, by Hawkins, Smith and Neville (Pitman & Sons), pp. 202–219.



(i) Armature ampere-turns purely cross-magnetising.



(ii) Armature ampere-turns, partly cross and partly direct-magnetising.



(iii) Armature ampere-turns purely de-magnetising.

FIG. 523.—The cross and direct magnetizing action of the armature ampere-turns on the salient-pole alternator.

party *cross* and partly *direct*; although not strictly an accurate description, it may be said that in virtue of the latter they directly affect the strength of the symmetrical field, while in virtue of the former they simply displace and distort the weakened or strengthened symmetrical field. The proportion of the directly magnetizing effect to the cross or distorting effect gradually increases until the axis of the coil is exactly over the centre of a pole when there is no cross effect, and all the ampere-turns are directly magnetizing (Fig. 523 (iii)). Later, as the coil passes under the next pole, the effect of the armature-turns is again divisible partly into direct and partly into cross magnetization. Thus in a single-phase alternator, or in one phase of a polyphase machine considered by itself, the groups of ampere-conductors have for each position which they occupy relatively to the poles a certain M.M.F., just as in the non-salient-pole machine, but the action of this M.M.F. must be separated into its cross and its direct effect.

Reckoning from an initial position of space or moment of time when the axis of a drum coil coincides with *IS*, the interpolar line of symmetry, and its sides are thus under the pole-faces, the direct effect from the M.M.F. of any current which it carries is zero, and gradually rises to a maximum when the axis of the coil coincides with the centre of a pole, while the cross effect changes in precisely the opposite way.

§ 23. The curves of the two effects as acting at any instant on a salient pole.—So far the magnetizing effect of a coil according to its position relatively to the salient poles has been described in general terms independently of the actual magnitude or direction of the current which may actually be flowing in it.

To obtain either the direct- or the cross-magnetizing effect separately, the effect from an assumed *constant* current of one ampere in the armature coils in different positions relatively to the poles will first be plotted as a curve upon a base-line representing a half space-period, *i.e.* a pole-pitch; when its ordinates are multiplied by the instantaneous value of the amperes which are flowing at each corresponding moment with their proper sign, the products will yield the actual direct or cross effect upon the pole as a whole. Owing to the presence of the salient poles and interpolar gaps the spacial curve of, say, the direct-magnetizing effect from a single-slot coil with a constant current is not immediately given by the pair of rectangles representing the magnetic potential due to it as in Fig. 509, but is a function of the angle between the axis of the coil and the interpolar line of symmetry.

The shape of the spacial curves for a constant current of one ampere will be further considered later in § 26. It will only here be pointed out that the direct-magnetizing effect will increase at first rapidly as the axis of the coil moves away from its central position on the interpolar line of symmetry between two poles, and then more slowly as the coil-sides emerge into the gap between the poles; after the latter stage the coil practically embraces the main magnetic circuit, and its effect cannot increase greatly, although the maximum is only reached when the axes of coil and pole coincide. Roughly speaking then, the spacial curve of the direct-magnetizing effect *mm* from a constant current will have more or less the shape of a sine curve (Fig. 524 *mm*), reckoning from the assumed initial position.

The reverse is true of the cross-magnetizing effect of the constant current, so that its curve *m'm'* will resemble a cosine curve (Fig. 524 *m'm'*), the maximum effect being produced at the position of Fig. 523 (i).

Each curve will alternate in sign, since the same constant current in the coil, if *e.g.* demagnetizing in relation to one pole for half a period, will assist in magnetizing the succeeding pole during the remaining half period.

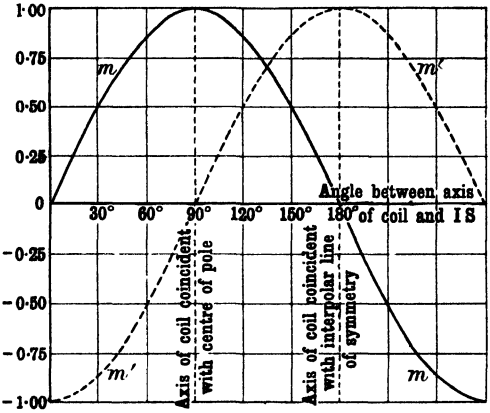


FIG 524.—*mm* = direct-magnetizing effect from a constant current.
m'm' = cross- „ „ „ „ „

To take into account the temporal variation of the current it is only necessary to plot the assumed sinusoidal current in its correct relation to the *mm* or *m'm'* curve, and to multiply the ordinates of the latter by the value of the current i which holds for each value of the angle α through which the axis of the coil has moved from IS, so as to give curves of *mi* or *m'i*.

It must be clearly borne in mind that the ordinates of the *mm*, *m'm'* or *mi* curves do not indicate the spacial or temporal distribution of the magnetic potential round the bore after the method previously adopted for the non-salient-pole machine. On the contrary they measure for each position or moment of time the effect of the armature ampere-turns upon the pole or upon its flux as a whole, either direct-magnetizing or cross-magnetizing as the case may be. Consequently the direct-magnetizing ampere-turns so obtained, being in line with the pole, can be treated as a *scalar* quantity which at the moment in question is immediately subtracted from or added to the pole-face potential AT_p, or if, as will be found to be the case in the polyphase machine, it becomes constant, it forms a permanent deduction therefrom, or addition thereto.

§ 24. The effect of the angle ϕ_0 between the vector of the current and *IS*.—The correct relation of the *mm* or *m'm'* curve to the curve of the current *i* is given by the angle ϕ_0 between the vector of the current and the interpolar line of symmetry, since, in accordance with the convention adopted in § 5, the vector of the current is made to coincide with the axis of the coil when carrying maximum current; *i.e.* the instantaneous value of the current for each value of a is $\mathbf{I} \cos (a + \phi_0)$. The effect of various values of ϕ_0 (positive and negative) can now be tried, and will be found to be of the utmost importance.

First, let the maximum value of the current occur at the moment of time when the axis of the coil coincides spacially with the interpolar line of symmetry, *i e.* in a clock diagram the vector of **I** coinsides with *IS* or $\phi_0 = 0$. The state of affairs is most easily followed by considering the special but common case of a full-pitch coil; then the maximum current occurs in the sides of the coil when they are exactly over the centres of poles. At the assumed starting-point, the current *i* is at its maximum value, and in a generator the direction of the current must be such that it agrees with the E.M.F. that would be generated at no-load or on open circuit in this position. It follows that immediately after this initial moment the direct magnetization begins by having a *de*magnetizing influence.

Whether the current be at its positive or its negative maximum at our starting-point, this remains true; it is a matter of indifference whether we take the alternating current at its positive maximum and so choose the direction of the constant current in the coil that *mm* is first negative and then becomes positive when the coil with the same constant current comes opposite the succeeding pole, or *vice versa* whether we start with the negative maximum of the current and with *mm* positive during the first half-period. The latter procedure is here adopted, and the thick curve *mi* in Fig. 525 (i), giving the product of corresponding ordinates of *mm* and *ii*, shows the direct-magnetizing ampere-turns to be at first demagnetizing and then forward-magnetizing in each half-period. The instantaneous direct-magnetizing effect thus yields a double wave for each period. There results a periodic weakening and strengthening of the main symmetrical field; but since the curve of *mm*, whatever its exact shape, must for a coil of mean span equal to the pole-pitch be symmetrical on either side of 90° and the curve of *ii* is in exact quadrature, the shaded areas must be equal. The *average* value of the direct-magnetizing turns is, therefore, in this case zero, and the average value of the flux is unaffected. Conversely the current *i* is in phase with the cross-magnetizing curve of the constant current; their product is, therefore, always of the same sign, and its effect is that in the leading part of the pole which is ahead of the coil-side in the direction of rotation of the moving field-magnet and which the coil-side has therefore passed, the density of the flux is weakened; in the trailing part of the pole which is behind the coil-side and into which it is moving, the flux is strengthened. That is, the resultant position of the flux is displaced backwards against the direction of the moving-field rotation towards the trailing pole-corner, although to a varying degree depending on the epoch in the period.

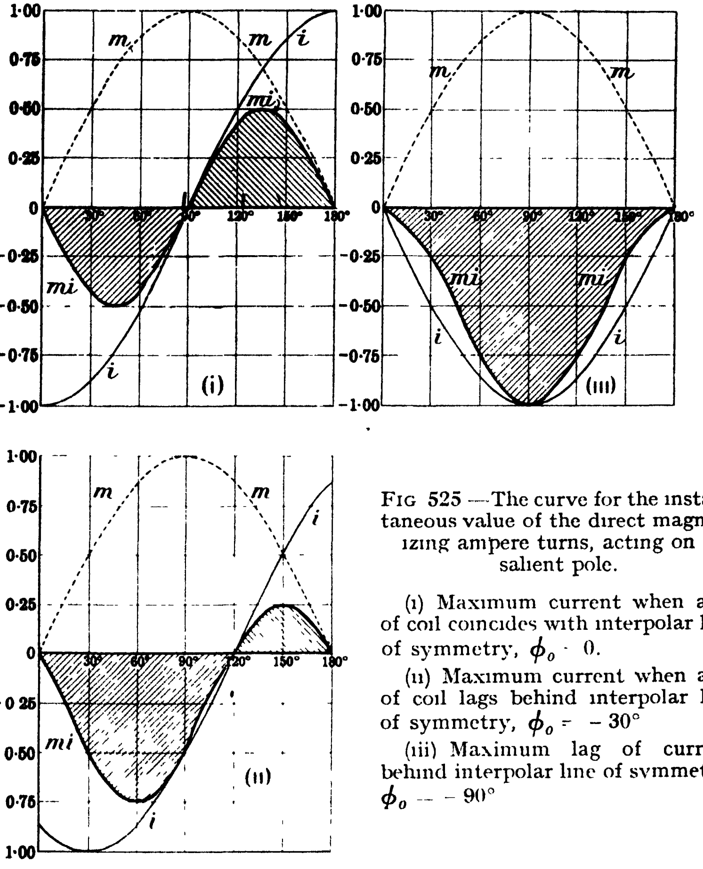


FIG 525 —The curve for the instantaneous value of the direct magnetizing ampere turns, acting on a salient pole.

(i) Maximum current when axis of coil coincides with interpolar line of symmetry, $\phi_0 = 0$.

(ii) Maximum current when axis of coil lags behind interpolar line of symmetry, $\phi_0 = -30°$

(iii) Maximum lag of current behind interpolar line of symmetry, $\phi_0 = -90°$

If the passage of a moving coil through the interpolar gap of a stationary field-magnet is considered and the portions of the pole are named in relation thereto, as was previously done in the case of continuous-current dynamos, the leading pole-corner is always weakened and the trailing pole-corner strengthened; the flux is then displaced forwards in the direction of rotation of the moving coil which gives it the same relative position in the two cases of a stationary and a rotating armature.

But the case supposed above, like the similar case in § 18, would be of rare occurrence. Normally, the vector of the current will lag behind the vector of the E.M.F. that would be generated on open circuit, or behind the interpolar line of symmetry, by some negative value of ϕ_0. That is, with long coils the maximum current in a coil-side will not occur when it is exactly over the centre of a pole.

Next, therefore, let the vector of the current *lag* behind *IS* by some angle ϕ_0, which in Fig. 525 (ii) is assumed to be $-30°$. The current at the starting-point has then not reached its negative maximum, and its instantaneous value is $-\mathbf{I}\cos(\alpha-30°)$ (see Fig. 526). It is seen that the weakening or negative effect of the direct turns is greater and lasts longer than the strengthening or positive effect, so that on the whole there is a back effect from the armature ampere-turns. On the other hand, the instantaneous cross-magnetizing effect from the alternating current now alternates in sign, and each pole-corner is first weakened and then strengthened.

The relative areas of the back and forward effects in each half-period, and the duration and amplitude of each oscillation of the flux centre due to the cross effect, will depend upon the value of ϕ_0. As ϕ_0 increases negatively, the area of the forward wave shrinks, until finally when $\phi_0 = -90°$ or the vector of the current coincides with the centre of a pole, there is only a weakening effect left (Fig. 525 (iii)), which waxes and wanes but never becomes reversed in sign. The current at our starting-point is now zero, or its phase is in exact agreement with the curve *mm*, and the demagnetizing effect is a maximum. As regards the cross-magnetizing effect with $\phi_0 = -90°$, the strengthening of one pole-corner is exactly balanced by its subsequent weakening, and there is no net effect during a half or a whole period, although the field so far as its centre of density is concerned oscillates on either side of the centre line.

If the current vector *leads* the interpolar line of symmetry by a positive angle ϕ_0, the forward effect of the direct-magnetizing turns exceeds the back effect, until when $\phi_0=90°$ there is only a forward effect left. The processes of displacing the flux ahead or backwards are reversed in order of sequence, and there is a strengthening of the leading pole-corner and a weakening of the trailing pole-corner, until when $\phi_0 = 90°$ there is again no net cross effect on the field over a whole period.

Thus in a single-phase alternator, or in a polyphase machine when only one phase is considered, the direct and the cross effects of the armature ampere-turns each vary periodically both in sign and amount as the conductors move relatively to the field, except in the two limiting cases of $\phi_0 = 0$, and $\phi_0 = 90°$, when the only variation is in amount and in synchronism with the period of the machine. The average value over a whole period is, however, in every case dependent upon ϕ_0, and the purport of the above has been to emphasize the importance of the angle ϕ_0 between the vector of the current and *IS*, or between the maximum current in the coil-side of a long coil and the centre of a pole. If there is no angle of lag or lead, the average cross effect reaches its maximum, and there is no average direct magnetizing effect ; if the angular displacement of the current vector approaches its maximum possible of 90° ahead of or behind *IS*, there is no average cross effect, but the average value of the direct magnetizing ampere-turns is a maximum, strengthening the field in the former case and weakening it in the latter case.

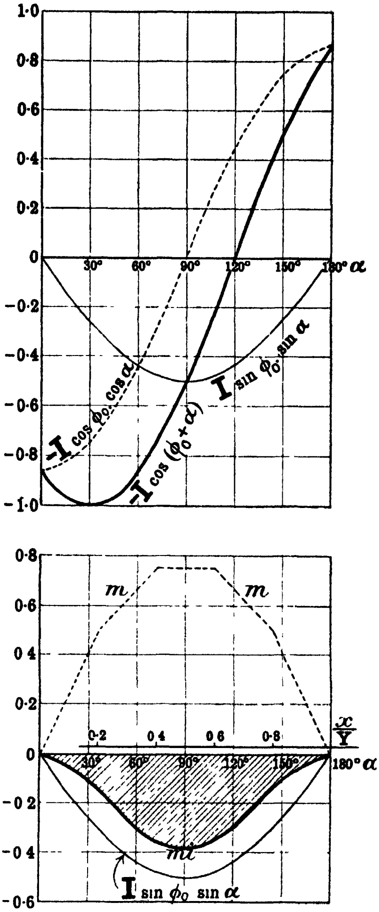


FIG 526.

(i) The resolution of the current into its direct and cross magnetizing components

(ii) Instantaneous value of direct magnetizing ampere-turns of coil.

§ 25. The use of the fundamental of the constant-current magnetizing curves and its average value.—Without an exact knowledge of the shape of either the *mm* or *m'm'* curve, a further conclusion can be drawn from certain simple considerations. It is assumed for the present that the constant-current magnetizing curves are symmetrical about the centre of each half-wave, so that they are composed only of a fundamental and of uneven harmonics. The *mm* curve will then have no cosine terms, and the *m'm'* curve no sine terms. Although the assumption is true for coils of mean span equal to the pole-pitch, its validity in other cases rests on certain further assumptions which are set forth in § 26. If now the current **I** is purely sinusoidal (which is assumed to be the case) so that it possesses no harmonics, and its vector is inclined at the angle ϕ_0 to *IS*, let it be resolved into a component **I** cos ϕ_0 along the interpolar line of symmetry and a component at right angles thereto of amplitude **I** sin ϕ_0. The maximum of the former component will occur when the axis of a coil is midway between the poles, and the maximum of the latter when the axes of the coil and pole coincide. From our assumed starting-point the instantaneous value of the former will be $-\mathbf{I}\cos\phi_o\,.\cos\alpha$, and of the latter $\mathbf{I}\sin\phi_o\,.\sin\alpha$ (Fig. 526 (i)). The product of any uneven harmonic of either the *mm* curve or *m'm'* curve at any instant with $i = -\mathbf{I}\cos(\alpha + \phi_o)$ must then be equal to the algebraic sum of its product with the two components of $i = -\mathbf{I}\cos\phi_o\,.\cos\alpha + \mathbf{I}\sin\phi_o\,.\sin\alpha$, the constant of the latter term becoming negative when ϕ_o is negative.

The former component, being in quadrature with the fundamental of the direct-magnetizing effect *mm*, when multiplied by either the fundamental or the *n*th harmonic must necessarily give the same net area from 0 to $\frac{\pi}{2}$ as it gives from $\frac{\pi}{2}$ to π, but of opposite sign, so that the two areas cancel out in each half-period, *i.e.* $\int_0^{\pi} \sin n\alpha\,.\cos\alpha\,.d\alpha = 0$ where $n = 1,\ 3,\ 5,$ etc. The latter component in phase with *mm* gives $\int_0^{\pi} \sin n\alpha\,.\sin\alpha\,.d\alpha$ which again is equal to zero for any uneven value of *n* except 1.

We thus reach the important result that the curve of the product of *i* with *the fundamental* of the actual *mm* or *m'm'* curve, even if there are uneven harmonics in the latter curves, reproduces all the net direct or cross reaction over a half period, and even as a representation of instantaneous values is, broadly speaking, true to the actual facts. On this account the working curve of *mm* in Fig. 525 has actually been shown as a sine curve, and the similar curve for *m'm'* would be a cosine curve.

§ 26. The joint fundamentals of the direct and cross M.M.F.'s from the coils of a single phase with a constant current of one ampere.—The next step will, therefore, be to determine the value of the amplitudes c_D and c_C for the fundamental curves of *mm* and *m'm'* for a constant current of one ampere with different numbers and arrangements of slots and proportions of polar arc to pole-pitch. So important is the accurate determination of either the direct or the cross ampere-turns to the designer that the subject must be treated in considerable detail. Some simple assumptions must be made in view of the broken nature of the surface which the rotor of a salient-pole alternator presents, and these assumptions being understood and granted, the treatment must be rigorously exact.[1]

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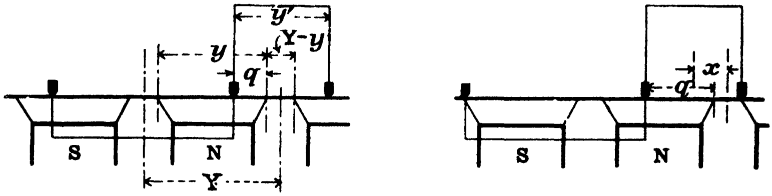
[1] *Cp.* the paper of Prof. A. Blondel on "The Methods of Calculation of the Armature Reactions (Direct and Transverse) of Alternators" at the St. Louis Congress of 1904 (*Trans Intern. Electr. Congress*, Vol. I, p. 635), and C. F. Guilbert "The Armature Reaction of Alternators" (*Electr. World and Eng.*, Vol. 40, p. 738 ff.), upon which the following treatment of the subject is based.

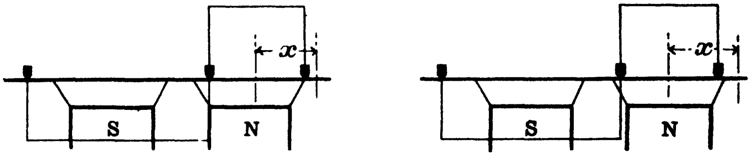
Let y = the effective width of the field, exceeding the width of the actual pole-face by an amount which gives the equivalent of the fringe of lines if concentrated to the same uniform density as under the pole ; thus y is less than the pole-pitch Y, by some amount, small or great, according to the ratio of the polar arc to the pole-pitch and the density of the fringe.

It is now assumed that neither the magnetic potential from the field excitation nor the magnetic potential from the armature ampere-turns takes effect upon the interpolar gap $Y - y$, but, so far as y falls short of Y, that their real effect upon the actual fringe of lines finds its equivalent in the effect upon the assumed concentrated fringe of width (y polar arc). The distribution of the magnetic potential from a single-slot coil as acting on the air-gap being, when plotted on a straight line base, a pair of rectangles if the coil carries a constant current, movement of the rotor causes the poles virtually to slide along relatively to the pair of rectangles In the general case when the pitch y' between the centre-lines of the two slots is different from Y, the two rectangles are of unequal height and width, as in § 14, but now these widths are not simply y' and $2Y - y'$ Whenever either rectangle covers part of the interpolar gap $Y - y$, any part so covered must be deducted from the geometrical width as not being available for the passage of flux upon the assumption mentioned above. The mean width of the two is always y. The relative heights of the two rectangles must accordingly be adjusted to suit the effective widths.



Beginning of 1st stage.
$x=0$

End of 1st stage,
Beginning of 2nd stage.
$x=\frac{y'-Y+y}{2}$



End of 2nd stage,
Beginning of 3rd stage.
$x=\frac{y'+Y-y}{2}$

End of 3rd stage.
$x=\frac{Y}{2}$

FIG 527.—The three stages in the effect of a coil in relation to salient poles.

Next, it is assumed that the average value of the flux-density in the air-gap at each instant, and therefore the average value of the air-gap ampere-turns, is reached by considering the joint magnetic potential from the main air-gap excitation and from the rectangles as acting on the two portions of the pole-face into which a slot may divide it, adding the two portions of the flux together and averaging them over the width of the field y. The flux over either pole would thus at times be sharply divided into bands of unequal density, and though these bands, positive and negative, would agree in their amounts over the double pole-pitch and are so far legitimate, the actual distribution of the flux in such bands could not strictly hold in nature. The assumption is however valid, so far as the discovery of the average flux, or rather of the average value of the direct-magnetizing ampere-turns in relation to the field excitation of the poles, is concerned. In other words, if the two rectangles at any moment have effective widths w_1 and w_2, and $t/2g'$ be their mean height, their respective heights are $\frac{t}{2g'}\,\frac{w_2}{y}$ and $\frac{t}{2g'}\,\frac{w_1}{y}$; then the former covers a distance q along a pole-face out of the total width y, the flux in the pole would be proportional to $\frac{t}{2g'} \times \frac{w_2}{y} \times q - \frac{t}{2g'} \times \frac{w_1}{y}(y-q)$, and the average flux-density which determines the ampere-turns would be proportional to $\frac{t}{2g'y^2}\{w_2 q - w_1(y-q)\}$

Starting with a position of the coil when its axis coincides with IS, each quarter of a period is divisible into three stages (Fig 527). In the first place let the width of the coil y' be less than the effective width y of the field.

(i) During the first stage from $x = 0$ to $x = \frac{y' - (Y-y)}{2} = \frac{y' + y - Y}{2}$, the trailing coil-side passes up to the edge of the field y (supposed of uniform density).

(ii) During the second stage from $x = \frac{y' - Y + y}{2}$ to $x = \frac{y' + Y - y}{2}$, the trailing coil-side passes through the interpolar gap of width $Y - y$.

(iii) During the third stage from $x = \frac{y' + Y - y}{2}$ to $x = \frac{Y}{2}$, the trailing coil-side passes into the succeeding field so far that the axes of coil and pole coincide.

Effective width of rectangle.		Values of	
Smaller, w_1	Larger, w_2	q	$y - q$
(i) $y' - Y + y$	$Y + y - y'$	$\frac{y' - Y + y}{2} + x$	$\frac{y + Y - y'}{2} - x$
(ii) $\frac{y' - Y + y}{2} + x$	$\frac{Y + 3y - y'}{2} - x$	$\frac{y' + y - Y}{2} + x$	$\frac{Y - y' + y}{2} - x$
(iii) y'	$2y - y'$	y'	$y - y'$

Considering the N pole of Fig 527

(i) During the first stage

$$\frac{t}{2g'y^2}\left\{\left(Y + y - y'\right)\left(\frac{y' - Y + y}{2} + x\right) - \left(y' - Y + y\right)\left(\frac{y + Y - y'}{2} - x\right)\right\}$$
$$= \frac{t}{2g'} \cdot \frac{2x}{y}$$

i e. since the M.M F.'s over the two portions into which the slot divides the pole-face are in opposite directions, the average effect depends upon their difference, and is, therefore, proportional to *twice* the displacement.

(ii) During the second stage

$$\frac{t}{2g'y^2}\left\{\left(\frac{Y + 3y - y'}{2} - x\right)\left(\frac{y' + y - Y}{2} + x\right) - \left(\frac{y' - Y + y}{2} + x\right)\right.$$
$$\left.\left(\frac{Y - y' + y}{2} - x\right)\right\} = \frac{t}{2g'} \times \frac{1}{y}\left(\frac{y + y' - Y}{2} + x\right)$$

This follows even more simply from consideration of the adjacent S. pole included entirely within the rectangle of lower height, for which we have $\frac{t}{2g'y^2}\left(\frac{y' + y - Y}{2} + x\right)y$.

The average effect at any instant during this stage, therefore, increases directly as the displacement, or the rate of increase is only half of that in the first stage.

(iii) During the third stage

$$\frac{t}{2g'y^2}\left\{(2y-y')\,y' - y'\,(y-y')\right\}$$

$$= \frac{t}{2g'} \times \frac{y'}{y}$$

which again also follows at once from consideration of the adjacent pole included within the rectangle, for which we have

$$\frac{t}{2g'y^2}\,(y' \times y)$$

The coil of pitch less than the effective width of the field falls entirely within the field, or the latter simply moves under it, and the effect is constant, or $\propto \frac{y'}{y}$

Thus the *mm* curve of the average effect from a constant current on the present assumption is, at any instant during a half period, as shown in Fig. 526 (ii) (which has been plotted for $\frac{y}{Y} = 0{\cdot}8$ and $\frac{y'}{Y} = 0{\cdot}6$). Since in practice the corners would be rounded off, it will be seen that its shape including all harmonics is not greatly dissimilar to the sine curve of *mm* in Fig. 524 or 525.

By the above assumptions the fundamental curve of *mm* has been rendered in every case symmetrical not only about the abscissa axis but also about the centre of each half-wave, so that there are no even harmonics ; its amplitude is, therefore,

$$\frac{4}{\pi}\int_0^{\frac{\pi}{2}} a \sin a \quad da$$

where a is the height of the ordinate at any angle a from the origin.

Any distance x is equivalent to an angle $a = \pi \cdot \frac{x}{Y}$, and when the linear expressions are thus converted into angles, we have for a single-slot coil

(i) from $a = 0$ to $a = \pi \frac{y + y' - Y}{2Y}$ $\qquad a = \frac{t}{2g'y} \times \frac{2Y}{\pi} \times a$

(ii) from $a = \pi \frac{y + y' - Y}{2Y}$ to $a = \pi \frac{y' + Y - y}{2Y}$ $\qquad a = \frac{t}{2g'y}\left(\frac{y'+y-Y}{2} + \frac{Y}{\pi} \cdot a\right)$

(iii) from $a = \pi \frac{y' + Y - y}{2Y}$ to $a = \frac{\pi}{2}$ $\qquad a = \frac{t}{2g'y} \times y'$

The amplitude is then

$$\frac{2}{\pi} \cdot \frac{t}{g'y}\left[\int_0^{\frac{\pi}{2} \cdot \frac{y + y' - Y}{Y}} \frac{2Y}{\pi} \cdot a \sin a \,.\, da \right.$$

$$+ \int_{\frac{\pi}{2} \cdot \frac{y + y' - Y}{Y}}^{\frac{\pi}{2} \cdot \frac{y' + Y - y}{Y}} \left(\frac{y' + y - Y}{2} + \frac{Y}{\pi} \cdot a\right) \sin a \,.\, da$$

$$\left. + \int_{\frac{\pi}{2} \cdot \frac{y' + Y - y}{Y}}^{\frac{\pi}{2}} y' \,.\, \sin a \,.\, da \right]$$

The integrals within the square bracket reduce to the very simple expression

$$\frac{2}{\pi} \times Y \sin\left(\frac{\pi}{2} \cdot \frac{y}{Y}\right) \times \sin\left(\frac{\pi}{2} \cdot \frac{y'}{Y}\right)$$

so that the amplitude is

$$\frac{t}{g'} \times \frac{4}{\pi^2} \times \frac{Y}{y} \times \sin\left(\frac{\pi}{2} \cdot \frac{y}{Y}\right) \times \sin\left(\frac{\pi}{2} \cdot \frac{y'}{Y}\right)$$

As y' is increased relatively to y, the first and second stages are prolonged, and the third stage shortened, until when $y' = y$, the third stage disappears, and the dotted curve of *mm* reaches an apex of height $\frac{t}{2g'}$. When y' is increased beyond the virtual width of the field, the length of the second stage is altered ; it now lasts until the leading coil-side reaches the edge of the field, or from $x = \frac{y' + y - Y}{2}$ to $x = \frac{y + Y - y'}{2}$, but with the same value for the effect as before. From this time, however, i e from $x = \frac{y + Y - y'}{2}$ to $\frac{Y}{2}$, a new third stage appears during which the coil embraces the entire field of a pole face. The effective width of each rectangle is then y, and $y - q = 0$; hence $a = \frac{t}{2g'y^2} \times y^2 = \frac{t}{2g'y} \times y$. The amplitude is thus

$$\frac{2}{\pi}\,\frac{t}{g'y}\left[\int_0^{\frac{\pi}{2} \cdot \frac{y' - Y + y}{Y}} \frac{2Y}{\pi} \,.\, a \sin a \;\; da \right.$$

$$+ \int_{\frac{\pi}{2} \cdot \frac{y' - Y + y}{Y}}^{\frac{\pi}{2} \cdot \frac{y + Y - y'}{Y}} \left(\frac{y' - Y + y}{2} + \frac{Y}{\pi}\, a\right) \sin a \,.\, da$$

$$\left. + \int_{\frac{\pi}{2} \cdot \frac{y + Y - y'}{Y}}^{\frac{\pi}{2}} y \sin a \,.\, da \right]$$

which reduces to exactly the same expression as before.

When the width of the coil is equal to the pole-pitch, the second stage entirely disappears, and also the term $\sin\left(\frac{\pi}{2} \times \frac{y'}{Y}\right)$ becomes unity in the amplitude. The *mm* curve for half a period is then a trapezium, with uniformly sloping sides joined by horizontal lines

When the width of the coil exceeds the pole-pitch, we return to the same *mm* curve as in Fig 526 (ii), and to precisely the same full expression for its amplitude. It is obviously immaterial whether the end-connexions of the coil span the shorter rectangle of width $y' = kY$ or the longer rectangle of width $(1 - k)Y$, or again whether the coil-connexions are actually divided.

Thus in all cases the amplitude of the fundamental of the direct-magnetizing ampere-turns from a single-slot component coil of t/g' turns carrying a constant current of one ampere is

$$\frac{t}{g'}\frac{4}{\pi^2} \times \frac{Y}{y} \sin\left(\frac{\pi}{2} \cdot \frac{y}{Y}\right) \sin\left(\frac{\pi}{2} \cdot \frac{y'}{Y}\right)$$

For the total number of g' slots in a belt per phase, whether the coils are concentric or lap-wound and whether g' be even or uneven, it is only necessary, as has already been done in § 14, to interpret y' as *the mean pitch between the two belts* and to multiply the expression for the component fundamental by $g'k_{s1}'$ to find the amplitude of the joint fundamental. The amplitude of the joint fundamental for the average effect of the direct-magnetizing ampere-turns of the belt corresponding to a pole or half magnetic circuit when carrying a constant current of one ampere is thus in general

$$t \times c_D = t\frac{4}{\pi^2}\frac{Y}{y}\sin\left(\frac{\pi}{2}\times\frac{y}{Y}\right)k_{s1}'\,k_{s1}''$$

since $\sin\left(\frac{\pi}{2}\times\frac{y'}{Y}\right)$ when y' is the *mean* pitch between the belts $= \cos\frac{\varepsilon}{2}$, as in eq. (33).

When the same procedure is applied to the calculation of the amplitude of the joint fundamental of the cross-magnetizing ampere-turns $m'm'$, for a constant current of one ampere in the belt of g' slots per phase, an equally simple expression is obtained, viz.,

$$t \times c_C = t\frac{4}{\pi^2}\frac{Y}{y}\left\{1-\cos\left(\frac{\pi}{2}\times\frac{y}{Y}\right)\right\}k_{s1}'\,k_{s1}''$$

Both expressions should be compared with the expression of eq. (235).

§ 27. The joint fundamentals respectively of the direct and cross M.M.F.'s from the coils of a single phase with an alternating current.—As the spacial position of the coil relatively to the salient poles alters, the direct and the cross magnetizing effects vary respectively as $c_D \sin a$ and $c_C \cos a$. But meantime, in the alternating case, the current has been varying in time as $\cos(a+\phi_o)$, reckoning from the same starting-point. Hence in the actual machine so far as the fundamentals of *mm* and I/q are concerned, the cases of Fig. 525 would be reproduced at any instant by the expression

$$\begin{aligned} mi &= -\sqrt{2}Jt\,c_D\sin a\,.\,\cos(a+\phi_o) \\ &= -c_D\,.\,v_o\,(\sin a\,.\,\cos a\,.\,\cos\phi_o - \sin^2 a\,.\,\sin\phi_o) \end{aligned}$$

The average value of $\sin a\,.\,\cos a$ between the limits of $a = 0$ and $a = \pi$ being zero, and the average value of $\sin^2 a$ between the same limits being $\frac{1}{2}$, the *average* value of the direct magnetizing effect from the armature coils of a single phase in ampere-turns as acting on a pole or one air-gap of the magnetic circuit is for a single phase $\overline{AT}_{DM} = \frac{1}{2}\,c_D\,v_o\sin\phi_o$ which is negative or demagnetizing when ϕ_o is negative and the current lags. Here $\overline{AT}_{DM}$ is comparable with AT_g, the excitation required over the single air-gap. As expressing the net average effect on a pole or on one air-gap, the above value is completely true under the given assumptions, whatever may be the harmonics of the *mm* curve, for the reason given in § 25.

Analogously since the instantaneous cross effect is $m'i = c_{\mathrm{C}} v_o \cos\alpha \,.\, \cos(\alpha + \phi_o)$, the *average* cross effect on a half pole from the coils of a single phase is

$$\overline{AT}_{\mathrm{CM}} = \tfrac{1}{2}\, c_{\mathrm{C}}\, v_o \cos\phi_o$$

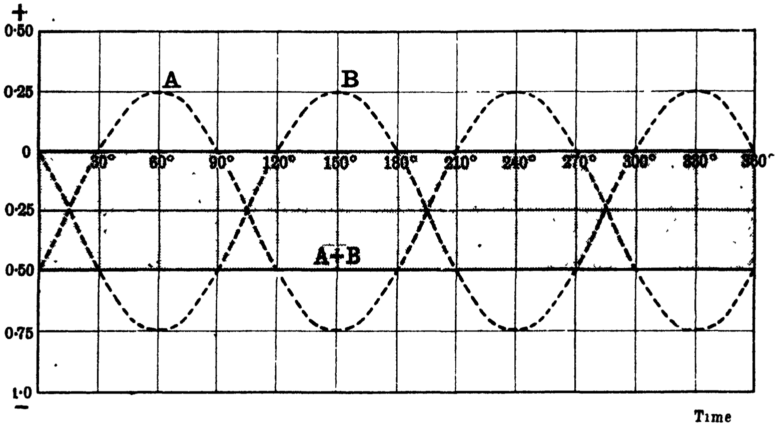


FIG. 528.—Constant value of demagnetizing armature ampere-turns of quarter-phase alternator, as acting on a salient pole.

Thus when the maximum alternating current **I** is resolved into the two components **I** sin ϕ_o and **I** cos ϕ_o, in accordance with § 24, only the former enters into the direct-magnetizing ampere-turns and only the latter into the cross-magnetizing ampere-turns.

§ 28. The constant value of both the direct and cross magnetizing M.M.F.'s in the polyphase alternator.—Since the *average* effect of one phase, whether cross or direct, does not vary in sign from one half period to another for a given value of ϕ_o, it is a matter of indifference between what points the period is reckoned. Consequently in the case of a polyphase machine with N_{ph} phases, the total average effect is simply N_{ph} times that of the single phase.

But this does not give any clue to the real instantaneous value. To discover this in the polyphase machine, it is necessary to repeat the curves, *e.g.* of *mi* in Fig. 525 for the direct effect from each phase at its correct interval, and to add them together algebraically. Thus in the quarter-phase machine Fig. 528 repeats the thick-line curve of Fig. 525 (ii) for a second set of coils displaced 90° relatively to the first, and Fig. 529 for a three-phase machine repeats the same curve at intervals of 120° and 240°. The shaded areas show the result.

Or analytically, the direct magnetizing ampere-turns in the quarter-phase machine are

$$-c_D v_o \{\cos(a+\phi_o)\sin a + \cos(a+\phi_o-90°)\cos a\}$$

which reduces to

$$c_D v_o \sin\phi_o (\sin^2 a + \cos^2 a) = c_D v_o \sin\phi_o$$

i.e. a constant value twice the average value from a single phase.

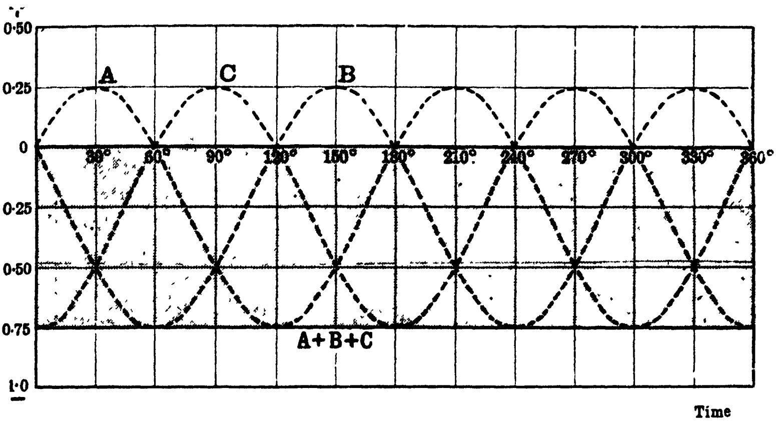


FIG. 529.—Constant value of demagnetizing armature ampere-turns of three-phase alternator, as acting on a salient pole.

In the three-phase machine

$$-c_D v_o \{\cos(a+\phi_o)\sin a + \cos(a+\phi_o-120°)\sin(a-120°) + \cos(a+\phi_o-240°)\sin(a-240°)$$

reduces to

$$\tfrac{3}{2} c_D v_o \sin\phi_o (\sin^2 a + \cos^2 a) = \tfrac{3}{2} c_D v_o \sin\phi_o$$

or a constant value three times the average value of the single phase.

Similarly the instantaneous cross-magnetizing effect becomes constant in the quarter-phase machine at double the average value of the cross ampere-turns of the single-phase machine, and in the 3-phase machine at treble the average value.

Thus in the polyphase machine with all phases equally loaded, so far as the fundamentals are concerned

$$AT_{DM} = \tfrac{1}{2} N_{ph} c_D v_o \sin\phi_o$$

and

$$AT_{CM} = \tfrac{1}{2} N_{ph} c_C v_o \cos\phi_o$$

In either case the effect becomes constant, *i.e.* in the polyphase machine there is a *permanent* weakening of the main field if ϕ_o is negative, or a permanent strengthening if ϕ_o is positive, and by the cross effect the field is permanently displaced to one or other side of the centre of a pole, so that the passage of a coil-side through maximum field occurs later or sooner than its passage past the centre of a pole, according as the vector of the current lags or leads relatively to the interpolar line of symmetry.

It is evident that in the special and ideal case when $y = Y$, $c_D = c_C$, and the two divisions of the armature reaction would only differ by the factors $\sin \phi_0$ and $\cos \phi_0$; they could then be represented by the two sides of a right-angled triangle. But the width y of the field in a salient-pole machine can never be equal to the pole-pitch, and as $\frac{y}{Y}$ is decreased, $1 - \cos\left(\frac{\pi}{2} \cdot \frac{y}{Y}\right)$ decreases much more rapidly than $\sin\left(\frac{\pi}{2} \cdot \frac{y}{Y}\right)$.

§ 29. The general expressions for the armature direct- and cross-magnetizing ampere-turns in relation to salient poles.—Recapitulating then the above results in the same form as in § 15, which was applicable to the non-salient-pole machine, we have found in a salient-pole machine that the amplitude of the fundamental of the direct-magnetizing ampere-turns as acting on a pole or on one-half of a magnetic circuit and as due to a single phase at the moment of maximum current is

$$\mathbf{a} = c_D v_o \sin \phi_o = \frac{4}{\pi^2} \frac{Y}{y} \sin\left(\frac{\pi}{2} \times \frac{y}{Y}\right) k_{s1}' k_{s1}'' t\sqrt{2}J \sin \phi_o$$

which should be compared with (236*a*).

The *average* value for a single phase is

$$\overline{AT}_{DM} = \frac{2}{\pi^2} \frac{Y}{y} \sin\left(\frac{\pi}{2} \times \frac{y}{Y}\right) k_{s1}' k_{s1}'' t\sqrt{2}J \sin \phi_o$$

which should be compared with (237).

The *constant* value of the joint fundamental amplitudes for a polyphase salient-pole machine with all phases symmetrically loaded is

$$AT_{DM} = N_{ph} \frac{\mathbf{a}}{2} = \frac{N_{ph}}{\pi} \times \frac{2}{\pi} \frac{Y}{y} \sin\left(\frac{\pi}{2} \times \frac{y}{Y}\right) k_{s1}' k_{s1}'' t\sqrt{2}J \sin \phi_o$$

or it may be expressed in terms of $JZ/4p$, the ampere-turns per pole, as

$$AT_{DM} = k_D \frac{JZ}{4p} \sin \phi_o \quad . \quad . \quad . \quad . \quad . \quad . \quad (242)$$

where

$$k_D = \frac{4\sqrt{2}}{\pi^2} \frac{Y}{y} \sin\left(\frac{\pi}{2} \times \frac{y}{Y}\right) k_{s1}' k_{s1}'' \quad . \quad . \quad . \quad (243)$$

which should be compared with expressions (238) and (239).

With long coils of mean span equal to the pole-pitch in a 3-phase alternator

$$\text{with 2 slots per pole per phase, } k_D = 0{\cdot}555\,\frac{Y}{y}\sin\left(\frac{\pi}{2}\times\frac{y}{Y}\right)$$

$$\text{,, 3 ,, ,, ,, ,,} \quad = 0{\cdot}55\,\frac{Y}{y}\sin\left(\frac{\pi}{2}\times\frac{y}{Y}\right)$$

$$\text{and with uniform distribution over } 60^\circ, = 0{\cdot}548\,\frac{Y}{y}\sin\left(\frac{\pi}{2}\times\frac{y}{Y}\right)$$

As a normal case, taking the *effective* pole-width y as $0{\cdot}7Y$ with 3 slots per pole per phase, $k_D = 0{\cdot}55 \times \frac{1}{0{\cdot}7} \times 0{\cdot}891.$

$$= 0{\cdot}701$$

This is so nearly $1/\sqrt{2}$, that AT_{DM} is often approximately identified with $\frac{1}{\sqrt{2}} \times \frac{JZ}{4p}\sin\phi_o$ as a convenient generalization.

Analogously to the above,

$$AT_{CM} = \frac{N_{ph}}{\pi} \times \frac{2}{\pi}\,\frac{Y}{y}\left\{1-\cos\left(\frac{\pi}{2}\times\frac{y}{Y}\right)\right\} k_{s1}'\,k_{s1}''\,t\sqrt{2}\,J\cos\phi_o$$

$$= k_C\,\frac{JZ}{4p}\cos\phi_o \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (244)$$

where

$$k_C = \frac{4\sqrt{2}}{\pi^2}\,\frac{Y}{y}\left\{1-\cos\left(\frac{\pi}{2}\times\frac{y}{Y}\right)\right\} k_{s1}'\,k_{s1}'' \quad . \quad . \quad . \quad (245)$$

with the same numerical values for $\frac{4\sqrt{2}}{\pi^2}\,k_{s1}'\,k_{s1}''$ in the same cases as above.

§ 30. The derivation of the expressions for the direct and cross turns in the salient-pole machine from the general expression for the armature magnetizing turns in the non-salient-pole machine.—The close correspondence between equations (236–9) and the expressions of § 29 for the two cases with non-salient poles and with salient poles is evident, and the connexion between the two can be shortly stated.

In the polyphase machine let the fundamental sine wave of the total armature reaction, F_{AR}, with amplitude A be resolved into two component sine-waves, F_{DM} and F_{CM}, with amplitudes $A\sin\phi_o$ and $A\cos\phi_o$ (Fig. 530). Each is stationary with respect to the poles, and the maximum ordinate of the former will stand centrally over the pole, while the maximum ordinate of the latter will coincide with the interpolar line of symmetry.

Now the assumptions underlying the treatment of the salient-pole case are that the main field is concentrated into the width y, and that the magnetic potential of the armature ampere-turns acts solely on this width. Let, therefore, the component fundamental ampere-turns be summed up over those portions only of the whole pole-pitch which are shown in Figs. 531 and 532, and let their respective average values over these portions be found. These values are then the required scalar values, AT_{DM} and AT_{CM}.

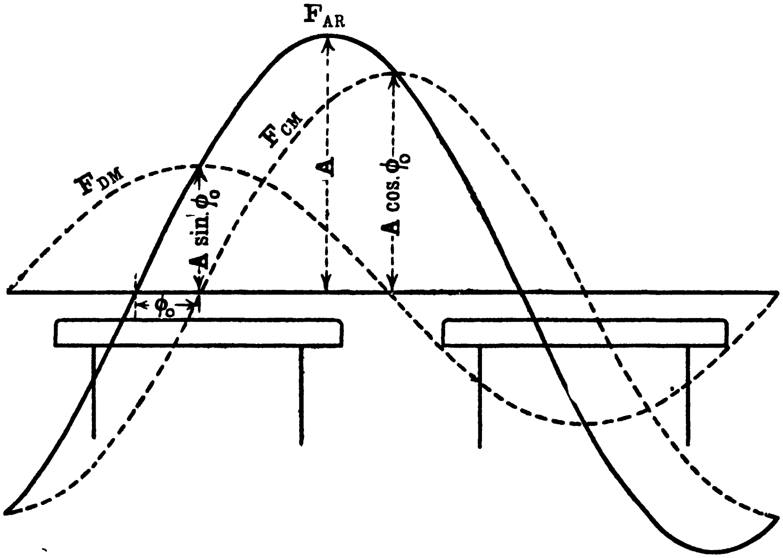


FIG. 530.—Resolution of fundamental curve of M.M.F. from armature ampere-turns into fundamentals of direct and cross turns.

Thus, in relation to the direct-magnetizing turns, the shaded area beneath the curve F_{DM} (Fig. 531) is

$$A \sin \phi_o \int_{\frac{\pi}{2} \times \frac{Y-y}{Y}}^{\frac{\pi}{2} \times \frac{Y+y}{Y}} \sin a \,.\, da = A \sin \phi_o \times 2 \sin \left(\frac{\pi}{2} \times \frac{y}{Y}\right)$$

Dividing by $\pi y/Y$, the averaged value, shown by the dotted line in Fig. 531, is

$$A \sin \phi_o \times \frac{2}{\pi} \cdot \frac{Y}{y} \sin \left(\frac{\pi}{2} \cdot \frac{y}{Y}\right) = AT_{DM}$$

Similarly the average ordinate of the shaded portion of Fig. 532, integrated over an angle $\frac{\pi}{2} \times \frac{y}{Y}$ reckoned from the centre of a pole, is

$$\frac{A \cos \phi_o}{\frac{\pi}{2} \times \frac{y}{Y}} \int_{0}^{\frac{\pi}{2} \times \frac{y}{Y}} \sin a \,.\, da = A \cos \phi_o \times \frac{2}{\pi} \times \frac{Y}{y} \left\{ 1 - \cos \left(\frac{\pi}{2} \cdot \frac{y}{Y}\right) \right\}$$

and this is the value of AT_{CM} as acting on one-half of the air-gap under a pole-face.[1]

In general, for any given arrangement of coils and number of slots, it is merely necessary in the salient-pole machine to take as the amplitudes of the fundamentals c_D and c_Q of the direct and cross magnetizing effects from a constant current of one ampere the fractions respectively

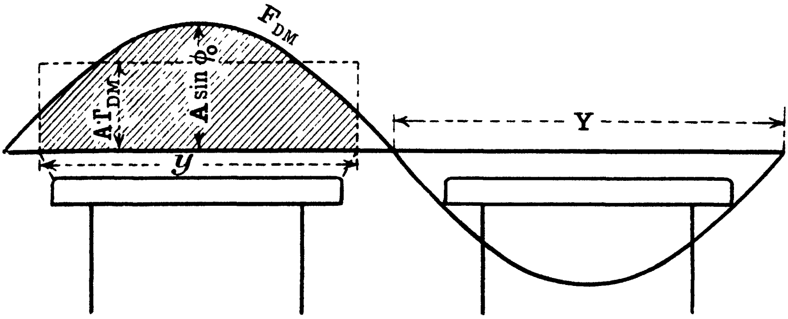


FIG. 531.—Direct-magnetizing armature ampere-turns on polyphase alternator with salient poles.

$$\frac{2}{\pi}.\frac{Y}{y}\sin\left(\frac{\pi}{2}.\frac{y}{Y}\right) \text{ and } \frac{2}{\pi}.\frac{Y}{y}\left\{1-\cos\left(\frac{\pi}{2}.\frac{y}{Y}\right)\right\}$$

of c_A, the amplitude from similar coils on a machine with non-salient poles. These fractions simply arise from the difference in the average ordinate of a sine-curve integrated over a portion only, instead of the whole, of a pole-pitch ; $\frac{2}{\pi}$ obviously is the averaged value for the whole sine curve, and the remaining term indicates the further lowering when $y < Y$. Thence

$$k_D = \frac{2}{\pi}.\frac{Y}{y}\sin\left(\frac{\pi}{2}.\frac{y}{Y}\right) \times k_A$$

$$k_C = \frac{2}{\pi}.\frac{Y}{y}\left\{1-\cos\left(\frac{\pi}{2}.\frac{y}{Y}\right)\right\} \times k_A$$

From the initial assumption that the rotor of Figs. 509–511 was a smooth cylinder with a continuous surface, the passage is thus made to the case when its surface is broken up into definite poles. In the ideal non-salient-pole machine the ampere-turns of field and armature were combined vectorially to give a resultant excitation ; with salient poles the calculations are so made that the average values of the direct-magnetizing and cross-magnetizing ampere-turns in quadrature are found separately. The former, being in line with the field excitation, can be immediately added to the air-gap ampere-turns or subtracted from AT_P as a scalar quantity.

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[1] *Cp.* J. B. Henderson and J. S. Nicholson, *Journ. I.E.E*, Vol. 34, pp. 469–471.

It has been shown in § 25 that the harmonics of the M.M.F. wave, although they may cause local variations of the flux, cannot exert any net permanent effect. Hence the isolation of the fundamental for sole consideration, as limited in its action to the effective width of field, must yield the same result as the method of § 26 based on the average effect of a coil when its width and that of the pole are duly taken into account.

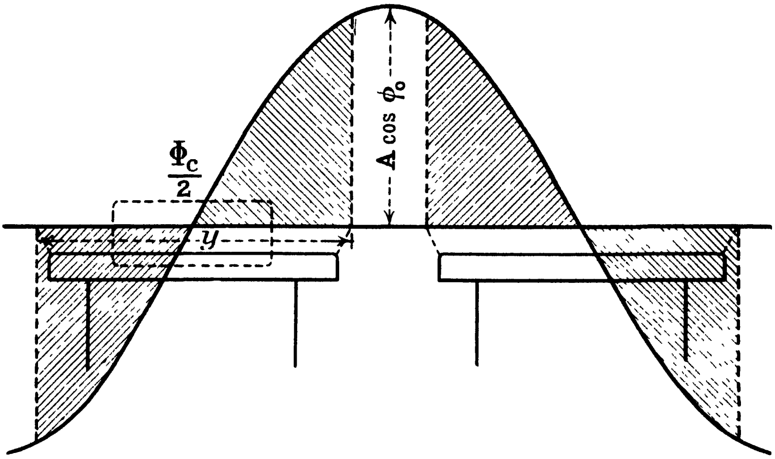

FIG. 532.—Cross-magnetizing armature ampere-turns on polyphase alternator with salient poles.

§ 31. The different nature of the effects from cross and direct armature ampere-turns.— The different character of the effects from the cross and direct magnetizing ampere-turns of the armature has important consequences. The cross turns increase the M.M.F. acting over the air-gap of one-half of the pole-face and diminish the M.M.F. acting over the air-gap of the other half. They thus simply alter the distribution of the main flux over the pole-face without sensibly altering its average value. They do not directly increase or decrease the resultant ampere-turns acting on the main magnetic circuit, but affect the shape of the E.M.F. wave and its form factor.

On the other hand, the third item of the armature reaction, or the direct magnetizing effect, is essentially magnetic, and must be taken into account by subtracting it from or adding it to the pole-face potential AT_P supplied by the field-winding. If ϕ_0 is negative, AT_{DM} are back ampere-turns, and increase the leakage of the field-magnet just as AT_b in the continuous-current dynamo when lead is given to the brushes.

§ 32. The conversion of the cross effect into a voltage by an assumed cross flux.—The effect of the cross ampere-turns upon the induced E.M.F. of the armature may be reproduced by superposing upon the undistorted main flux a local cross flux passing through one-half of the air-gap, along the iron of the pole-shoe, across the second half of the same air-gap, and so completing its magnetic circuit through the iron of the armature (Fig. 532). This local flux must be of such magnitude and distribution as to reproduce the actual distribution when compounded with the main flux.[1] Since the assumed cross flux either varies in amount or, if constant, as in polyphase machines, travels with the rotating poles, it will cut the armature conductors and thence arises a self-induced E.M.F. in the armature. By calculating this E.M.F. we account for one element of the armature reaction corresponding to the second item in the group of effects covered by the generic term "inductance" of the armature, and the reason for thus converting it into an electric voltage will appear shortly.

If $4\mathfrak{R}_{cc}$ be the reluctance of the complete cross-circuit under *one* pole as shown on the left side of Fig. 532, the flux crossing the air-gap under the right half of that pole is $1{\cdot}257\,\dfrac{AT_{CM}}{2\mathfrak{R}_{cc}}$. In the similar half of the adjacent pole the flux has the same direction and the same amount. The total flux forming one cross field under the adjacent halves of *two* poles is, therefore,

$$\Phi_c = \frac{1{\cdot}257\,AT_{CM}}{\mathfrak{R}_{cc}}$$

The reluctance of the cross-circuit under one pole-face $4\mathfrak{R}_{cc}$ is practically determined by the reluctance of the two air-gaps in series, each of virtual length $K\,l_g$, and of area equal to the breadth of half the virtual pole-face, $y/2$, multiplied by the axial length L_f of the pole-face. The reluctance of two such circuits in parallel, or $2\mathfrak{R}_{cc}$, may, therefore, be identified in practice with $\dfrac{2Kl_g}{yL_f}$, or $\mathfrak{R}_{cc} = \dfrac{Kl_g}{yL_f}$, the reluctance of an air-gap. On this account the reluctance of one cross-circuit was expressed as 4 times $\mathfrak{R}_{cc}$, since the air-gap portion of the path would have twice the length and half the area of the normal single air-gap. The effect of any saturation of the teeth through the crowding of the actual flux to one or other corner of the pole would be to shift the virtual centre of the field towards the unsaturated side, as described in Chapter XIX; this would correspond to an increase of the angle ϕ_0 when the current lags, or to a decrease of ϕ_0 when the current leads. The effect is, however, except in extreme cases, negligible in view of other sources of error which are present,[1] or an approximate allowance may be made for it by a reduction of some 3 to 5 per cent. in the calculated value of Φ_c. Thus, approximately, it may be taken that

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[1] For the approximate plotting of the cross flux and also of the main flux, see "E.M.F. Wave Forms," by E. A. Biedermann and J. B. Sparks, *Journ. I.E.E.*, vol. 35, p. 493.

$$\Phi_c = \frac{1{\cdot}257\, AT_{\text{CM}} \times yL_f}{Kl_g}$$

$$= 1{\cdot}257\, k_C \frac{JZ}{4p} \times \frac{yL_f}{Kl_g} \cos \phi_0 \qquad . \quad . \quad . \quad (246)$$

For chamfered pole-faces in which the length of air-gap increases from a minimum in the centre to a maximum at the pole-edge, a convenient correction in the value of AT_{CM} has been given by Mr. N. Shuttleworth, *Journ. I.E.E.* Vol. 50, p. 551.

§ 33. The virtual value of the cross E.M.F.—In order to determine the virtual E.M.F. from the cross flux, the accurate process will be to convert the flux curve of Fig. 532 into an E.M.F. curve in terms of the volts produced by one component coil of the armature winding in a single pair of slots; then to plot one or more such curves spaced at the correct distance apart according as there are one, two, or more slots per pole per phase, and finally to add together algebraically the ordinates of the component curves, rounding off the sharp corners to form a resultant instantaneous E.M.F. curve.[2] The differential coefficient k_{dc} is then

$$\frac{\text{area of resultant E.M.F. curve}}{\text{area of single E.M.F. curve} \times \text{number of slots per pole per phase}}$$

Converting the curve of resultant E.M.F. plotted by rectangular co-ordinates into a polar curve, and finding the mean radius of the polar curve, the factor is

$$k_{fc} = \frac{\text{mean radius of polar curve}}{\text{mean ordinate of Cartesian curve}}$$

and $K_c = k_{fc} k_{dc}$.

Just as the virtual value of the main E.M.F. of the armature in relation to the resultant flux is by equation (38*a*)

$E_a = 2k_f \,.\, k_d \times f \times$ no. of active conductors in series in one phase $\times\ \Phi_a \times 10^{-8} = m \,.\, \Phi_a$,

where $\quad m = 2k_f \,.\, k_d f \times \dfrac{Z}{N_{ph} q} \times 10^{-8}$

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[1] Henderson and Nicholson, *Journ I.E.E.*, Vol. 34, p. 482.

[2] An example of the process is given in the above-cited paper of Messrs. Henderson and Nicholson, p. 472.

is the factor converting total flux of a pole-pitch into volts per phase, so the virtual value of the cross E.M.F. induced per phase in the armature conductors as they are cut by the cross flux is

$$E_c = m \frac{K_c}{k_f \, . \, k_d} \Phi_c$$

The virtual cross E.M.F. as above obtained includes the harmonics of which that of triple frequency is usually strongly pronounced. Strictly then to compare with the E.M.F. E_a from the main undistorted field for which the fundamental B_{g1} is obtained, the fundamental of the non-sinusoidal cross flux should be extracted, in order that the E.M.F. from it may be treated as a vector.

The value of K_c in a particular machine[1] with 3 slots per pole per phase and a pole-width 0·75 of the pole-pitch was 0·89 × 1·06 = 0·945, and in general it does not differ very greatly from $k_f \, . \, k_d$ in the equation for the virtual E.M.F. E_a from the main field. If the two are taken as identical, and $\mathfrak{R}_{cc}$ is identified with the reluctance of an air-gap, $\frac{E_c}{E_a} = \frac{AT_{\text{CM}}}{AT_g}$,

$$\text{or } E_c = m \, . \, \frac{1{\cdot}257 \, k_C \, JZ \times yL_f}{4p \times Kl_g} \cos \phi_o \quad . \quad . \quad (247)$$

§ 34. The single-phase alternator.—It has been shown that in the single-phase alternator the armature reaction pulsates as the armature coils and poles change their relative position through rotation, and such pulsation can be rendered evident by experiment as "beats" on ammeter and voltmeter.[2] The pulsation affects both the direct and the cross magnetizing turns, and due to this the determination of the actual effect of either the one or the other is open to much more uncertainty than in the polyphase machine But in the single-phase alternator the alternating M M F may be resolved into two equal constant M M F.'s, each of half the amplitude, one rotating in the same direction and at the same speed as the poles (and, therefore, fixed in relation to them), and the other in the opposite direction and also at the same speed. The latter by itself would cause a pulsating flux through the field-magnet of double the machine frequency. The above expressions for the average effect $\overline{AT}_{\text{DM}}$ when applied to the single-phase alternator thus involve the assumption that the latter component may be completely neglected, and the expression is only true if the armature is provided with a practically perfect amortisseur or damper winding If the pole-pieces and magnet-cores are solid, eddy-currents are set up which do very appreciably limit the amplitude of the variations ; and even if the iron is laminated, the exciting coils will partially act as a secondary. In the polyphase machine the same objection only applies to the higher harmonics.

In order, then, to render the method based upon average values for the direct and cross-magnetizing effects more true in the case of the single-phase alternator, it is necessary to increase the value of the self-inductance of the armature $\mathcal{L}_a$ as compared with its value under otherwise similar conditions in the polyphase machine. Taking the theoretical values for AT_{DM} and AT_{CM}, calculated from equations (243) and (245) with $N_{ph} = 1$, the fluxes which they yield are respectively $\frac{1{\cdot}257 \, AT_{\text{DM}}}{\mathfrak{R}_d}$ and $\frac{1{\cdot}257 \, AT_{\text{CM}}}{\mathfrak{R}_{cc}}$, where $\mathfrak{R}_d$ and $\mathfrak{R}_{cc}$ are the reluctances of the direct and cross-circuits. In the same way as the equivalent cross reactance x_b and cross inductance $\mathcal{L}_c$ are obtained in the next section, so from the equivalent direct reactance x_d of § 42 is obtained the apparent direct self-inductance of a phase (or of a branch of a complete phase when $q > 1$), viz.,

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[1] Henderson and Nicholson, *loc. cit.*

[2] Atchison, *Journ. I.E.E.*, Vol. 33, p. 1085.

$$\mathcal{L}_d = \frac{pt^2}{q} \frac{4\pi\, k_{s1}'\, k_{s1}''\, k_D\, N_{ph}}{2\sqrt{2}\, \mathfrak{R}_d} \times 10^{-9} \text{ henrys}$$

The *additional* parasitic inductance is then equal to the mean of the two values $\mathcal{L}_d$ and $\mathcal{L}_c$, reduced by some coefficient k less than unity to allow for the greater or lesser amount of damping by surrounding solid masses or by actual damping circuits, so that $\mathcal{L}_a' = \mathcal{L}_a + k \cdot \frac{\mathcal{L}_d + \mathcal{L}_c}{2}$. Finally, in proportion as k is decreased the resistance of the armature must be increased to some value greater than r_a in accordance with the energy which is lost by the eddy-currents, for this energy must be supplied by the armature. If the actual values of the armature inductance are measured with the magnet poles first opposite to the armature poles and then crossed, half of the mean value or $\frac{\mathcal{L}_1 + \mathcal{L}_2}{4}$ may be taken as the equivalent of $\mathcal{L}_a + k \cdot \frac{\mathcal{L}_d + \mathcal{L}_c}{2}$. Account is taken of the remaining half in the average direct and cross ampere-turns. It is evident that in the matter of its regulation the single-phase alternator is at a disadvantage as compared with the polyphase machine, since any additional self-inductance will further prejudice the regulation.

§ 35. The vector diagram of E.M.F.'s in the salient-pole machine.—The complete vector diagram for the salient-pole machine can now be drawn, and therefrom can be determined the ampere-turns of excitation required on the field-magnet to produce a given terminal voltage under load, the magnetization curves of the alternator being assumed to be known. The two vectors E_c and E_a (Fig. 508) will be in quadrature, since the cross flux is in quadrature with the main field flux, and by combining the two at right angles to one another the virtual value of the real induced E.M.F. E_i which is greater than E_a is obtained. Although the diagram of vectors is based on and presupposes sinusoidal fluxes, the increase of E_i as compared with E_a, although the total value of the flux Φ_a remains constant, does in fact correspond to the more peaked shape of the resultant flux curve, which increases the form factor and the virtual E.M.F. for the same value of the average E.M.F.

But a difficulty would still appear to remain in that the angle of lag of the current vector behind the interpolar line of symmetry is not yet known and, therefore, the cross E.M.F. from the transverse reaction is unknown. The difficulty, however, disappears through a simple geometrical construction, when the cross effect has been converted into an electric voltage, and it is through its enabling the value of ϕ_0 to be closely determined that the special advantage of the two-reaction method of Prof. A. Blondel lies.

$$\text{Let } x_b = \frac{m}{q} \cdot \frac{1{\cdot}257\, k_C\, Z\, yL_i}{4p \times Kl_g} \quad . \quad . \quad . \quad . \quad . \quad (248)$$

which for a steady sinusoidal flux becomes

$$= 8f\left(\frac{Z \,.\, k_{s1}' \, k_{s1}''}{q}\right)^2 \frac{YL_f}{2p\, N_{ph}\, Kl_g}\left\{1 - \cos\left(\frac{\pi}{2} \times \frac{y}{Y}\right)\right\} \times 10^{-9} \qquad (248a)$$

All the factors in this are known from the constructional data of the machine, and when written as $x_b = 2\pi f\, \mathcal{L}_c/q$ it is evidently of the nature of a reactance arranged to be multiplied by $I = qJ$, the corresponding inductance per branch of a phase being

$$\mathcal{L}_c = \left(\frac{Z}{N_{ph}\, q}\right)^2 \frac{(k_{s1}' \, k_{s1}'')^2\, 0{\cdot}636\, q\, N_{ph}}{p\, \mathcal{R}_{cc}} \frac{Y}{y}\left\{1 - \cos\left(\frac{\pi}{2} \times \frac{y}{Y}\right)\right\} \times 10^{-9}$$

or

$$= \frac{2p}{q} t^2 \times 4\pi \frac{(k_{s1}' \, k_{s1}'')^2\, N_{ph}}{\pi^2\, \mathcal{R}_{cc}} \frac{Y}{y}\left\{1 - \cos\left(\frac{\pi}{2}\frac{y}{Y}\right)\right\} \times 10^{-9}$$

$$= \frac{p}{q} t^2 \times \frac{4\pi\, k_{s1}' \, k_{s1}'' \, k_c\, N_{ph}}{2\sqrt{2}\, \mathcal{R}_{cc}} \times 10^{-9} \text{ henrys}$$

where the intermediate part corresponds to the effective permeance if all the cross flux were linked with all the *t* turns (*cp*. the expression for $\mathcal{L}_a$ in § 48 at foot of p. 197).

Now suppose that $\mathcal{L}_c/q$ is acted upon by the full value I of the current per phase, instead of by $I \cos \phi_o$ only. Then Ix_b is an imaginary self-induced voltage at right angles to the phase of the current and in phase with Ix_a. But though Ix_b is imaginary, it is equal to $E_c/\cos \phi_o$, where E_c is the real cross E.M.F., and ϕ_o is obtainable by the simple process of producing $BC = Ix_a$ (Fig. 508) by a length $CD = Ix_b$, and joining OD. Upon OD let fall the perpendicular CF from C; then the angle between OD and OI is the required angle ϕ_o by which the vector of the current OI lags behind the interpolar line of symmetry, and $FC = E_c$, while $OF =$ the E.M.F. which is caused by the main field flux Φ_a, if this were undistorted, and to which Φ_a must be proportional. That this construction fulfils all the requirements can be seen from the following considerations. If DB is produced to meet OI at G, it is evident that the triangle DOG is similar to the triangle DCF, and angle $DCF =$ angle DOG, or ϕ_o. Hence $FC = DC \,.\, \cos \phi_o = E_c$, and this is at right angles to OF, the vector of the E.M.F. which is due to the main flux, and which coincides in phase with the interpolar line of symmetry.

The angle ϕ_o is thus given by the relation

$$\tan \phi_o = \frac{DG}{OG} = \frac{GB + BC + CD}{OG}$$

$$= \frac{V_e \,.\, \sin \phi_e + I(x_a + x_b)}{V_e \,.\, \cos \phi_e + Ir_a'} \quad . \quad . \quad . \quad . \quad (249)$$

$$= \frac{V_e \,.\, \sin \phi_e + 2\pi f\, (\mathcal{L}_a + \mathcal{L}_c)\, I/q}{V_e \cos \phi_e + Ir_a'} \quad . \quad . \quad (249a)$$

$k_f \,.\, k_d$ having been assumed the same for both E.M.F.'s, and $\mathfrak{R}_{cc}$ having been identified with the reluctance of the air-gap, the angle between E_a and E_t or $\sin^{-1} \frac{E_c}{\sqrt{E_a^2 + E_c^2}}$ is also

$$= \sin^{-1} \frac{AT_{\mathrm{CM}}}{\sqrt{AT_g^2 + AT_{\mathrm{CM}}^2}}$$

but this latter equality does not necessarily hold except upon the above assumptions.

In the case of a finished machine, x_b may be obtained approximately by referring to the no-load characteristic and taking any two corresponding values of E and AT on the straight part of the curve before saturation becomes appreciable to find E/AT, the vólts per ampere-turn of excitation for a half magnetic-circuit. Hence the volts per one virtual ampere in the armature phase with its given disposition of winding are

$$x_b = \frac{E}{AT} \times \frac{K_c}{k_f k_d} \times k_c \cdot \frac{Z}{4pq}$$

The angle ϕ_o for a given value of ϕ_e is thus chiefly dependent upon the cross E.M.F. or inductance $\mathscr{L}_c$. The cross magnetization so far from having any direct effect in causing the fall of terminal voltage under load, slightly increases the voltage (in the actual machine through the form factor being increased by the distortion); yet it has a most important indirect effect, inasmuch as it partly determines the angle ϕ_o upon the sine of which depends the value of the *direct-magnetizing* ampere-turns.

Although the field after distortion by the cross ampere-turns (even when the distortion remains practically constant as in a polyphase machine) cannot as a matter of fact give a sine curve of E.M.F., so that no deductions as to the instantaneous values of the field or of the power can be based upon the relative positions and values of the vectors of E.M.F. and current, yet such a diagram as that of Fig. 508 does give a clue to the actual state of the case. If an instantaneous photograph of the flux of an alternator when running under load could be taken, the lines of the field crossing the air-gap from pole to armature core would be found to correspond to E_t; their number would correspond to E_a, but the effect of their non-uniform distribution or concentration under the distorting effect of the cross ampere-turns is given in E_t. The diagram shows that the maximum of the displaced field preceding E_t by a right angle would no longer coincide with the centre of the pole as given by a line at right angles to E_a, except in the special case of short-circuit when the lag is a maximum and the distortion is a minimum. Upon the angle of lag of the current behind the actual distribution of the field which is the same as the angle ϕ_i between E_t and the current vector I depends the magnetic pull upon the armature wires or torque on the machine as a whole, and any deductions as to its amount would have to be based upon E_t rather than upon E_a.

§ 36. The field excitation of the alternator under load.—The diagram of E.M.F.'s is thus complete, and we pass to the question of the ampere-turns of excitation. In order to embrace the case of well saturated field-magnets, it is essential to employ two partial characteristics, the one connecting Φ_a with the excitation required over the armature core, teeth, and air-gap, *i.e.* over the half of the magnetic circuit, excluding the magnet pole and yoke which are affected by the field leakage, and the other a curve connecting the total flux in the magnet system Φ_m with the excitation required over the pole and yoke only (Fig. 533). The useful flux Φ_a to be supplied by the field-magnet within the air-gap is proportional to E_a, and is thence determined; the net ampere-turns over the air-gap, stator teeth and core, which must be supplied by the field-magnet at the point of emergence of the lines from the pole into the air, are then found from the partial characteristic as $OH = AT_c + AT_t + AT_g$. But the ampere-turns between the poles must also counterbalance the direct reaction. Let $HK = AT_{DM}$, as above calculated in § 29. Therefore $OK = AT_c + AT_t + AT_g + AT_{DM} = AT_P$, the pole-face potential in AT.

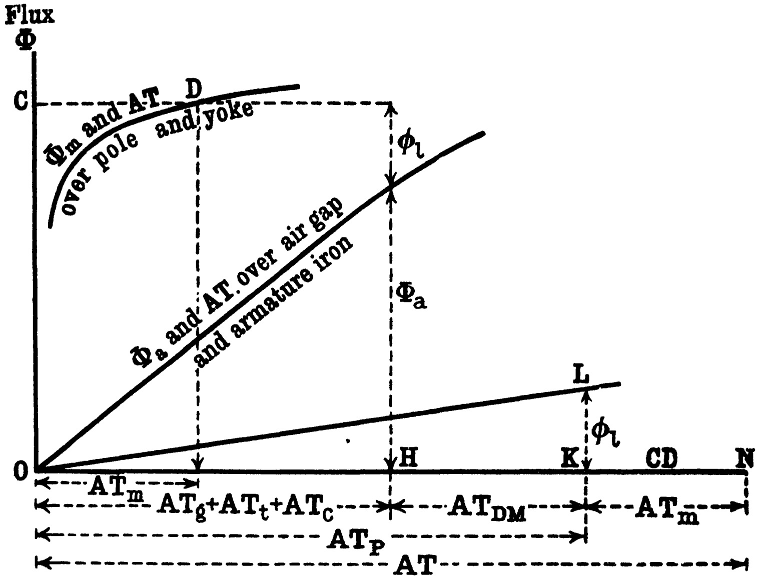
Flux
Φ
C
D
Φm and AT over pole and yoke
φl
Φa and AT over air gap and armature iron
Φa
L
φl
O
H
K
CD
N
ATm
ATg+ATt+ATc
ATDM
ATm
ATP
AT

FIG. 533.—Partial magnetic characteristic curves of alternator.

The leakage flux is $\phi_l = 1{\cdot}257 \times 2AT_P \times \mathfrak{S}_l$, where $\mathfrak{S}_l$ is the leakage permeance of the air-paths reduced to a quantity that may be regarded as in parallel with the armature path. The value of ϕ_l may be conveniently shown in relation to the value of AT_p measured along the axis of abscissæ as the straight line OL. For any particular value of Φ_a a number of different values of $\Phi_m = \Phi_a + \phi_l$, and of the density in the pole and yoke may thus be obtained according to the power-factor and consequent value of the angle ϕ_o. Exactly as in the continuous-current dynamo (without commutating poles) the total ampere-turns of excitation for a given Φ_a and armature current are really dependent upon the position of the brushes and the consequent value of AT_b as affecting the leakage, so in the alternator the values of Φ_a and of the armature current do not determine the necessary excitation. The phase of the current vector relatively to the interpolar line of symmetry must also be known, and it is on this account that the total characteristic of excitation cannot be drawn once and for all. The amount $CD = AT_m$ for pole and yoke must, therefore, be added to AT_p, and the total excitation on a pole required under the particular circumstances is $ON = AT_p + AT_m = AT_f$. The only difference from the continuous-current dynamo is that the brushes in the latter usually have a fixed position for each value of the current, namely, that of sparklessness, so that, in addition to the total excitation curve for no-load, similar curves, say, for half- or full-load, can also be calculated, which under ordinary conditions will remain true, and are, therefore, of real service.

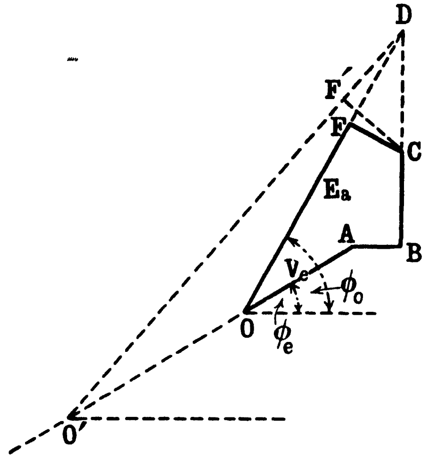

FIG. 534 —Construction to find armature E.M F. for given current with varying external voltage at terminals.

§ 37. The external characteristic under load with different power-factors.—To calculate a full-load characteristic or the complete curve connecting terminal voltages as ordinates with the AT of total excitation (or field-current) as abscissæ for any particular armature current and power-factor of the external circuit (or value of ϕ_e) is now a simple matter. In Fig. 534 $AB = Ir_a'$, as also $BC = Ix_a$, and $CD = Ix_b$ are then constant. From A a line is drawn inclined at an angle of $180° - \phi_e$ to AB, and along this line the terminal voltage V_e per phase is measured. Taking any value for this, say, AO, join OD, whence ϕ_o is found, and upon OD let fall from C the perpendicular CF. Then $OF = E_a$, to which the flux through the armature or Φ_a is proportional. Another value for $V_e = AO'$ is then taken, and the corresponding E_a' and Φ_a'

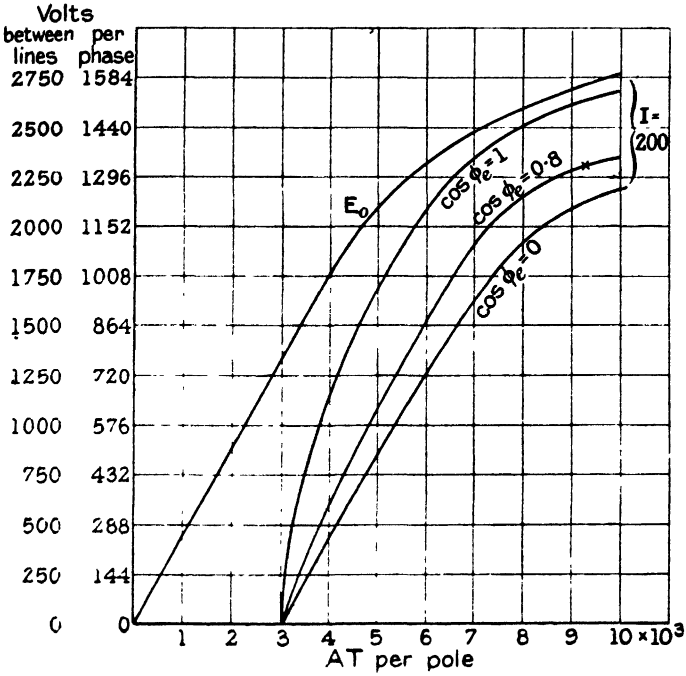

FIG. 535.—External characteristic curves of voltage of alternator.

determined, and so on. The calculation is as easily made analytically as graphically, from equation (249); for each value of V_e, DG and OG (Fig. 508) are calculated, and from $\tan \phi_o = DG/OG$, ϕ_o is found. Further, $OD = \sqrt{(DG)^2 + (OG)^2}$, and[1]

$$E_a = OD - x_b I \sin \phi_o \quad . \quad . \quad . \quad . \quad . \quad . \quad (250)$$

From the lower curve of Fig. 533 the ampere-turns over an air-gap, teeth and armature core for each value of Φ_a are found, and AT_{DM} is calculated from the known value of $I \sin \phi_o$. The two added together give AT_P, whence the leakage is calculated and added to Φ_a. The corresponding value of AT_m is then read off from the upper curve of Fig. 533, and when added to AT_P gives the total excitation per pole, AT_f. A number of points having been obtained for, say, I = full-load current and $\phi_e = 0$, a curve is drawn which will fall below the no-load characteristic; below this again will fall a similarly drawn curve for $\phi_e = 90°$, and between these two limiting cases will lie the terminal voltage for all values of ϕ_e with the given current I.

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[1] Here and subsequently in similar expressions ϕ_o is for practical convenience treated as positive when as usual it is an angle of lag.

When the process is applied to the alternator designed in Chapter XXIX, § 10, the curves so obtained for the three cases of unity power-factor, cos $\phi_e = 0{\cdot}8$, and zero power-factor are plotted in Fig. 535. It will be seen that for unity power-factor, as the terminal voltage rises from zero the decreasing value of ϕ_0 and so of AT_{DM} causes the terminal voltage with full armature current to approach gradually the no-load curve of E.M.F., but that as saturation of the field-magnet begins the increasing excitation required by the pole and yoke causes the two curves again to draw farther apart. With the intermediate power-factor of 0·8 the latter of these two effects has begun to show, and with zero power-factor the two curves throughout diverge horizontally farther from one another.

So far certain values of the terminal voltage have been taken, and the necessary excitation calculated under the given conditions of current and power-factor. When it is required to solve the converse problem, namely, with a constant excitation to find the terminal voltage or the fall of volts as a function of the load, it is best to proceed systematically by first determining the working parts of the curves exactly as above, from which the further results which are required may be deduced or interpolated.

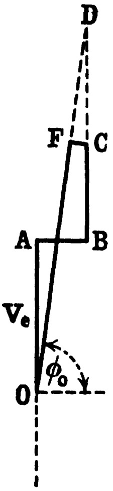

FIG. 536. Vector diagram of wattless current, cos $\phi_e = 0$.

§ 38. The wattless current characteristic with zero power-factor.—If cos $\phi_e = 0$, *i.e.* if the current lags relatively to the terminal voltage by 90°, and so is wattless, Fig. 536 shows that the angle ϕ_0 is always large, and as the terminal voltage is raised, still further increases until it nearly approaches 90° Sin ϕ_0 may, therefore, be at once identified with unity, and $AT_{DM} = k_D \cdot JZ/4p$, or the maximum direct magnetizing ampere-turns of the armature. $FC = E_C$ practically vanishes, and E_a is nearly $= V_e + Ix_a$. The volts required to pass the current over the resistance of the armature have practically no effect, since they are entirely out of phase with the terminal voltage and the self-induced E.M F. due to the armature reactance.

The quantity $k_D\ JZ/4p$ or the *magnetizing ampere-turns of the armature*[1] as resulting from the composition of the several phases, and as affecting a half magnetic circuit (without reference to the actual angle of lag or on the assumption that it is a maximum) may be symbolized as AT_a. Neglecting, then, the *difference* produced in the field leakage, and therefore, in the reluctance of the field-magnet by the addition of AT_{DM} or AT_a, if OB ampere-turns (Fig. 537) produce a flux corresponding to an E M F. BP on the open-circuit curve, a resultant excitation $AT_R = AT_f - AT_a$ will produce the same flux, so that if a length $BA = AT_a$ is added to OB, a total excitation of $OA = AT_f$ will with a wattless current of value I lagging 90° produce the same E.M.F. BP. From this must be deducted an amount $PQ = e_{sa} = Ix_a$, so that the actual terminal voltage $V_e = BQ$. Since this value occurs with a total field excitation OA per pole, BQ must be transferred horizontally across as shown at AP'. Thus P' is one point on the curve of terminal voltage for a wattless current of value I, and is seen to be obtained by shifting the point P from the curve of the open-circuit voltage E_0 vertically downwards by an amount

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[1] The maximum magnetizing ampere-turns of the armature per pole may also be expressed through the *ampere-conductors* per pole $JZ/2p$; to give ampere-turns per pole, k_D must then be halved, or retaining its former value $AT_a = \frac{1}{2}k_D\ JZ/2p$. But $JZ/4p$ is throughout retained here, as being strictly ampere-turns per pole.

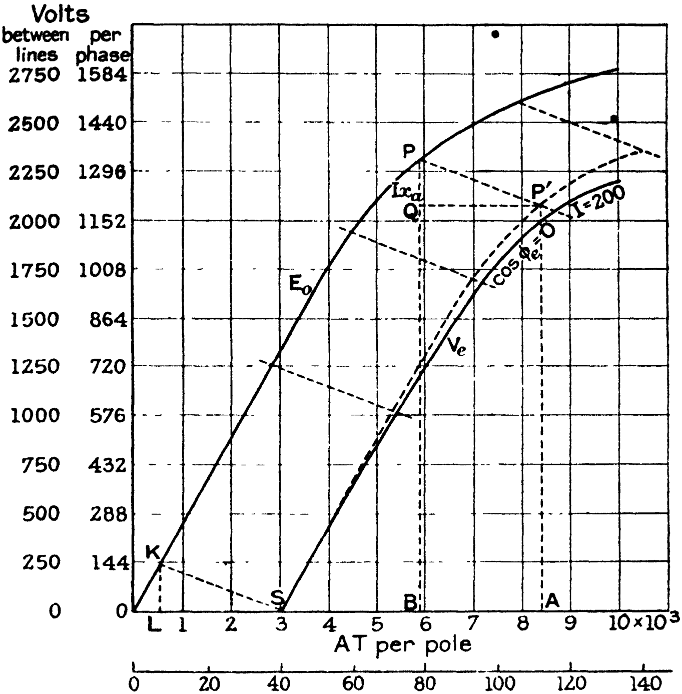

FIG. 537 —Analysis of terminal voltage curve for wattless current.

Ix_a, and horizontally along towards the right through a distance $AT_a = k_D JZ/4p$. Any other point may be similarly treated as shown by the triangle KLS, where the special value of the field ampere-turns is that which gives the value I for the armature current on the short-circuit curve. The whole curve of the open-circuit E.M.F. when shifted aslant through a distance PP', as shown by the inclined dotted lines, would then give a characteristic of terminal voltage for a given wattless current. When the machine is running as a synchronous motor, and taking a wattless current, the curve is shifted along the same inclined lines, but in the opposite direction, so that it lies above the curve of E_0. The expression for the terminal voltage with a wattless current is, under the above assumptions,

$$V_e = m \cdot \frac{1{\cdot}257\,(AT_f - k_D JZ/4p)}{\Re_d} - Ix_a,$$

where m is the factor converting flux into volts. When the abscissæ are in amperes of exciting current I_f flowing through T_f turns wound on each pole, $V_e = \frac{m}{\Re_d} \cdot 1{\cdot}257\, T_f\left(I_f - \frac{k_D JZ/4p}{T_f}\right) - Ix_a$, and in the expression for the horizontal distance $QP' = kI$, k becomes the constant by which the magnetizing ampere-turns of the armature can be converted into the equivalent current through the T_f turns on a half magnetic circuit; its value is then $k = \frac{k_D . Z}{4pq\,T_f}$.

The above analysis of the terminal voltage curve for a constant wattless current, identified with the name of M. Potier[1] and based upon the previous work of Prof. Kapp and A. Blondel, will be found to yield valuable consequences in the practical work of designing, about which more will be said in § 49 The amount of error involved in the assumption that the reluctance of the field-magnet is a constant, and that the *alteration* in the field leakage may be neglected, depends entirely upon the degree of saturation at which the magnet system is normally worked, and its extent is indicated in a particular case in Fig 537. The more correct curve which takes into account the increased field leakage due to the additional of AT_a is repeated in full line from Fig 535, and it is seen that the approximate curve (shown dotted) while agreeing for low values of the excitation, draws slightly away at high values.

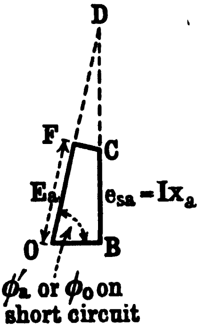

FIG. 538.—Short-circuit diagram of alternator.

§ 39. The short-circuit characteristic.—On short-circuit $V_e = 0$,

$$\tan\phi_o = \frac{I_{sc}(x_a + x_b)}{I_{sc}\,r_a'} = \frac{x_a + x_b}{r_a'}$$

becomes a constant fixed by the constructional data of the machine, and the particular value which ϕ_o then always assumes is the internal angle of lag due to the armature alone, and may be specially symbolized as ϕ_a'. The special value of OD on short-circuit is

$$O'D' = I_{sc}\sqrt{(x_a + x_b)^2 + (r_a')^2} = I_{sc}\,z_a'$$

where z_a' is practically a constant for the machine. It follows that the triangle which is the form that the vector diagram of E M F 's then takes is always similar to that which has been drawn in Fig 538 for one particular value of the short-circuit current. $E_a = O'D' - x_b . I_{sc} . \sin\phi_a'$ is now $=$

$$I_{sc}(z_a' - x_b . \sin\phi_a').$$

But $$\frac{E_a}{m} = \Phi_a = \frac{1{\cdot}257\,(AT_c + AT_t + AT_g)}{\Re_c + \Re_t + \Re_g}$$

Therefore $$AT_c + AT_t + AT_g = 0{\cdot}8\,I_{sc}\,\frac{(z_a' - x_b \sin\phi_a')}{m}\,(\Re_c + \Re_t + \Re_g)$$

Also $$AT_{DM} = k_D\,\frac{Z}{4pq}\,I_{sc}\sin\phi_a', \text{ so that}$$

$$AT_P = I_{sc}\left\{k_D\,\frac{Z}{4pq}\sin\phi_a' + 0{\cdot}8\,\frac{(z_a' - x_b\sin\phi_a')}{m}\,(\Re_c + \Re_t + \Re_g)\right\}$$

The field leakage is $\phi_l = 1{\cdot}257 \times 2\,AT_P \times \mathfrak{P}_l$ Therefore the flux in the magnet on short-circuit is

$$\Phi_{msc} = I_{sc}\left[1{\cdot}257\,k_D\,\frac{Z}{4pq}\sin\phi_a' \times 2\,\mathfrak{P}_l + \frac{(z_a' - x_b\sin\phi_a')}{m}\left\{1 + (\Re_c + \Re_t + \Re_g)\,2\,\mathfrak{P}_l\right\}\right]$$

$= \nu\,\Phi_a$, where ν is the leakage coefficient under the given conditions.

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[1] From his article " Sur la Reaction d'Induit des Alternateurs " in *L'Éclairage Électrique*, Vol. 24, p. 133 ff.

All the quantities are thus expressed in terms of the short-circuit current, and since $AT_m + AT_y = f'\left(\frac{\Phi_{msc}}{a_m}\right) l_m + f'\left(\frac{\Phi_{msc}}{a_y}\right) \frac{l_y}{2}$

$$\begin{aligned} AT_{fsc} &= AT_{\mathrm{P}} + AT_m + AT_y \\ &= I_{sc}\left\{k_{\mathrm{D}} \frac{Z}{4pq} \sin \phi_a' + 0{\cdot}8 \frac{(z_a' - x_b \sin \phi_a')}{m} (\mathfrak{R}_c + \mathfrak{R}_t + \mathfrak{R}_g)\right\} \\ &\qquad + f'(B_{msc})\, l_m + f'(B_{ysc}) \frac{l_y}{2} \quad . \quad . \quad (251) \end{aligned}$$

or since $\nu \Phi_a (\mathfrak{R}_m + \mathfrak{R}_y) = 1{\cdot}257\,(AT_m + AT_y)$

$$AT_{fsc} = I_{sc}\left\{k_{\mathrm{D}} \frac{Z}{4pq} \sin \phi_a' + 0{\cdot}8 \frac{(z_a' - x_b \sin \phi_a')}{m} \times \mathfrak{R}_{do}\right\}$$

where $\mathfrak{R}_{do}$ = the sum of the direct reluctances $\mathfrak{R}_c + \mathfrak{R}_t + \mathfrak{R}_g + \nu(\mathfrak{R}_m + \mathfrak{R}_y)$ for the low value of the saturation which holds in the magnet-core and yoke whether under short-circuit or open circuit.

The angle ϕ_a' or ϕ_o on short-circuit is in almost all cases greater than 80°, and sin ϕ_a' is but little different from unity. If ϕ_a' be identified with 90°, *i.e.* if r_a' is negligible by comparison with $x_a' = x_a + x_b$

$$z_a' - x_b \sin \phi_a' = (x_a + x_b) - x_b = x_a$$

and E_a becomes simply $= e_{sa} = I_{sc}\, x_a$, or proportional to I_{sc}.

Hence very closely

$$AT_{fsc} = I_{sc}\left(k_{\mathrm{D}} \frac{Z}{4pq} + 0{\cdot}8 \frac{x_a}{m} \mathfrak{R}_{do}\right) . \quad . \quad . \quad . \quad . \quad . \quad (251a)$$

The fact that AT_{DM} is really $k_{\mathrm{D}} \frac{Z}{4pq} I_{sc} \sin \phi_a'$, say $0{\cdot}985\, k_{\mathrm{D}} \frac{Z}{4pq} I_{sc}$, may be set against the fact that the actually existing field lines must correspond to $I_{sc}(z_a' - x_b \sin \phi_a')$ which exceeds $I_{sc}\, x_a$.

For the low values of the flux-densities which are alone in question under short-circuit conditions, $\mathfrak{R}_t + \mathfrak{R}_c$ is so nearly constant, and in any case bears so small a proportion to $\mathfrak{R}_g$ that AT_{P} increases almost strictly in proportion to I_{sc}; consequently ϕ_l increases in the same proportion, as also does Φ_a. The ratio $(\Phi_a + \phi_l)/\Phi_a$ thus retains the same value, or ν is constant at such a value as 1·87. The only error, therefore, is in the assumption that $\mathfrak{R}_m + \mathfrak{R}_y$ remains constant, *i.e.* the *change* in the magnet reluctance due to increasing Φ_m is neglected. But although this may be considerable, yet so long as the absolute value of the magnet reluctance bears but a small ratio to the reluctances of air-gap, stator teeth and core, the approximation involves but little final error. It thence results that the short-circuit current when plotted in relation to the exciting current or exciting ampere-turns as abscissæ will be a straight line, and such is usually found to be the case (Fig. 618*b*), so long as the flux through the iron magnet on short-circuit remains comparatively small. The value of $0{\cdot}8 \frac{I_{sc}\, x_a}{m} . \mathfrak{R}_{do}$ may then be identified with the no-load excitation $AT_{fo} = 0{\cdot}8 \frac{E_a}{m} R_{do}$ as obtained from the no-load curve connecting E_a and Φ_a.

Having now described the chief features of the characteristic curves of alternators, we pass to the question of their " regulation."

§ 40. Values of inherent regulation.—For the purpose of estimating the degree to which different alternators tend to maintain a constant terminal voltage under varying loads, their ***inherent regulation*** may be compared. The definition of this, authoritatively adopted by the British Standardisation Rules for Electrical

Machinery[1] (§ 18), is the *rise* in volts when the full rated output at the specified power-factor and rated pressure is removed, the speed and excitation being maintained constant at the values for the rated output, and the rise being expressed as a percentage of the rated pressure of the full output. "Regulation up," as this has been called, indicates roughly how far the alternator may be left without attention under varying loads. An alternative definition is the *fall* in volts which occurs if the machine is excited to give its rated pressure at no-load and full-load is then switched on while the excitation and speed are kept constant, *i.e.* "regulation down"; the fall, being expressed as a percentage of the no-load terminal volts, then rather indicates the amount by which the excitation must be increased by hand or automatically in order to maintain a constant terminal voltage. Owing to the greater degree of saturation and greater field leakage in the former case, the first definition gives appreciably smaller values than the second.

Thus the inherent regulation varies according to the nature of the external full-load, and the power-factor of this must be stated. Two figures are usually given for this, viz., $\cos \phi_e = 1$ and $\cos \phi_e = 0{\cdot}8$, and under these conditions normal values for the inherent regulation up will be of the order of, say, 8 per cent. and 22 per cent. respectively, while the regulation down in the former case will be, say, 12 to 15 per cent. The above figures presuppose a polyphase generator with well-distributed winding, and equally good inherent regulation can only be obtained in the single-phase alternator at greater cost of iron and copper in the field-magnet.

In a machine with a large amount of armature reaction and correspondingly poor inherent regulation the permanent short-circuit current might only exceed the full-load current by 50 per cent. even with the normal excitation for full-load. It would then be impossible for the machine to be very seriously overloaded or for its armature to be immediately burnt out through an accidental short-circuit. This advantage is, however, bought at the expense of the regulation, and when this is important, a compromise must be struck between the two desiderata, such a rise of voltage being then called for in the specifications as can be obtained with reasonable cost of copper and manufacture. For a few seconds a current several times the normal will do no serious harm, and beyond this reliance must be placed on fuses or automatic circuit-breakers to protect the machine in the case of a prolonged short-circuit. Within limits the greater the ratio which the short-circuit current of the alternator bears to the full-load current, the better is the machine, and in modern machines it is usually from 3 to 4 times the full-load current.

The importance formerly attaching to close inherent regulation has, however, been largely reduced by the greater use in central stations of automatic voltage regulators—so much so that while formerly regulations of 5 per cent. on unity power-factor and of 17 per cent. on a power-factor of 0·8 were frequently called for, the limits previously given are now often extended to 10 per cent. and 25 per cent., or even to 20 per cent. with $\cos \phi_e = 1$ and 40 per cent. with $\cos \phi_e = 0{\cdot}3$.[1] Automatic voltage regulators can now deal with a range of 1 : 2 or even 1 : 2½ in the exciting current, and this would permit of even such coarse regulations as 25 per cent. on unity power-factor and 40 per cent. at a power-factor of 0·8.

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[1] No. 168, 1923. (Crosby Lockwood and Son.)

§ 41. Direct calculation of the regulation.—Although it is true, as stated by Prof. Miles Walker,[2] that " no method " of predetermining the regulation " known to the author is perfectly accurate when put to the practical test," yet the accuracy obtainable by the two-reaction method meets every practical requirement even under extreme conditions.[3] Other simpler methods based either on what is termed the " synchronous-impedance " of the machine or on the vectorial addition of ampere-turns are sometimes sufficient for preliminary calculations, but since they are confessedly only an approximation and require to be corrected to allow for iron saturation, the accurate calculation, even if more complex, may as well be taken in hand at the outset as in the end. The other methods as of lesser value are, therefore, postponed until the accurate method has here been described. For the latter, as for the former, much of the initial work can be done once for all and tabulated in curves for practical use, so that in the end it involves but little more calculation.

Thus if it is assumed that in normal cases Ir_a' is about 1·2 per cent. of V_e and that Ix_a, the true reactance voltage, is about 10 per cent. of V_e, then for different values of $\cos \phi_e$ the ratio of E_i to V_e in Fig. 508 is as shown by the upper (*b*) curve of Fig. 522 which is again repeated as the upper curve of Fig. 539.

Next upon the same assumptions, if a number of different values are assigned to the ratio Ix_b/V_e, (or if preferred to $Ix_a'/V_e = I(x_a + x_b)/V_e =$ upon the given assumption $Ix_b/V_e + 0{\cdot}1$), the corresponding ratio E_a/V_e can be obtained trigonometrically or graphically from Fig. 508, and also the value of ϕ_0. The values of the former ratio are plotted in Fig. 539, and can be similarly determined and plotted, if so desired, for other proportions of resistance and reactance drops to suit any standard line of machines. It will be seen that for high values of the power factors E_a is less than V_e owing to the action of the cross ampere-turns, which increase the height of the fundamental. Proportional to E_a is AT_{gl}, the air-gap ampere-turns under load. A normal ratio of pole-arc to pole-pitch being assumed, k_n is known and ϕ_o has been found; AT_{DM} can then be calculated and added to AT_{gl}, and thus

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[1] *Cp.* Prof A. Gray, *Franklin Inst. J.* Vol. 183, 1917.

[2] *Specification and Design of Dynamo-Electric Machinery*, p. 278.

[3] *Cp.* J. B. Henderson and J. S. Nicholson, *Journ. I E.E.*, Vol. 34 (Fig. 12), p. 483, and V. Karapetoff, *Trans. Amer. I.E.E.*, Vol. 42, pp. 144–156, where the results obtained by a mechanical device, the Blondelion, based on this method are compared with tests.

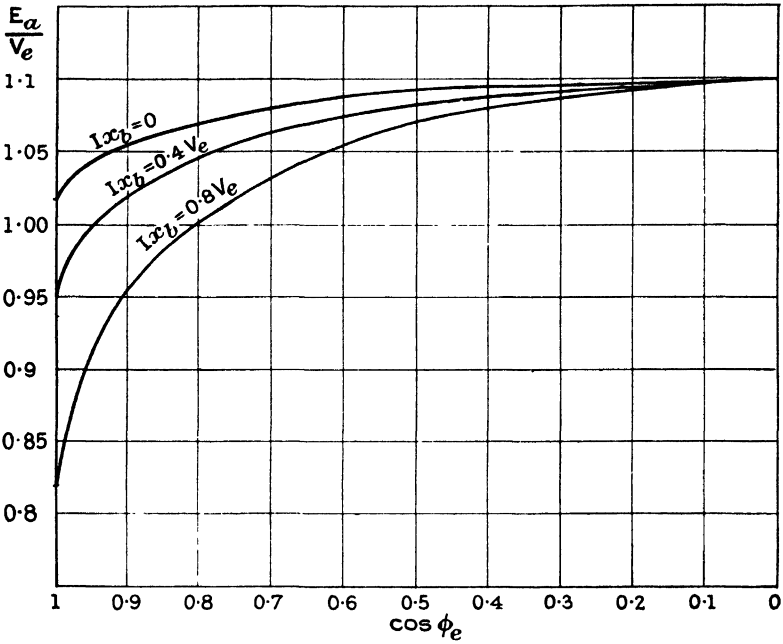

FIG. 539.—The ratios E_l/V_e and E_a/V_e on the assumption that the ohmic drop of volts and the true reactance voltage are 1·2 and 10 per cent. respectively of V_e

finally the ratio $(AT_{gl} + AT_{DM})/AT_{go}$ can be plotted once and for all (Fig. 540) in relation to the ratio of armature ampere-turns per pole $JZ/4p$ or ampere-conductors per pole $JZ/2p$ to the air-gap ampere-turns AT_{go} on no-load. Given an open-circuit or no-load magnetization curve, calculated or obtained by test, the use of the above ratio is simple. Let OB (Fig. 541) be the air-gap line, OC the same with addition of the ampere-turns expended over stator teeth and core. Let AE be the horizontal line marking the terminal voltage V_e and normal flux *at no-load.* Upon this line mark off a length $AB = AD\dfrac{AT_{gl} + AT_{DM}}{AT_{go}}$, the ratio being derived from Fig. 540. E_a may be a little more or a little less than V_e in accordance with Fig. 539, and B_g under load may be a little more or less than corresponds to no-load V_e; this has been taken into account in the calculation of AT_{gl} as $= AT_{go} \times E_a/V_e$. Hence, using the ratio E_a/V_e of Fig. 539, mark off on the air-gap line a point B above or below or on D, and thence find BC, the ampere-turns expended on stator teeth and core under load. Extend the horizontal line AB by this amount; then $AC = AT_{pl}$, the pole-face potential in AT under load. Thence is found ϕ_l and $CE = AT_m + AT_y$, and finally the full-load excitation $AT_{fl} = AT_{pl} + AT_m + AT_y$. The increase of volts when the full-load is thrown off is then EF, and the regulation up as a percentage is $100 \times FE/AO$. It will be seen from Fig. 539 that for $\cos \phi_e = 0{\cdot}8$ between wide limits of Ix_b the E_a is not greatly different from V_e, so that it is only when the teeth are highly saturated that the refinement of taking BC higher up the air-gap line becomes

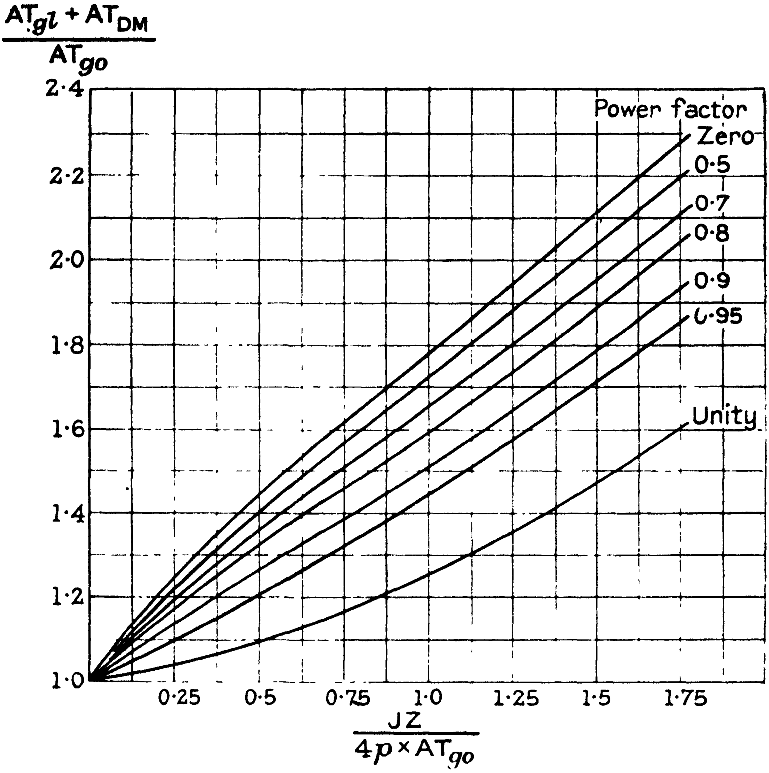

FIG. 540.—The ratio $\frac{AT_{gl} + AT_{DM}}{AT_{go}}$ on the assumptions of Fig. 539 and of a ratio of pole-arc/pole-pitch, $\beta = 0{\cdot}69$ with salient poles. ($y/Y = 0{\cdot}75$)

necessary; for such a power factor therefore the simple addition to the no-load AT_{g+t+c} (= say, AG) of a length =

$$AD\left(\frac{AT_{gl}+AT_{\text{DM}}}{AT_{go}}-1\right)$$

will often serve as a provisional estimate of $AT_{\text{P}l}$ or AC. If ϕ_{lo} be the leakage flux at no-load, the leakage flux under full-load

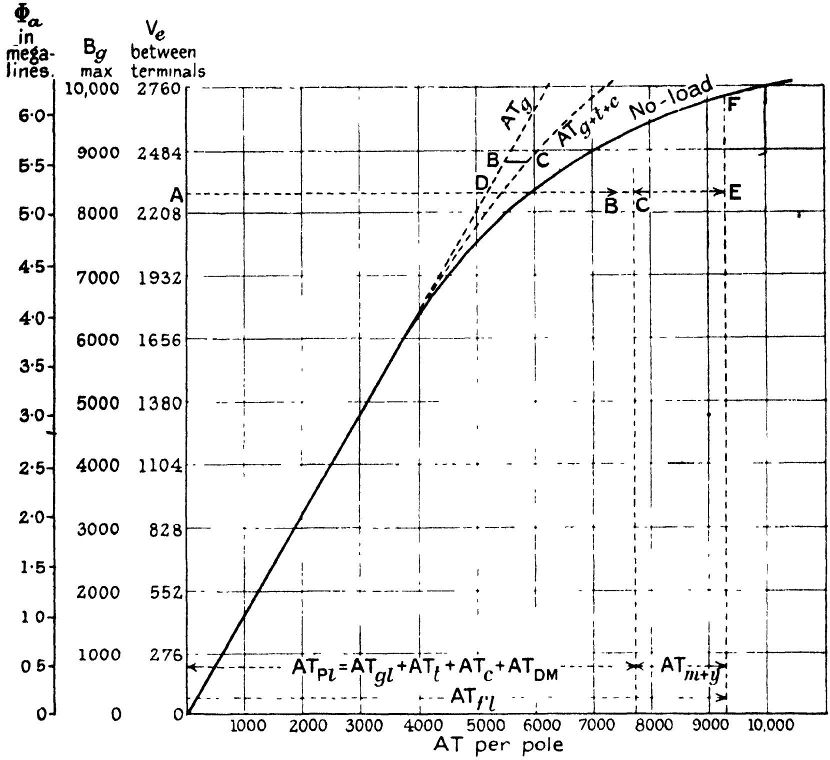

FIG. 541 —Predetermination of regulation of alternator.

is $\phi_{lo}\dfrac{AC}{AG}$, and this again is but little different from ϕ_{lo} multiplied by the ratio $(AT_{gl}+AT_{\text{DM}})/AT_{go}$ obtained from Fig. 540.

§ 42. Conversion of the vector diagram of E.M.F.'s into terms of synchronous impedance. Identification of *OD* with E_o.—The justification for expressing the effect of the cross-reaction of the armature ampere-turns by means of a fictitious flux and consequent cross E.M.F. is that the cross-reaction in practice has very little effect on the actual reluctances of the iron portions in the direction of the main field as dependent on the resultant main excitation.

The component E.M.F. $FC = x_b I \cos\phi_o$ is, however, obtained in Fig. 508 through the intermediate step of producing Ix_a by a length

$CD = Ix_b$. The same point D and the same diagram is, therefore, obtained by crediting the armature with an apparent reactance $(x_a + x_b) = x_a'$; this single quantity is then called the *synchronous reactance*, and the E.M.F. required to overcome it is the synchronous reactance E.M.F., $BD = Ix_a'$. The synchronous reactance again may be combined with r_a' to give the *synchronous impedance* $\sqrt{(x_a')^2 + (r_a')^2} = z_a'$, and the E.M.F. required to overcome it is the synchronous impedance E.M.F. $e = Iz_a' = AD$, which when combined with the terminal voltage gives the E.M.F. OD and the angle ϕ_o (*cp.* Fig. 542).

Now the dotted portion of FDC of Fig. 508 is strictly speaking fictitious, and OD has been called the " nominal generated E.M.F." The question then arises since OD does not exist simultaneously with E_a, Does it correspond to any physical reality ? The answer is that it *would be* the voltage E_0 given on open circuit by the same field-magnet excitation AT_f as is actually present under load, provided that the effect of saturation is exactly balanced by another cause of divergence in the opposite direction.

$$OD = E_a + x_b I \sin \phi_o.$$

If m be the coefficient converting flux into volts, and $\mathfrak{R}_d$ = the sum of the direct reluctances, $\mathfrak{R}_c + \mathfrak{R}_t + \mathfrak{R}_g + \nu(\mathfrak{R}_m + \mathfrak{R}_y)$ under the given conditions of load, and $AT_f = AT_{\mathrm{P}} + AT_m + AT_y$,

$$E_a = m \frac{1{\cdot}257\,(AT_f - AT_{\mathrm{DM}})}{\mathfrak{R}_d}$$

Now just as the electric effect FC of the cross AT_{CM} acting on the cross circuit of reluctance $\mathfrak{R}_{cc}$ has above been expressed through a supposed reactance $x_b = m \dfrac{1{\cdot}257\ k_c Z}{4pq\,\mathfrak{R}_{cc}}$, so the electric effect of the direct-magnetizing turns AT_{DM} acting upon the direct circuit of reluctance $\mathfrak{R}_d$ may be expressed through the supposition of a superposed back or forward flux, and a direct reactance

$$x_d = m \frac{1{\cdot}257\,k_{\mathrm{D}} Z}{4pq\,\mathfrak{R}_d}$$

requiring an E.M.F. $x_d I \sin \phi_o$, proportional to AT_{DM}. Therefore

$$OD = m \frac{1{\cdot}257\,AT_f}{\mathfrak{R}_d} - x_d I \sin \phi_o + x_b I \sin \phi_o.$$

When the load is removed, let E_0 be the open-circuit voltage, and let the direct reluctance then have risen to $\mathfrak{R}_d'$; for in spite of the lesser leakage coefficient ν', $\mathfrak{R}_m + \mathfrak{R}_y$ will have risen more than ν has fallen, so that $\nu'(\mathfrak{R}_m + \mathfrak{R}_y)'$ will exceed $\nu(\mathfrak{R}_m + \mathfrak{R}_y)$ even if $\mathfrak{R}_a + \mathfrak{R}_t$ remained practically constant.

Hence $E_0 = m \dfrac{1{\cdot}257\,AT_f}{\mathfrak{R}_d'}$, and the first term in the above expression for OD is $E_0\,\mathfrak{R}_d'/\mathfrak{R}_d$ Therefore

$$OD = E_0 \frac{\mathfrak{R}_d'}{\mathfrak{R}_d} - I \sin \phi_o\,(x_d - x_b) = E_0 \frac{\mathfrak{R}_d'}{\mathfrak{R}_d} - \frac{m \times 1{\cdot}257\,IZ \sin \phi_o}{4pq}\left(\frac{k_{\mathrm{D}}}{\mathfrak{R}_d} - \frac{k_{\mathrm{C}}}{\mathfrak{R}_{cc}}\right)$$

Whether the E.M.F. upon withdrawal of the armature current will rise to a value above or below OD or equal to it turns upon how far the second term balances the increase due to the ratio $\mathfrak{R}_d'/\mathfrak{R}_d$ arising from the changed saturation and reluctances.

To compare the two more closely, we have

$$OD = E_0 \frac{\mathfrak{R}_d'}{\mathfrak{R}_d}\left\{1 - \frac{JZ \sin \phi_o}{4p\,AT_f}\left(k_{\mathrm{D}} - \frac{\mathfrak{R}_d}{\mathfrak{R}_{cc}}\,k_{\mathrm{C}}\right)\right\}$$

Since $\mathfrak{R}_d'$ must exceed $\mathfrak{R}_d$, it follows that for exact equality of OD to E_0, $k_D - \frac{\mathfrak{R}_d}{\mathfrak{R}_{cc}} k_C$ should yield a positive difference. Since $y < Y$ (except in a truly non-salient-pole rotor), k_D is always greater than k_C in the ratio

$$\sin\left(\frac{\pi}{2} \times \frac{y}{Y}\right) \Big/ \left\{1 - \cos\left(\frac{\pi}{2} \times \frac{y}{Y}\right)\right\}$$

and especially with chamfered poles of small width, k_D greatly exceeds k_C. On the other hand, $\mathfrak{R}_d$ is always greater than $\mathfrak{R}_{cc}$, so that the difference between k_D and $k_C\ \mathfrak{R}_d/\mathfrak{R}_{cc}$ may not be very great. It results finally that if there is very little saturation and $\mathfrak{R}_d'$ is nearly $= \mathfrak{R}_d$, the error in identifying OD with E_0

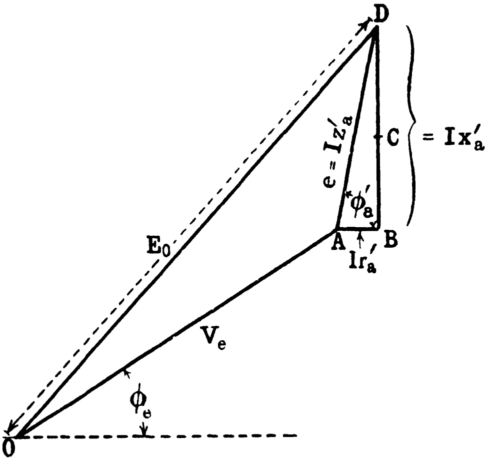

FIG. 542—Diagram based on synchronous impedance and open-circuit E M F.

increases as the load becomes more inductive and ϕ_0 increases. But more usually when there is an appreciable amount of saturation with high power-factors and small values of ϕ_0, the second term does not sufficiently balance the increase $\mathfrak{R}_d'/\mathfrak{R}_d$. OD then exceeds E_0, and the E M F. will not rise so high as OD upon withdrawal of the armature current.

Thus, the fact that the direct- and cross-magnetizing ampere-turns of the armature do not act on precisely the same magnetic circuit partially balances the effect of saturation of the iron parts.

When OD may be identified with E_0, Fig. 508 takes the form of Fig. 542, in which the true reactance E.M.F. Ix_a and Ix_b are added together as $BC + CD = e_{sa} + e_{sb} = Ix_a'$, and the synchronous impedance E M F, $e = AD = Iz_a'$ is combined with the terminal voltage to give the open circuit E.M.F. E_0.

§ 43. Experimental determination of short-circuit characteristic, and synchronous impedance.—Since within certain limits the quantities x_a' or z_a' may be regarded as physical constants of the alternator, methods of calculation based on them may legitimately be used in the approximate solution of many alternator problems, *e g.* those connected with machines working in parallel The determination of the synchronous impedance is, therefore, of importance, but it cannot be measured directly by voltmeter and ammeter. All that can be experimentally observed is the relation between given values of the field excitation, AT_f per pole, and the short-circuit current I_{sc}, *i.e.* the " short-circuit characteristic."

This relation has already been given in equations (251) and (251*a*) which may also be re-written as

$$m \frac{1{\cdot}257\, AT_{fsc}}{\mathfrak{R}_{do}} = I_{sc} \left\{ z_a' + m \frac{1{\cdot}257\, Z \sin \phi_a'}{4pq\, \mathfrak{R}_{do}} \left(k_{\mathrm{D}} - \frac{\mathfrak{R}_{do}}{\mathfrak{R}_{cc}} k_{\mathrm{C}} \right) \right\} \quad . \quad (252)$$

$$= \text{approximately}\, I_{sc}\, x_a' = I_{sc}\, (x_a + x_b)$$

when r_a' is negligible and k_{C} is not far different from k_{D}, $\mathfrak{R}_{do}$ being also but little greater than $\mathfrak{R}_{cc}$.

It is evident that a knowledge of the value of AT_f to give $I_{sc} = I_n$, the normal current, or of I_{sc} under full-load excitation AT_{fl}, or in general of the relation of I_{sc} to AT_{fsc} would be a valuable aid in design, as furnishing the key to the value of the synchronous impedance, if z_a' remained practically constant under all conditions of load. It has been largely used on this assumption, and from this fact the "short-circuit characteristic" of the machine derives most of its importance.

Now the left-hand term in the above expression is the open-circuit E.M.F. that would result if $\mathfrak{R}_{do}$ remained unaltered. When the diagram of Fig. 542 is reduced to the case of short-circuit, *e* is also $E_0 = O'D' = I_{sc}\, z_a'$, so that $E_0/I_{sc} = z_a'$. But the accuracy of this entirely depends upon the degree of accuracy with which the particular value of $OD = O'D'$ may be identified with E_0. In fact, even with small excitations, $\mathfrak{R}_{do}$ will on open circuit rise to some higher value $\mathfrak{R}_d'$, and in accordance with § 42

$$O'D' = E_0 \frac{\mathfrak{R}_d'}{\mathfrak{R}_{do}} \left\{ 1 - \frac{ZI_{sc} \sin \phi_a'}{4p\, AT_f \times q} \left(k_{\mathrm{D}} - \frac{\mathfrak{R}_{do}}{\mathfrak{R}_{cc}} k_{\mathrm{C}} \right) \right\}$$

whence $$\frac{E_0}{I_{sc}} \times \frac{\mathfrak{R}_d'}{\mathfrak{R}_{do}} = z_a' + m \frac{1{\cdot}257\, Z \sin \phi_a'}{4pq\, \mathfrak{R}_{do}} \left(k_{\mathrm{D}} - \frac{\mathfrak{R}_{do}}{\mathfrak{R}_{cc}} k_{\mathrm{C}} \right)$$

and $$\frac{E_0}{I_{sc}} = z_a' \frac{\mathfrak{R}_{do}}{\mathfrak{R}_d'} + m \frac{1{\cdot}257\, Z \sin \phi_a'}{4pq\, \mathfrak{R}_d'} \left(k_{\mathrm{D}} - \frac{\mathfrak{R}_{do}}{\mathfrak{R}_{cc}} k_{\mathrm{C}} \right)$$

$= z_{sc}$, the *inferred short-circuit impedance*, when the open-circuit E.M.F. due to some excitation AT_{fo} is divided by the short-circuit current due to the same excitation. Beginning by exceeding z_a', the quotient $E_0/I_{sc} = z_{sc}$ falls off as the excitation is increased owing to the rise of $\mathfrak{R}_d'$ as compared with $\mathfrak{R}_{do}$ and the bending over of the curve of E_0 in relation to AT_f. Thus if $\mathfrak{R}_{do}$ is known or can be obtained, to deduce z_a' from E_0 and I_{sc} rather than from AT_{fsc} and I_{sc} is to introduce an unnecessary source of error. The quotient $E_0/I_{sc} = z_{sc}$ has, however, often been used as and called the "synchronous impedance."

In order to determine the short-circuit characteristic of an alternator a low-resistance alternating-current ammeter is placed across the terminals of the armature of a single-phase machine. Or in a polyphase machine the primary of a series transformer is placed between the terminal of each phase of the winding and a common connexion which short-circuits all the phases; the secondaries of the three transformers are then by means of switches successively applied to one and the same alternating-current ammeter.[1] The values of I_{sc} or the short-circuit current per phase at the rated speed of the alternator are then plotted as ordinates with corresponding values of the exciting current I_f or field ampere-turns as abscissæ (Fig. 618*b*). The curve of I_{sc} as already mentioned, is usually nearly a straight line passing through zero if residual magnetism has been eliminated; at strong excitations it tends to bend over slightly towards the horizontal. If the teeth of the armature are highly saturated by the transverse flux across the slots, this transverse flux will increase less than in proportion to the current, and with large values of I_{sc} the tendency of the curve to bend downwards may be more than counterbalanced, so that it becomes slightly concave. During the short time necessary for each observation of I_{sc} and I_f the armature current may safely be allowed to rise to two or three times the full-load normal current. During the readings exact constancy of the speed is not of essential importance, since, as the speed varies, the reactance varies equally, and the only effect is to alter the relative amount of the eddy-currents and of Ir_a' to Iz_a'. Since Ir_a' is small as compared with Iz_a', the current is almost entirely determined by the reactance, and is, therefore, independent of the speed for the given excitation. It is, however, important to take the short-circuit current as high as is permissible so as to reproduce as nearly as may be the full-load excitation and obtain as high a degree of saturation of the field-magnet as possible.

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[1] See Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, p. 209.

§ 44. The ratio of field ampere-turns to armature ampere-turns on short-circuit.—From its use in the process of designing to be described later, the diagram representing the conditions of short-circuit deserves further consideration.

On permanent short-circuit an alternator resembles a transformer, in so far as there are on it two opposing sets of ampere-turns, on the field-magnet and armature respectively. The maximum permanent short-circuit current occurs when the poles formed on the armature are almost exactly opposite to the poles of the magnet-system. The demagnetizing ampere-turns are then nearly equal to the magnetizing ampere-turns of the field-winding, and the difference between them merely corresponds to the flow of a small magnetizing current and a small number of lines.

In the ideal case of a machine without ohmic resistance and with no true armature reactance, AT_{DM} on short-circuit would be $= AT_a$, since the angle of lag of the current vector behind the interpolar line of symmetry would be 90°. These ampere-turns, embracing the main magnetic circuit, would exactly balance the interpolar excitation AT_{P}, and there would be no resultant armature flux.

But in reality there must on short-circuit be some surplus of ampere-turns on the field to provide the flux necessary to counterbalance the reactance from the end-connexions of the coils and slot inductance, and also to provide the volts expended over the ohmic resistance and by eddy-currents Of the two the former is by far the most important, since ϕ_a' although never reaching 90° usually exceeds 80°, and the total number of lines to be provided in the armature will only exceed the true self-induced flux in a proportion slightly greater than cosec 80° : 1 or 1·015 : 1, *i e* by an addition of rather more than $1\frac{1}{2}$ per cent.

Thus, instead of the ideal ratio of 1, we have $AT_{fsc}/AT_a = k_o$, where k_o has some value greater than unity. The short-circuit characteristic, in fact, measures the combined effect of the maximum direct magnetizing ampere-turns of the armature and of the true armature reactance expressed in ampere-turns. From § 39 it is seen how the field-current is balanced by the effect of the short-circuit current with its two components into which this effect is divisible, the one due to the back ampere-turns and the other to the armature reactance. The last term in eq. (251) dealing with the loss of magnetic potential over the reluctance of the iron magnet-cores and yoke is in most cases quite negligible as compared with the first two terms, since the actual flux when the alternator is short-circuited is but small.

When a sinusoidal wave of armature M.M.F. is balanced against a flat-topped wave of field M.M.F., the difference-curve yields a resultant M.M.F. wave having two peaks in each pole-pitch, one at each edge of the pole with a deep depression between them,[1] and in the absence of any approach to saturation a similar flux curve. Thence results a highly distorted wave of E.M.F. in the armature winding, and the relative proportions of fundamental and harmonics which hold on open-circuit or under normal loads are entirely altered. The amplitude of the 3rd harmonic may be largely in excess of that of the small fundamental, or usually is at least equal to it, and the amplitudes of the 5th and 7th may be from 25 to 70 per cent. Yet in spite of this, the short-circuit current remains not far from sinusoidal, with the possible exception of a moderately large 3rd harmonic in a mesh-connected armature. The reason is that the reactance presented by the armature (which practically determines the magnitude of any component of the current, the resistance being negligible) to the nth harmonic E.M.F. is n times its reactance to the fundamental E.M.F. The assumption of a sinusoidal wave of armature M.M.F. is thus not invalidated.

The value of k_0 or the ratio AT_{fsc}/AT_a on short-circuit for various types of machines and arrangements of winding has been tabulated by several observers from experiments, and is found to differ considerably from unity. In a given machine, if the short-circuit characteristic bends over and becomes convex, k_0 is increasing with increasing values of the excitation. For small values of the exciting current and of I_{sc}, k_0 may in some cases be less than unity and as low as 0·9, but this result is due to residual magnetism in the iron cores, which in effect makes the apparent ampere-turns of the field greater than the actual product of the exciting current and the field turns.

The relation between AT_{fsc} and AT_a as affecting a half magnetic circuit may also be expressed in terms of the total number of ampere-turns on the whole field-system and armature respectively. Since AT_a on short circuit $= k_D \frac{Z}{4pq} I_{sc}$,

$$AT_{fsc} = k_0 AT_a = k_0 k_D \frac{J_{sc}Z}{4p}$$

$$2p \cdot AT_{fsc} = k_0 k_D \frac{J_{sc}Z}{2}$$

But $2p \cdot AT_{fsc}$ = the total number of ampere-turns on the field-system, and $J_{sc}Z/2$ = the total virtual AT on the armature, all reckoned as of equal value.

The ratio $\frac{AT \text{ on field}}{AT \text{ of the armature on short-circuit}}$, whether total or per pole, $= k_0 k_D$, and these factors may be grouped into a single quantity, ζ, so that

$$\zeta = \frac{2p\, AT_{fsc}}{J_{sc}Z/2}$$

It is then sufficient for the designer to refer to tabulated values of ζ which previous experiments have shown to hold for machines of similar type and of similar proportions, and from it can be estimated the synchronous impedance, as a datum for the preliminary work of design. Thus, if the value of ζ be assumed from previous machines, or, when the number of slots per pole per phase and the pole-width ratio, etc., are known so that k_D can be calculated, if the value of k_0 be approximated, the short-circuit current with the full excitation is $I_{sc} = \frac{AT_{fl} \times 4pq}{\zeta Z}$, and the apparent synchronous impedance is E_0/I_{sc}. The smaller the value of ζ and, therefore, with a given value of AT_{fl}, the larger I_{sc} as compared with the normal full-load current, I_n, the better the regulation of the machine.

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[1] See A. E. Clayton, *Journ. I.E.E.*, Vol. 54, p. 84, where the wave-shapes of E.M.F. and current of alternators working under permanent short-circuit conditions are fully investigated and explained: *cp.* also N. S. Diamant, *Trans. Amer. I.E.E.*, Vol. 37, Part II, pp. 1175–1184.

§ **45. The value of ζ.**—The value of k_0 ranges in practice from 1·08 to 1·5 in 3-phase machines, and on an average may be taken as about 1·2 to 1·3. As a normal case in such machines, taking k_D for 3 slots per pole per phase $= 0{\cdot}701$ and the value of 1·25 for k_0,

$$\zeta = k_0 k_D = 1{\cdot}25 \times 0{\cdot}701 = 0{\cdot}875 \text{ approximately,}$$

which may be regarded as a fair average.

In the single-phase alternator, the pole-ratio is usually rather larger and with the coil-side divided between four out of six slots per pole, $k_D =$ say, 0·6. The average value $\overline{AT}_{DM}$ as reckoned in § 29 being taken as AT_a, k_0 varies from 2 to 3, and in general is about 2·45. Whence

$$\zeta = 2{\cdot}45 \times 0{\cdot}6 = 1{\cdot}47.$$

But it is very dependent on the presence and the efficacy of an amortisseur winding to damp out the pulsation of the single-phase armature reaction.

The ratio of the field ampere-turns giving the same short-circuit current J_{sc}, first in a single phase out of the three in a 3-phase alternator (*i.e.* with $Z' = Z'''/3$), and then with all three phases short-circuited, would thus be $2{\cdot}45/(1{\cdot}25 \times 3) = 0{\cdot}655$.[1]

In general the value of ζ is influenced by the number of slots per pole per phase, and is less the greater the number of such slots, since the reactance is then less and the back ampere-turns are not so concentrated on the main magnetic circuit.

The absolute value of the pole-pitch, and so indirectly the frequency, has an appreciable influence upon ζ. The leakage reactance is largely a constant whatever the pole-pitch, and, therefore, its ratio to the main flux varies inversely as the pole-pitch. Further, the value of ζ depends not only upon the actual number of the demagnetizing turns of the armature, but also upon their effect in increasing the primary leakage. The greater the permeance $\mathfrak{P}_l$ of the leakage paths, the higher the density of the resultant flux in the field-magnet; its reluctivity rises and $f'(B_m)$ on the right-hand side of eq. (251) increases. But with reduced value of the pole-pitch, $\mathfrak{P}_l$ rather tends to increase, and in consequence with an increase of frequency unless the pole-pitch can be maintained constant, AT_m increases and the value of ζ is higher.

§ **46. The synchronous-impedance method for predetermining regulation.**—The essential basis underlying the use of the synchronous impedance for calculating the regulation is the identification of OD with E_0, and if this were strictly true under all conditions, the prediction of the inherent regulation of an alternator up or down would present no difficulty. If in the triangle of Fig. 542 V_e was the specified terminal voltage under load, throwing off the load would cause the steady voltage to rise from V_e to E_0. If E_0 in Fig 542 was the specified terminal voltage, throwing on the load would cause the voltage to fall from E_0 to V_e. Further, the method presupposes that z_a' retains the same value, even when pushed to the extreme of short-circuit, so that if the full-load excitation which gives E_0 on open circuit gives a short circuit current I_{scf},

$$\frac{E_0}{e} = \frac{I_{scf}\, z_a'}{I_n\, z_a'} = \frac{I_{scf}}{I_n} = Q$$

where Q is the multiple which the steady short-circuit current under full-load excitation is of the known full-load current I_n. Giving various values to the ratio of, say, V_e to E_0, a solution of the triangle of Fig. 542 for various values of the power-factor ϕ_e will enable the value of Q for the given cases to be calculated, and tabulated or plotted as curves once and for all.

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[1] *Cp.* Prof. Miles Walker (*Journ. I.E.E*, Vol. 57, p. 129), who gives the ratio as from 0·65 to 0·75. In a case cited by B A. Behrend " Factors which determine the Design of Monophase and Polyphase Generators," *Electr. Eng.*, Vol. 26, p. 727, ζ' for a single-phase alternator with one slot per pole was 1·56; when the same type and size of machine was wound as a 3-phase generator with 3 slots per pole, ζ''' was 0·966, so that the ratio was $1{\cdot}56/(0{\cdot}966 \times 3) = 0{\cdot}54$.

It will be seen that no further reference to any open-circuit curve of magnetization is required, in which lies the convenience of the method but also its inaccuracy.

From the fact that z_a' is determined when only a comparatively small flux is passing and the magnet is not saturated as it would be under working conditions, the synchronous impedance method with z_a' obtained through AT_{fsc} and still more with $z_{sc} = E_0/I_{sc}$ loses its accuracy as the load becomes more nearly non-inductive. The percentage rise of volts when full-load is thrown off as deduced by it is so uniformly too high by some 5 or 10 per cent. that it has been called the " pessimistic " method. But the supposition on which it is based is so far from the truth, that it is of little use, and is only here introduced as leading up to the second approximate method of greater accuracy.

§ 47. The vectorial addition of ampere-turns.—If field and armature ampere-turns acted upon identical circuits, corresponding to the three sides, E_0, e, and V_e of the triangle of Fig 542 there would be a similar triangle of fluxes, in which the largest flux would contain a component nullifying the apparent back flux from the armature ampere-turns and leaving as the resultant the actually existing flux which causes the terminal voltage V_e *plus* the ohmic loss Ir_a'. Granting then the use of the same final resultant reluctance in each of the three cases, there would also be another similar triangle of ampere-turns (Fig. 542*a*). Its sides, taken in the same order and each at right angles to the E M.F. to which it gives rise, would be the field ampere-turns AT_f, $k_0 AT_a = \zeta \, J_{sc} Z/4p$ representing the whole of the armature reaction and covering also the ampere-turns required for the flux that provides the volts expended over the ohmic resistance of the armature, and lastly AT_r giving the net voltage at any load (*cp.* Fig. 519). The angle between AT_r and $k_0 AT_a$ would be $180° - \phi_a' + \phi_e$, slightly exceeding, but practically identifiable with, $90° + \phi_e$. If then AT_r was AT_{f0}, the ampere-turns giving the specified terminal voltage V_e on open circuit and AT_a was the armature magnetizing ampere-turns with full current, the third or longest side would be AT_{fl}, the field ampere-turns on load. By reference to the open-circuit curve of magnetization, above the abscissa AT_{fl} there would then be found E_0, the voltage to which V_e would rise when the full-load was thrown off. If the longest side was AT_{f0}, throwing on the load would cause the resultant ampere-turns to sink to AT_r, and again by reference to the open-circuit magnetization curve, the new terminal voltage V_e would be found.

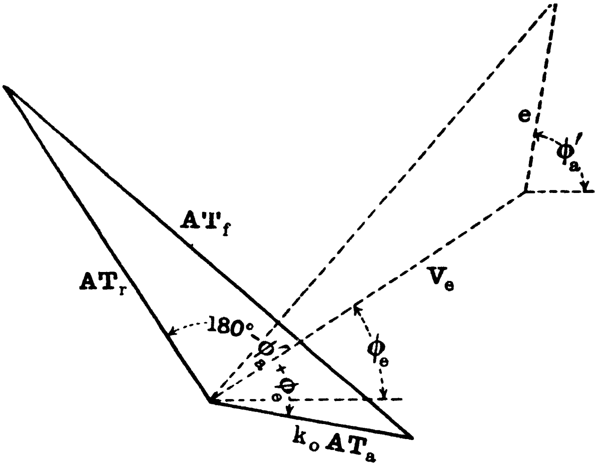

FIG. 542*a*.—Vectorial addition of ampere-turns.

It will be seen that, as contrasted with the previous case, for numerical results the open-circuit curve of magnetization is required, and that in place of $E_0/e = I_{scf}/I_n$ we now have

$$\frac{I_{scf}}{I_n} = \frac{AT_{fl}}{k_0 AT_a}$$

A value is thus obtained for the rise of voltage on throwing off the load which takes more or less into account the increasing saturation of the magnet system. If now ζ be known by experiment or $k_0 AT_a$ is calculated, the ratio of two sides of the triangle will be known, *i.e.*

either $$\frac{\text{full-load excitation, } AT_{fl}}{AT_{fsc} \text{ to give } I_n \text{ under short-circuit}}$$

or $$\frac{\text{open-circuit excitation, } AT_{fo}}{AT_{fsc} \text{ to give } I_n \text{ under short-circuit}}$$

Again, therefore, a solution of the triangle will enable curves to be plotted for the remaining side with given values of ϕ_e (*cp.* § 20) Thus

$$\begin{aligned} AT_f &= \sqrt{AT_r^2 + (k_0 AT_a)^2 - 2\,AT_r \,.\, k_0 AT_a \cos(180^\circ - \phi_a' + \phi_e)} \\ &= \sqrt{AT_r^2 + (k_0 AT_a)^2 + 2\,AT_r \; k_0 AT_a \cos(\phi_a' - \phi_e)} \end{aligned}$$

and since ϕ_a' is nearly 90°,

$$= \sqrt{AT_r^2 + (k_0 AT_a)^2 + 2\,AT_r \,.\, k_0 AT_a \sin\phi_e}$$

Analogously

$$AT_r = \sqrt{AT_f^2 - (k_0 AT_a)^2 \sin^2(\phi_a' - \phi_e)} \quad k_0 AT_a \cos(\phi_a' - \phi_e)$$

and approximately

$$= \sqrt{AT_f^2 - (k_0 AT_a)^2 \cos^2\phi_e} - k_0 AT_a \sin\phi_e$$

which for the limiting cases of unity power-factor and zero power-factor

$$= \sqrt{AT_f^2 - (k_0 AT_a)^2} \text{ and } AT_f - k_0 AT_a \text{ respectively.}$$

The regulation up is now more nearly accurate, but is "optimistic," the defect in the method of calculation being that AT_{fl} is brought out too low owing to the fact that the increased field-magnet leakage due to the presence of AT_a on full-load is not taken into account [1]

§ 48. Calculation of armature reactance.—In the process of designing a new alternator the value of $x_a = 2\pi f\, \mathcal{S}_a/q$ must be determined by calculation of $\mathcal{S}_a$. The shape of the slots and the degree of saturation in the teeth have considerable effect in modifying the inductance, but the general principles on which it must be estimated remain the same as have already been employed in Chapter XX, §§ 23–24 in connexion with the inductance of the short-circuited section of a continuous-current armature.

If t = the number of active conductors in a belt per pole per phase, the inductance of the belt with its share of the end-connexions may be expressed just as in Chapter XX, § 23, through a term $\mathfrak{S}$, or the equivalent permeance surrounding them when it is assumed that every line of flux is linked with all the turns; *i.e.* as

$$\mathcal{S} = 4\pi t^2 \mathfrak{S} \times 10^{-9} = t^2 \Lambda \times 10^{-9} \text{ henrys}$$

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[1] On §§ 46, 47 *cp.* especially A. G. Warren, *Electr.*, Vol. 86, p. 581 ff.

The permeance will again require to be considered under divisions analogous, although not precisely similar, to those of the continuous-current armature. The divisions appropriate to the alternator will correspond respectively to—

(1) the *side of a coil* embedded in the length of the armature core; this will require to be subdivided again into

(*a*) the portion corresponding to the flux *within the slots*, proportional to the net iron length l cm. of the core,

and (*b*) the *core-surface permeance*, proportional to the over-all length L,

(2) the *straight projections of the coil-sides beyond the slots* of the core, and proportional to $l_1 - l$, where l_1 is the total length in cm. of the coil-side (Fig. 543*b*), and the width of the radial air-ducts is thereby included,

and lastly (3), the true *end-connexions* of the coil, of length l_3 cm. along the circumference of the stator. In the case of rectangular coils with more than one slot per pole per phase the periphery of the packet forming a coil-side as it comes straight out of the slots over the length $l_1 - l$ is usually different from the periphery of the same packet as it stretches across the span over the length l_3, since the slot insulation and intervening iron teeth add to the former dimension; hence for this reason the end-connexions of the coils require to be considered under the above two headings (2) and (3).

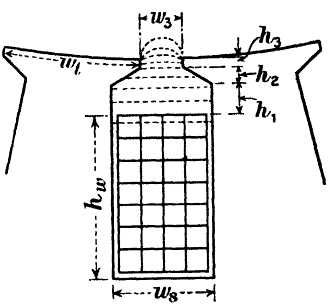

FIG. 543*a*.—Slot inductance.

If λ_s, λ_c, λ_p, and λ_e are the four quantities corresponding to the four divisions of the permeance for *one* side and end of a complete coil or pair of divided coils, corresponding to a pair of poles, the inductance for the *two* sides and ends of the *two* belts of wires giving t turns is

$$t^2 \times 2 \{l\lambda_s + L\lambda_c + (l_1 - l)\lambda_p + l_3\lambda_e\} \times 10^{-9}$$

No distinction is now required between the two sides of the coil, so that the simple multiplier 2 can be at once introduced. The inductance of a complete phase of p/q such coils corresponding to p/q pole-pairs when the q branches are assumed to be assigned to separate sets of poles is

$$\mathcal{L}_a = \frac{p}{q} t^2 \times 2 \{l\lambda_s + L\lambda_c + (l_1 - l)\lambda_p + l_3\lambda_e\} \times 10^{-9} \text{ henrys}$$

and its reactance is $2\pi f$ times this quantity. But in order that the self-induced E.M.F. may be given as the product Ix_a instead of as

$2\pi f\,.\,\mathscr{L}_a I/q$, it is convenient to divide the true inductance by q when there are two or more paths in parallel; since $t = \dfrac{Z}{2p\,.\,N_{ph}}$, we then have $x_a = 2\pi f\,\mathscr{L}_a/q$

$$= 2\pi f\left(\frac{Z}{N_{ph}\,q}\right)^2 \frac{1}{2p}\,\{l\lambda_s + L\lambda_c + (l_1 - l)\lambda_p + l_3\lambda_e\} \times 10^{-9}$$

$$= \pi\frac{N}{60}\left(\frac{Z}{N_{ph}\,q}\right)^2 \{l\lambda_s + L\lambda_c + (l_1 - l)\lambda_p + l_3\lambda_e\} \times 10^{-9}\ . \qquad (253)$$

(1*a*) *Slot inductance.* Taking a half-closed slot of shape such as that of Fig. 543*a*, the slot-inductance may be divided into the portions corresponding to the flux within the winding space up to height h_w and to the flux between the overhanging edges and across the actual opening. Up to the height h_w, assuming that the slot has straight parallel sides and square corners and that it is filled with layers of small conductors, the M.M.F. increases uniformly from the bottom of the slot as we proceed upwards; the M.M.F. acting over an elementary strip across the slot at a height x from its bottom with one C.G.S. unit of current flowing in each conductor is $4\pi\dfrac{t}{g'} \times \dfrac{x}{h_w}$, where g' is the number of slots per pole per phase. The self-induced flux is similarly increasing, so that by proposition (I) of Chapter XX, § 23,

$$\lambda = \frac{4\pi}{3} \times \frac{h_w}{w_s} \text{ per cm. length of core.}$$

Since there are g' such slots for each belt of conductors, their joint inductance up to the height h_w and per cm. of axial length is $\dfrac{t^2}{g'}\,\lambda \times 10^{-9}$ henrys, or $\lambda' = \dfrac{4\pi}{g'} \times \dfrac{h_w}{3w_s}$. The assumptions are not correct in so far as only flux outside of a copper conductor is concerned in true inductance, and the inaccuracy thereby introduced increases as the height of the conductors increases and the number within a slot decreases in machines for large currents. But merely for purposes of comparison the formula serves.

The flux above the coil is linked with all the t/g' wires, so that, analogously to Chapter XX, § 24, for g' slots the second item in the slot inductance is

$$\frac{t^2}{g'} \times 4\pi\left(\frac{h_1}{w_s} + \frac{2h_2}{w_s + w_3} + \frac{h_3}{w_3} + 0{\cdot}2\right) \times 10^{-9} \text{ henrys,}$$

or

$$\lambda'' = \frac{4\pi}{g'}\left(\frac{h_1}{w_s} + \frac{2h_2}{w_s + w_3} + \frac{h_3}{w_3} + 0{\cdot}2\right)$$

where the constant 0·2 to take into account the semi-circular zone over the mouth of the slot has been already explained in Chapter XX, § 24.

Thus for substitution in equation (253) with straight-sided slots

$$\lambda_s = \frac{12{\cdot}57}{g'}\left(\frac{h_w}{3w_s} + \frac{h_1}{w_s} + \frac{2h_2}{w_s + w_3} + \frac{h_3}{w_3} + 0{\cdot}2\right) \quad . \quad (254)$$

(1*b*) In the *surface of the core inductance* it is not desired to include any part of the true cross or back effects which have been otherwise taken into account separately on another principle as magnetomotive-force or cross E.M.F. effects. As an approximation therefore, these are best eliminated by assuming the field-magnet rotor to be removed, so that the completion of the magnetic circuit after the flux leaves the stator iron is entirely in air. As in the continuous-current armature, the assumption leads to fairly accurate results, and its advantage is that the calculated figures can readily be checked by actual experiment.

It may here be emphasized that for the same number of poles and the same number of slots with the same relative proportions of slot-opening w_3 to tooth-pitch t_1, the inductance per cm. of core-length due to the effective bore permeance λ_c is independent of the internal diameter of the stator—more so than in the analogous case of the continuous-current rotor, since with poles and slots the same in number and in proportions, the areas and lengths of paths of the flux of the alternator stator in air increase with increasing diameter of the bore in the same proportion, so that one diagram of the flux is true for all diameters.

Approximate calculation on the lines explained in the Note appended to this Chapter shows that with different numbers of poles the value for any given number of slots is practically constant, and that for different numbers of slots, λ_c only varies within comparatively small limits in practical cases. As general expressions, when the rotors are removed will, therefore, be adopted for a single-phase alternator

$$\lambda_c = 10 + \frac{9{\cdot}2}{(g')^2}\log\frac{t_1}{2w_3} \quad . \quad . \quad . \quad . \quad . \quad (255)$$

and for one phase of a 3-phase alternator, when two phases are in series,

$$\lambda_c = 13 + \frac{9{\cdot}2}{(g')^2}\log\frac{t_1}{2w_3}$$

In the process of designing, for reasons explained in § vi of the appended Note, the values of λ_c that are required are less, and with 2 or 3 slots per pole per phase in medium-speed machines with short cores the constant may be taken as 7·75 for a single-phase machine and 10 for a 3-phase alternator in 3-phase work (thus returning to equation 255). These again fall with larger numbers of slots, so that with 6 or more slots per pole per phase in turbo-alternators with comparatively long cores, it suffices to take

$$\lambda_c = 5 + \frac{9{\cdot}2}{(g')^2} \log \frac{t_1}{2w_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (255a)$$

and the last term depending upon g' becomes itself relatively unimportant.

(2) *The straight projections of the coil-sides.* Maxwell's formula for the inductance of two parallel cylindrical conductors of length l cm. and radius r, conveying equal currents in opposite directions and situated at a distance b apart in air, viz., $2l\,(4{\cdot}6 \log_{10} \frac{b}{r} + 0{\cdot}5)$ can also be written

$$4{\cdot}6 \log \frac{1{\cdot}284\, b}{r} \text{ per cm. length.}$$

Treating, therefore, the straight projections of the two belts as parallel conductors, each having an equivalent radius of $\frac{a+b}{\pi}$, separated by a *direct* distance l_2 between the axes of the belts (Fig. 543*b*) where a and b are the two dimensions of the packet (*cp.* Fig. 391), we have

$$\lambda_p = 4{\cdot}6 \log \frac{4{\cdot}04\, l_2}{a+b} \quad . \quad . \quad . \quad . \quad . \quad . \quad (256)$$

The direct distance l_2 between the axes of the belts is $Y \,.\, \frac{2p}{\pi} \,.\, \sin \frac{\pi}{2p}$, where Y is the pole-pitch. The product of the last two factors rises from 0·637 for a two-pole machine through 0·9 for a 4-pole and 0·955 for a 6-pole, so that it approximates closely to the pole-pitch with 6 or more poles.

(3) *The end-connexions.* If there is one large undivided coil in a phase per pair of poles, all the end-connexions, with an equivalent radius $\frac{a'+b'}{\pi}$ (Fig. 543*b*), on one side of the core follow the same path. Their actual length l_3 as they span the pole-pitch bears no direct relation to their direct distance apart or to the pole-pitch, since they may be bent up into different planes and may differ at the two ends, and one phase may be different from another phase; with a small number of poles l_3 is very different from l_2. Owing to the wide distance separating the two sets of ends and the interposition of the core, they are best treated separately and the flux simply integrated up to the core over the distance $\frac{l_1 - L}{2}$ in circles round each end. Thence

$$\lambda_e = 4{\cdot}6 \log \frac{\pi\,(l_1 - L)}{2\,(a' + b')} = 4{\cdot}6 \log \frac{1{\cdot}57\,(l_1 - L)}{(a' + b')} \quad . \quad (257)$$

If bent up close to the iron in one or more phases, the first constant may require to be increased, and it is also increased by the proximity of iron end-shields. In 3-phase work with undivided coils, there is practically no mutual inductance in the end-connexions.

With divided coils, *i.e.* with $2p$ coils per phase and with the turns equally divided between them, so that only $t/2$ conductors follow the same path at the ends, then with a large number of poles and a nearly flat coil, or in a two-pole machine if the end-connexions, which are actually bent up over the rotor, were opened out flat at each end, the divided connexions become practically independent of one another; their joint inductance is then $\propto \left(\frac{t}{2}\right)^2 \times 2$, and it would only be necessary to divide the above expression (257) by 2. But with a small number of poles and especially in a 2-pole machine where the divided ends carrying current in the same direction surround the whole bore, there is always a certain amount of mutual inductance between the divided connexions of the coil which has the effect of raising the constant 2·3 somewhat. Its amount varies greatly with the shape and distance between the coils, the increase averaging from 15 to 30 per cent.

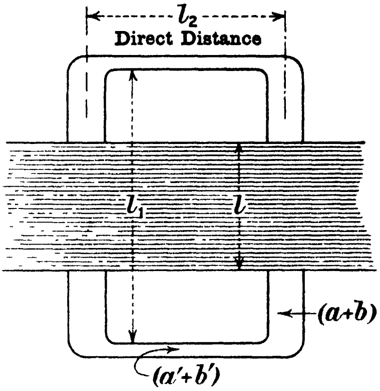

FIG. 543*b*.—Projection and end-connexion inductance.

For the V-shaped end-connectors of a bar-wound alternator with barrel-winding in two layers, may be used the expression (*cp.* Chapter XX, p. 169, Vol. II)

$$\lambda' = 4{\cdot}6 \log \frac{l'}{a' + b'} \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (258)$$

to be multiplied by l'. Here a' and b' are the two dimensions of the complete rectangular sheaf of one phase. When the V-shaped end-connexions of a bar-wound alternator are examined by means of sections as in Fig. 394 (Vol. II, p. 170), the case differs from that of the continuous-current armature owing to the fact that the current is not the same in each end-connexion, but varies in those of different phases. Following the sheaf of end-connexions of one phase along the two legs of the V, it will be found that there are as many crossings in which the currents in the two layers have a similarly directed axial component as there are crossings with oppositely directed axial component currents and the magnitudes of the currents are similar; the increased inductance due to the mutual induction of the former is, therefore, cancelled by the latter, and there is left practically the simple inductance of the single phase even in 3-phase work. The full expression previously obtained for the continuous-current case has, therefore, been adopted above without any deduction.

In the case of definite coils, if $a' + b' = a + b$ of the projecting ends, and the coils are flat, so that $l_3 = l_2$, and further, if the grouped ends yield a square with $l_1 - l = l_2$, the two expressions given in (256) and (257) multiplied by their respective lengths, viz.,

$$4{\cdot}6\left[(l_1 - l)\log\frac{4{\cdot}04\, l_2}{a + b} + l_2 \,.\, \log\frac{1{\cdot}57\,(l_1 - l)}{a' + b'}\right]$$

become

$$4{\cdot}6\, l'\left(\log\frac{l'}{a' + b'} + 0{\cdot}1\right)$$

where $l' = l_1 - l + l_2 = 2l_2$. This is not greatly different from (258), but errs by being too high.

So far it has been assumed that any branches of a single phase have been assigned to separate pole-pairs. If the divided coils are allotted to separate branches of a phase, and t or g' are retained as before as the turns and slots per pole per phase, the same fluxes are linked with only $t/2$ turns in a phase. In relation to the single branch of the phase the slot and core inductances are halved as well as the end-connexion inductance, but $2p/q$ coils being now in series, the original expression for L_a (p. 197) still holds.

K. Pichelmayer[1] has shown that in spite of the wide diversities in the dimensions and disposition of the armature slots and coils, the inductance of good modern 3-phase alternators may approximately be represented as simply proportional to the length of the armature core. Adopting the present symbols, he thus finds that the term within the bracket in equation (253) may be expressed simply as $L\lambda$, where $\lambda = 27{\cdot}5$ for open and 30 for half-closed slots.

Many experimental measurements have been recorded of the inductance of armature coils under different conditions and in different positions relatively to the poles, but in any deductions therefrom care must be taken to consider how far the measured inductance includes not only that due to the slot and end-connexion fluxes, but also that which corresponds to the cross-magnetizing and demagnetizing effects which are to be estimated by other methods.[2]

From data given by Mr. Hobart,[3] the flux of the free end-connexions of the coils is of the order of 8 lines per centimetre length and per C.G.S. current-turn for one slot per pole per phase, sinking to 7, 6, and 5 respectively for 2, 3, and 4 slots per pole per phase. The flux per centimetre length of an embedded coil and per C.G.S. current-turn varies so greatly with the proportions of the slot and the grouping, that experimental values are seldom immediately applicable.

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[1] *Der Dynamobau*, p. 207.

[2] *Cp.* Chap. XXIX, § 11, R. Pohl, *E.T.Z.*, Vol. 42, p. 1057, and Togna *Rev. Gén. d'Él.*, Vol. 5, p 779 ff.

[3] *Journ. I.E.E.*, Vol. 31, p. 192 ff.

§ 49. Separation of direct armature reaction from true armature reactance.—Equations (251) (251*a*), and (252) appropriate to short-circuit conditions enable the combined effect of the direct-magnetizing ampere-turns of the armature and of the true armature reactance to be deduced from the experimentally obtained short-circuit characteristic of a machine that has already been built. But to divide their sum and obtain k_D separately, x_a must be assumed from calculation, or *vice versa.* Thus in the latter case, if AT_{DM} on short-circuit $= AT_a = k_D \frac{Z}{4pq} I_{sc}$ is assumed as known, and is deducted from AT_f, the difference $AT_f - AT_{DM}$ is found from the no-load characteristic to yield E_a volts, and this is identified with $e_{sa} = I_{sc} x_a$, whence x_a is found.

There are, however, elements of doubt in the predetermination of the exact numerical value of either of the two quantities concerned. Some further datum is, therefore, very desirable to add to the knowledge that can be derived from the mere short-circuit characteristic. Such an additional datum is found in the approximate separation of the two effects of the armature back ampere-turns and the true armature reactance, which is based upon the analysis of a wattless-current curve as described in § 38. Although not completely true when the magnet is very highly saturated, yet such a condition rarely occurs with very low power-factors, so that the results obtained are found to be in close agreement with observed facts. A curve of terminal voltage for a constant wattless current, *i e.* a current of considerable magnitude but with a power-factor cos ϕ_e, say less than 0·20, is in the first place required. Such a curve may be obtained by working the alternator on a highly inductive load such as induction motors running light, or in connexion with a synchronous motor under-excited so as to shift the phase of the current by a large angle of lag relatively to the E.M.F. ; or lastly, the alternator may itself be operated as a synchronous motor. Even if a lower power-factor than 0·5 cannot be obtained, the drop of volts is but little less than with a truly wattless current. The field-exciting current I_f is progressively reduced while the armature current I is maintained as nearly as may be constant, until, if the machine is short-circuited, the final value of I_f is one reading of the field current in a short-circuit curve. The latter may, in fact, then be entirely dispensed with, the necessary data being all obtainable from a complete curve of the terminal voltage for different field-currents with a wattless current in the armature of constant value. Or, if only a few readings for the upper portion of the wattless-current curve are obtained, one supplementary reading giving the point where the curve cuts the horizontal axis may always be added from the short-circuit characteristic if this has been independently taken.

If now the open-circuit curve is drawn on tracing paper and is shifted parallel to itself in a slanting direction, a direction may be found such that it fits fairly well over the lower curve of terminal voltage A line joining the old and new positions of any one point, as *PP′* (Fig 537), will then give the hypotenuse of the right-angled triangle, whence Ix_a and the remaining side are at once determined. From the former is found the true armature reactance, and from the latter the magnetizing ampere-turns of the armature.[1]

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[1] The direct reaction may also be separated from the effect of the leakage reactance by a construction due to M Blondel (*E.T.Z*, Vol. 22, p. 474; and described in the 5th edit. of this book). For a further improvement, see A. G. Warren, *Electr.*, Vol. 86, pp. 654-6. See also A. Blondel and F. Carbenay, *Rev. Gén. d'Él.*, Vol. 5, p. 811 ff., and G. Sentenac, *Rev. Gén. d'Él.*, Vol. 11, p. 813.

Following upon a paper[1] describing the separation of the armature reactance and armature reaction by a method requiring characteristics at zero power-factor, *both lagging and leading*, M. J. le Monnier[2] has described a simple graphical construction for the same purpose. In addition to the usual open-circuit curve of excitation, two readings are taken of the exciting current for rated voltage, one for zero power-factor *leading*, the ordinate to which will terminate at A to the left of the open-circuit curve, and the other for zero power-factor *lagging*, the ordinate to which will terminate at B to the right, Join AB and about its centre C turn the no-load voltage curve through 180°, so that it now cuts the original no-load curve at K and H. The perpendiculars let fall on AB, viz., KP and HM, are then equal and measure the true reactive drop, while $AM = PB$ measures the exciting current equivalent to armature reaction.

Another method for the same purpose has been devised by Dr. G. Kapp;[3] for the same speed and armature current per phase, *e.g.* of a star-connected armature with centre accessible, two readings of the field ampere-turns are taken, one AT_{f1} with a single phase short-circuited and the other two phases open, and the second AT_{f3} with all three phases short-circuited.

Thence

$$AT_{f1} = \overline{AT}_{DM} + \frac{0{\cdot}8\, I_{sc}\, x_a}{m}\, \Re_{do}$$

$$AT_{f3} = 3\,\overline{AT}_{DM} + \frac{0{\cdot}8\, I_{sc}\, x_a}{m}\, \Re_{dt}$$

By subtraction $\overline{AT}_{DM} = \frac{1}{2}\,(AT_{f3} - AT_{f1})$, and the last term which is $= \frac{1}{2}\,(3AT_{f1} - AT_{f3})$ ampere-turns is found by reference to the no-load characteristic to correspond to E_o volts $= I_{sc}\, x_a$, whence x_a is determined.

§ 50. Means for controlling the inherent regulation.—Apart from arrangement of the winding so as to secure a low value of x_a and of the cross magnetizing coefficient k_C, the only method of securing a good inherent regulation is to make the ratio of AT_f to AT_a as high as possible. The pole-pitch being usually more or less fixed by the considerations of frequency and speed, economy of manufacture dictates that it should be reasonably loaded with ampere-conductors over its whole distance, so that it is desired to keep AT_a high. The remaining factors which can be varied are the length of the air-gap and the degree of saturation of the teeth and magnet-circuit in general. To absorb the ampere-turns AT_{DM} which are released when the full-load current is extinguished with a minimum variation of voltage, the most effective method would be to work the stator teeth and core at a high density, so that they are strongly saturated and can receive but little additional flux, but this is forbidden by the heating and loss of efficiency that would result. Two alternatives, then, remain open to the designer, viz. (1) to employ a long air-gap, so that AT_g is high and, therefore, AT_{DM} bears a small proportion to the total AT_f, or (2) to saturate the magnet cores, so that AT_m is large, and any increase of the flux will soon absorb a large number of surplus ampere-turns.

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[1] By A. Curchod, *Rev. Gén d'Él*, Vol. 14, p. 829.

[2] *Rev. Gén. d'Él*, Vol 15, p. 531.

[3] *Journ. I.E.E.*, Vol. 42, p. 703. But by it the inductance of a single phase is assumed to be unaffected by mutual inductance from the other phases when the alternator is working as a 3-phase machine.

Adopting the former course, we approach what may be called an " air " machine, and in the ideal case of such a machine of constant reluctance, AT_{DM} becomes directly related to AT_f in dependence on the permissible rise of volts.

But even in this case it must not be supposed that if the full-load current be removed, the fractional increase of the E.M.F. above that which is generated on full-load will be only $\frac{AT_{\mathrm{DM}}}{AT_r}$, still less that it will be $\frac{AT_{\mathrm{DM}}}{AT_{fl}}$; the influence of the field leakage forbids this, and even without change of the iron reluctance, the increase above the full-load E.M.F. E_a is if $\mathcal{R}_l = 1/\mathcal{P}_l$

$$\frac{E_0 - E_a}{E_a} = \frac{\frac{AT_{\mathrm{DM}}}{AT_{fl}}\left(1 + \frac{2\,(\mathcal{R}_m + \mathcal{R}_y)}{\mathcal{R}_l}\right)}{1 - \frac{AT_{\mathrm{DM}}}{AT_{fl}}\left(1 + \frac{2\,(\mathcal{R}_m + \mathcal{R}_y)}{\mathcal{R}_l}\right)}$$

While the first method is objectionable from its effect on the cost or efficiency of the machine, the second method has the apparent advantage that it reduces the weight and cost of the iron magnet and reduces the length of the inner turn of the exciting coil. But on the other hand it involves a great variation in the amount of the exciting current for different loads, and it cannot be pushed to an extreme without danger of failure to reach the required voltage through the materials being less permeable than the calculation has assumed. In this inability of the designer to forecast the exact behaviour of the iron and steel at high densities, so that he cannot work confidently to a specified maximum permissible rise, lies its grave disadvantage. Further, a small percentage rise or fall of volts, if obtained by the employment of a very high degree of saturation in the poles, may render the alternator unable to meet an emergency over-load, or even the normal load with a power-factor lower than usual without an undue increase of the field excitation—unless it has been designed at the start for a low power-factor, and it will then be unnecessarily large for its normal load and power-factor, or uneconomical in exciting energy. In the highly saturated machine the value of the armature reaction must be very carefully estimated at the outset. A judicious combination of both methods is, therefore, best in practice.

In either case the change in the reluctance of the iron is prejudicial, and both methods tend to defeat partially their own ends. Upon removal of the full-load current a redistribution of the drops of magnetic potential over the circuit ensues. The ampere-turns $2AT_{\mathrm{P}}$ expended between the poles fall when the full-load current is extinguished, and the field leakage decreases; the reluctance of the magnet cores rises, but this is due to the total flux, and more of this flux reaches the armature. Roughly described, the operation is that the armature flux must rise by so much that, although combined with a reduced field leakage, the total exceeds appreciably the full-load total flux. It is, therefore, of the utmost importance to keep down as far as possible the leakage permeance of the field, so that the ampere-turns balancing AT_{DM} on full-load do not help much to saturate the magnet cores when in full work.

With low frequencies or very high revolutions per minute, either of which have a general tendency to increase the pole-pitch, it becomes increasingly difficult to counterbalance the effect of the large number of ampere-conductors per pole, at least when the power-factor is low. This difficulty is especially felt in the case of turbo-alternators;[1] their slot and end-connexion inductance may be low, and their regulation good on non-inductive loads, but on partially inductive loads recourse must be had to the first of the two alternatives to assist in securing a good regulation. A large air-gap is, however, here advantageous as assisting in the ventilation of both stator and rotor.

With either method economy of exciting watts as affecting the efficiency must be given its due weight, and if the alternator is to be employed to supply induction motors, a considerable overload capacity is even more important than good inherent regulation. For electric lighting the drop of volts is always more than can be permitted at the consumers' terminals, so that the excitation must necessarily be adjusted, and the amount of the adjustment within reasonable limits becomes more or less a matter of indifference. Finally, with large turbo-alternators it has in many cases been considered so desirable to limit the instantaneous rush of current in the event of accidental short-circuit that external reactances have been added for this express purpose without regard to their effect in increasing the regulation, particularly with 25-frequency machines.

§ 51. Constant-power alternators.—A special case in which a high inherent regulation is desirable is that of an alternator used for electric smelting or electro-metallurgical work, for which only the heating and not the chemical properties of the current are required, as for instance in the production of calcium carbide. For such use an approximately constant power is needed in order to maintain the temperature constant; as fusion proceeds in the electric furnace, the resistance of the circuit rapidly falls; a greatly increased current with violent fluctuations would result, but is held in check by the drop in volts which proceeds *pari passu* with the increase of the current, so that the alternator delivers an approximately constant watt output.

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[1] See especially Dr. M. Kloss, *Journ. I.E.E.*, Vol. 42, pp. 163–176.

NOTE TO CHAPTER XXVII

THE SURFACE-OF-BORE INDUCTANCE OF ALTERNATOR ARMATURES

Analogously to the Note appended to Chapter XX, the present Note deals with the value of λ_{cb} and λ_{ca} for the inductance due respectively to flux linked with all the t conductors of a belt, and to flux embracing two or more slots but not the whole belt of a phase, the two together making up the item λ_c for insertion in equation (253).

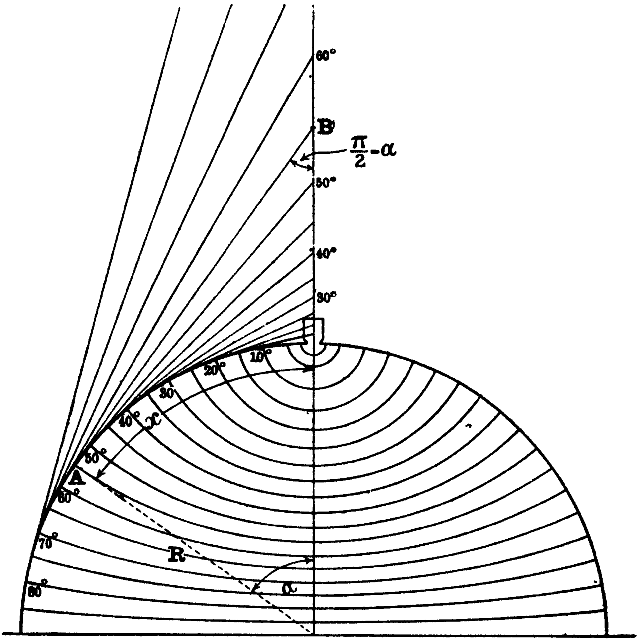

FIG 544 —Construction for calculation of permeance of bore with a single slot per pole in the two-pole machine.

In contrast to the surface of the continuous-current armature which was convex, it will be necessary in connexion with the external stators of alternators to consider the effect of the concave curvature of the bore, with the rotor removed.

(i) *The joint permeance relatively to the belt of a phase as a whole.*—In the two-pole case the construction shown in Fig. 544 affords a means of estimating the effect when there is only a single slot per pole. It will be seen that the tubes of flux straighten out as they cross the bore, until they finally pass straight across the diameter; the radii of the curved portions of the paths are at the iron core tangents to the circle of the bore and gradually increase in length, the centres from which they are struck receding up the centre line. If R is the radius of the bore, any radius AB is $R \tan \alpha$. The length of the path in air is then

$$2\left(\frac{\pi}{2}-\alpha\right) R \tan \alpha = (\pi R - 2x) \tan \frac{x}{R}$$

where α is in radians and $= x/R$. The widths of the tubes gradually contract as they pass inwards from the sides; their central section also diminishes as we proceed downwards along the centre line towards the horizontal diameter, Calculating the mean width of each tube from a large-scale drawing, its permeance per cm. length of core may be estimated as

$$\frac{\text{mean width}}{(\pi R - 2x)\tan\dfrac{x}{R}}$$

The values of this expression multiplied by 4π and summed up between appropriate limits will give the numerical value of λ_{cb} that is required. Starting from a point 90° from the centre of the slot, the integrated permeance $4\pi\mathfrak{P}$ increases at an increasing rate as the slot is approached, and its total value at any given angle is shown by the lowest curve of Fig 547 This figure is analogous to Fig. 384, and the approach to infinity has the same explanation that has already been given in connexion with that figure.

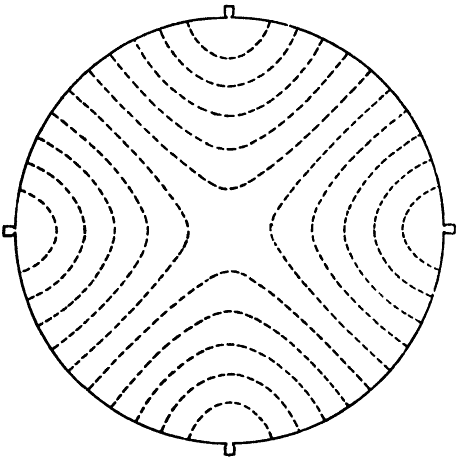

FIG. 545 —Paths of tubes of flux within the bore of a 4-pole machine

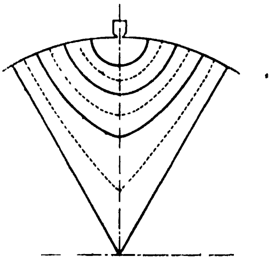

FIG 546 —Paths of tubes of flux within the bore of a 6-pole machine.

In the 4-pole machine, with a single slot per pole, while one coil-side by itself would repeat Fig. 544, the return coil-side at right angles thereto would by itself cause a similar flux but intersecting the first in curved paths crossing the horizontal axis. The composition of the two leads to a resultant distribution such as is shown in Fig 545, the flux being divided into four quadrants. Thus the final flux linked with one coil-side as given by one of the four sections is the result as it were of the M.M.F.'s of both coil-sides combined. For a given value of a the greater length of the paths when calculation is made of the flux due to $4\pi t$ finds compensation in the greatly increased area of the longer tubes towards their apices ; but the true cause which necessitates that the flux should follow the longer downward path towards, and the longer upward path away from, the centre of the bore is to be found in the magnetic effect of the side wires.

Analogously the actual resultant field in the 6-pole machine (Fig. 546) is divided into six sections, each linked with one coil-side, and so on. The greater the number of poles the greater the density of the flux across the radius to the middle of the coil-side near the bore as compared with the density as we approach the centre where it vanishes.

Figs. 545–6 indicate how the permeance of the assumed paths may be estimated from their mean area and length, and if for 4 or more poles the results of the summation beginning from a point midway between two coil-sides (*i.e.* from 90 electrical degrees) are plotted on a horizontal axis of electrical degrees, ready comparison can be made of any number of poles with the two-pole case. As in the case of the convex continuous-current armature, the conclusion is that the number of poles makes little difference ; the greater the number of poles, the less the permeance near the dividing lines, but this difference is gradually made up farther away, and the total remains practically identical. In consequence the lower curve of Fig. 547 may be taken as applicable to all numbers of poles with a single slot per pole per phase.

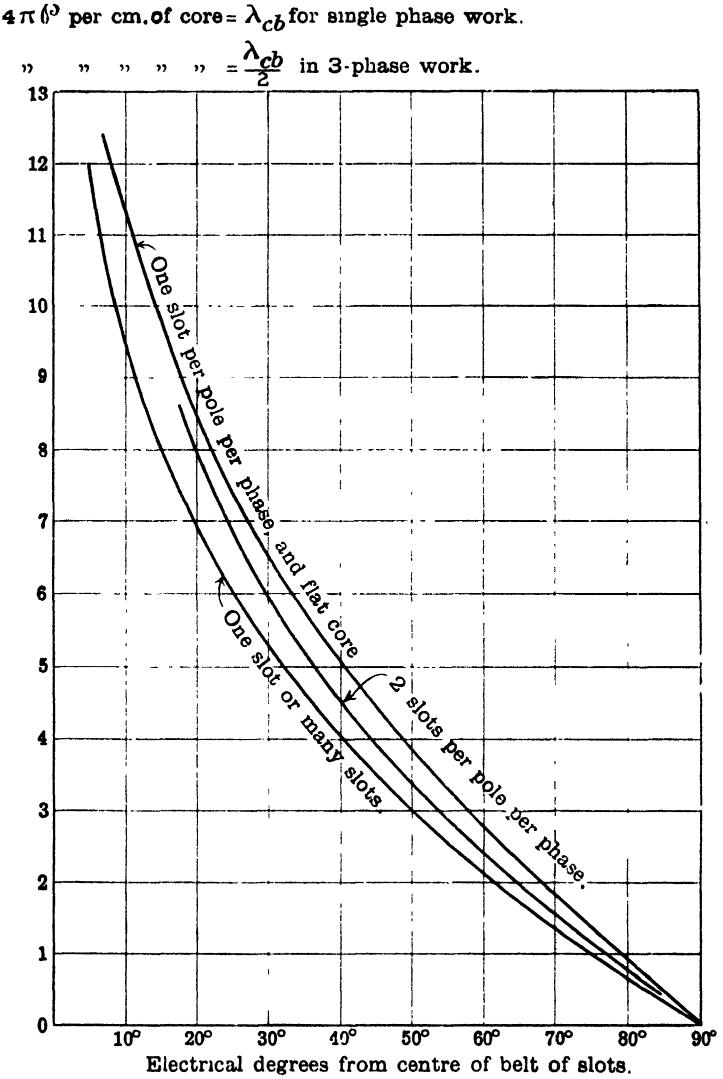

FIG. 547.—Values of $4\pi\mathfrak{P}$ integrated from a starting-point 90° from the centre of a coil side.

It is also of interest to compare the permeance of the concave bore with that of the straight flat core with semicircular paths and with the addition, as in the continuous-current case, of the permeance of the corresponding arcs past 90 electrical degrees and reckoned onward towards the opposite coil-side. This addition has been made in the upper curve of Fig. 547, which thus repeats the upper dotted curve of Fig. 384, and it will be seen that the permeance of the curved bore is less than that of the flat core. This result may be followed when it is remembered that the whole of the resultant flux linked with one coil-side is compelled to pass within a certain prescribed area across a radius, and cannot branch out in ever widening circles as it can in the case of the flat core.

When the t turns of a coil-side are distributed uniformly or in groups in two or more slots, the effect must in every case be to reduce the *inductance* as compared with that of the single slot, owing to the fact that the whole of the flux is not linked with all the turns. But there is a further effect on the distribution of the flux; the permeance corresponding to any given arc round the surface of the bore may be altered as compared with the single-slot case, owing to the path and area traversed by the flux of the arc being altered. With a 3-phase machine and numerous slots per pole per phase, within a belt of 30° on either side of the centre of the coil-side, the M M F acting across the bore, instead of being uniformly equal to that of the t turns, only gradually grows to this value. The result is that the flux that would otherwise occupy a zone near to the coil-side is weakened. The secondary effect of this is that the bands of flux embracing the central turns pass in a straighter path across the bore, being confined to a narrower area. The joint flux beyond the outer turns also passes in straighter lines from side to side, and finding more room between the central turn and the centre of the bore, is increased in amount. Thus the effective *permeance* from 30° onwards or λ_{cb} at 30° is increased as compared with the single-slot case. And this will also be the case if the turns of the phase are concentrated into a few small groups within 2, 3, or 4 slots The change of the permeance for any given arc as compared with the single-slot case is, however, not so important as the fact that the central flux is reduced.

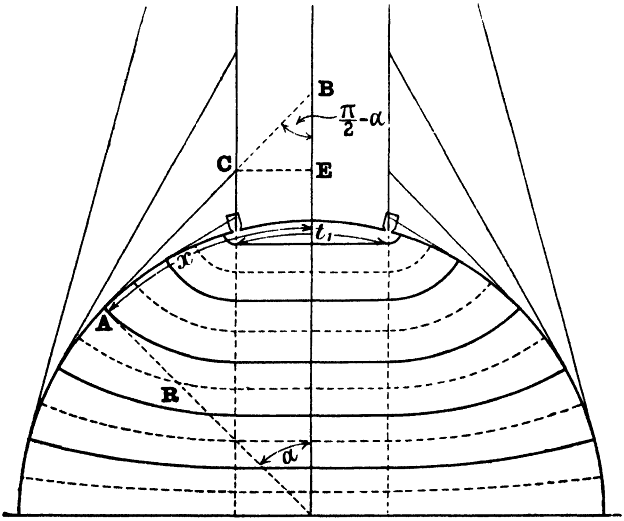

FIG. 548.—Construction for calculation of permeance with 2 slots per pole per phase in a 2-pole machine.

To represent the case of 2 or more slots, and to calculate numerical values, *e.g.* in the two-pole machine, we may, as in Fig. 548, shift the lines on which fall the centres of the radii to a distance of either $\frac{1}{2}t_1$ with an even number, or of t_1 with an uneven number, of slots to the left and right of the centre line. The new value of the radius of any path is then $AC = AB - BC$. If γ = the real angle corresponding to one tooth-pitch, $= \frac{360°}{S}$, $CE = R \sin \frac{\gamma}{2}$

with an even number of slots, or $R \sin \gamma$ with an uneven number, while $BC = \dfrac{CE}{\cos a}$. The length of the path in air at any angle a or arc x from the centre line is then composed of the two arcs each of length $\left(\frac{\pi}{2} - a\right)AC$, *plus* t_1 or $2t_1$, according as the number of slots is even or uneven, *i.e.* in the two cases the total length is

$$2\left(\frac{\pi}{2} - a\right) R \left(\tan a - \frac{\sin (180°/S)}{\cos a}\right) + t_1$$

or

$$2\left(\frac{\pi}{2} - a\right) R \left(\tan a - \frac{\sin (360°/S)}{\cos a}\right) + 2t_1$$

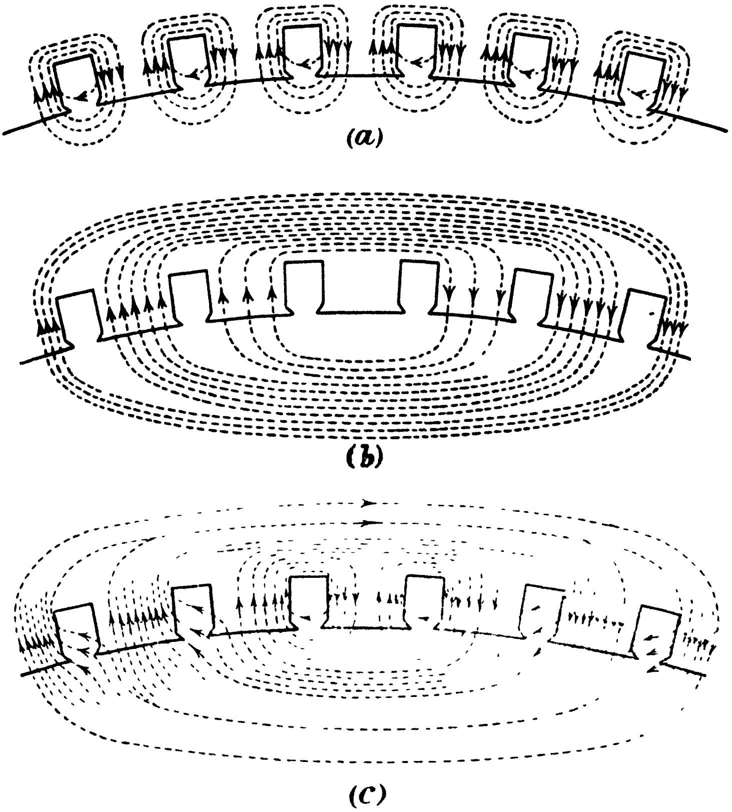

FIG. 549.—Paths of self-induced flux within the coil-side of a phase (*a*) corresponding to slot inductance ; (*b*) linked symmetrically in bands with two or more slots ; (*c*) resultant.

The contraction of the tubes is evidently less, and the resulting increase in the joint permeance as compared with the case of the single slot is greater the smaller the number of poles, and the greater the arc corresponding to a tooth-pitch. An extreme case of a 2-pole 3-phase machine with only 2 slots per pole per phase giving $\gamma = 30°$ is, therefore, chosen for investigation in Fig. 548. The summation of the permeances gives the intermediate curve of Fig. 547. At 20° from the centre line (the slot centre being 15° from the centre), $4\pi\mathfrak{S} = 8$ as compared with 7 for a single slot. The ratio of the total area of the tubes at their narrowest section to the area from which they spring has risen from 0·636 to 0·7. The suppression of the local flux near the centre of the coil is at its maximum with only 2 slots per pole, and in a 2-pole machine; the intermediate curve of Fig 547 may therefore be taken as giving the maximum values of λ_{cb} for the joint flux due to the coil as a whole. With 3 or any larger uneven number of slots there still remains some local flux linked with the central slot, and the distribution of the flux more nearly repeats that of the single-slot case. Hence on the average the lower curve of Fig. 547 may also be used for calculating λ_{cb} with several slots, and has so been marked

(ii) *The local permeance relative to portions of the belt of a single phase.*—Within the band of slots containing the coil-side of a phase, the combined action of slot and symmetrical field is precisely the same as that already described in the Note to Chapter XX in connexion with continuous-current armatures,

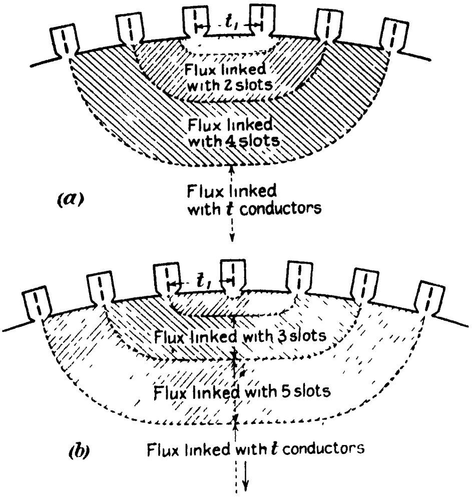

FIG 550—Linkages with (*a*) even; (*b*) uneven number of slots.

the only difference being in the concave surface of the stator bore instead of the convex surface of the rotor (Fig 549) The slot inductance having been allowed for, it only remains to consider as in the Note to Chapter XX the symmetrical bands of flux

Owing to the very different values of the ratio w_t/w_3 according to whether the slots are open, half-closed or almost entirely closed, it is not possible with alternators to calculate the permeance for the local fluxes once and for all, and to express the whole in terms of $D/2p$ w_3 as can with fair accuracy be done with continuous-current armatures (Chap XX, § 24*b*) In the latter w_t/w_3 seldom varies beyond a range of 1·5 to 3, but in alternators it may range from 1 to 10, or $t_1/w_3 = 2$ to 11.

With an uneven number of slots per pole per phase, in order to complete the region over the central slot, we may assume a flat core and either semicircular paths up to a distance of $t_1/4$ from the centre-line or paths made up of quadrants and straight lines up to a distance $\frac{1}{\pi}(t_1 - w_3/2)$. The former assumption yields a local value of $\lambda = \frac{9\cdot2}{(g')^2} \times \log \frac{t_1}{2w_3}$, and the latter a value of $\lambda = \frac{9\cdot2}{(g')^2} \log \frac{t_1}{3\cdot14\, w_3}$, to which in either case must be added the difference in the values from Fig. 547 for the angle $\frac{180^\circ}{4g}$ or $\frac{180^\circ}{\pi g} \times \left(1 - \frac{w_3}{2t_1}\right)$ and $\frac{180^\circ}{g}$.

Either assumption leads to nearly the same result, so that the former as the simpler will serve. Thence on the lines of Fig. 550*b* with 3 slots per pole per phase

$$\lambda_{ca} = \left(\frac{1}{3}\right)^2 \left(9{\cdot}2 \log \frac{t_1}{2w_3} + 5{\cdot}5\right) = \left(\frac{1}{3}\right)^2 \left(9{\cdot}2 \log \frac{t_1}{2w_3}\right) + 0{\cdot}61$$

the value of $4\pi\mathfrak{S}$ from $t_1/4$ to t_1 or from 5 to 20 electrical degrees being $12{\cdot}5 - 7 = 5{\cdot}5$.

With 5 slots per pole per phase, from 3° to 12°, $4\pi\mathfrak{S} = 13{\cdot}5 - 8{\cdot}7 = 4{\cdot}8$ and from 12° to 24° $= 8{\cdot}7 - 6{\cdot}3 = 2{\cdot}4$, so that

$$\lambda_{ca} = \left(\frac{1}{5}\right)^2 \left(9{\cdot}2 \log \frac{t_1}{2w_3} + 4{\cdot}8\right) + \left(\frac{3}{5}\right)^2 \times 2{\cdot}4$$

and similarly for higher uneven numbers of slots per pole per phase.

With an even number of slots per pole per phase, as contrasted with the continuous-current case of p. 166, Vol. II, the greater distance between the slots in the alternator necessitates that the curvature of the stator face should be taken into account in determining the distance from the centre at which the limiting line of the local flux linked only with a slot falls (Fig. 550*a*). On the basis of Fig. 548 the particular angle α' is fixed by the relation

$$\frac{1}{\pi\,(x\,w_t + 0{\cdot}5\,w_3)} = \frac{2}{2R\left\{\left(\frac{\pi}{2} - \alpha'\right)\left(\tan\alpha' - \frac{\sin\,(180^\circ/S)}{\cos\alpha'}\right) + \sin\,(180^\circ/S)\right\}}$$

and since $xw_t + 0{\cdot}5\,w_3 = R\,(\alpha' - \pi/S) = R\,(\alpha' - \pi/2pg)$, the angle α' in radians can be found from

$$\alpha'\left\{\pi + \left(\tan\alpha' - \frac{\sin\frac{\pi}{2pg}}{\cos\alpha'}\right)\right\} - \pi/2\left(\tan\alpha' - \frac{\sin\frac{\pi}{2pg}}{\cos\alpha'}\right) = \frac{\pi^2}{2pg} + \sin\frac{\pi}{2pg}$$

In a two-pole 3-phase machine this gives with 2 slots per pole per phase $\alpha' = 23^\circ = 0{\cdot}766\,t_1$ and $\alpha' - \pi/S = 8^\circ = 0{\cdot}266\,t_1$. The permeance for the local flux is then $\frac{2{\cdot}3}{\pi}\log\frac{0{\cdot}266\,t_1}{0{\cdot}5\,w_3} = \frac{2{\cdot}3}{\pi}\log\left(0{\cdot}53\,\frac{t_1}{w_3}\right)$, and

$$\lambda_{ca} = \left(\frac{1}{2}\right)^2 \times 2 \times 9{\cdot}2\log\left(0{\cdot}53\,\frac{t_1}{w_3}\right)$$

With increasing even numbers of slots, the dividing line of the local fluxes linked with one slot only falls nearer and nearer to $0{\cdot}818\,t_1$, the value for a flat core. Thus with 4 slots per pole per phase $\alpha' = 12^\circ = 0{\cdot}8\,t_1$ and $\alpha' - \pi/2pg = 4{\cdot}5^\circ = 0{\cdot}3\,t_1$, whence

$$\lambda_{ca} = \left(\frac{1}{4}\right)^2 \times 2 \times 9{\cdot}2\log\left(0{\cdot}6\,\frac{t_1}{w_3}\right) + \left(\frac{1}{2}\right)^2 \times 2{\cdot}5$$

(iii) *The combined permeances.*—We thus have in 3-phase machines for the sum of the joint and local effects in combination with 2 slots per pole per phase

$$\begin{aligned}\lambda_c &= \lambda_{cb} + \lambda_{ca}\\ &= 7{\cdot}4 + \frac{1}{2} \times 2 \times 9{\cdot}2\log\left(0{\cdot}53\,\frac{t_1}{w_3}\right)\\ &= 7{\cdot}5 + \frac{1}{2} \times 9{\cdot}2\log\frac{t_1}{2w_3}\end{aligned}$$

with 3 slots per pole per phase

$$= 7 + 0{\cdot}61 + \left(\frac{1}{3}\right)^2 \times 9{\cdot}2\log\frac{t_1}{2w_3}$$

with 4 slots per pole per phase

$$\begin{aligned}&= 6{\cdot}8 + \left(\frac{1}{2}\right)^2 \times 2{\cdot}5 + \left(\frac{1}{4}\right)^2 \times 2 \times 9{\cdot}2\log\left(0{\cdot}6\,\frac{t_1}{w_3}\right)\\ &= 7{\cdot}5 + \frac{1}{8} \times 9{\cdot}2\log\frac{t_1}{2w_3}\end{aligned}$$

with 5 slots per pole per phase

$$= 6{\cdot}3 + \left(\frac{3}{5}\right)^2 \times 2{\cdot}4 + \left(\frac{1}{5}\right)^2 \times 4{\cdot}8 + \left(\frac{1}{5}\right)^2 \times 9{\cdot}2 \log \frac{t_1}{2w_3}$$

$$= 6{\cdot}3 + 0{\cdot}86 + 0{\cdot}19 + \left(\frac{1}{5}\right)^2 \times 9{\cdot}2 \log \frac{t_1}{2w_3}$$

$$= 7{\cdot}35 + \left(\frac{1}{5}\right)^2 \times 9{\cdot}2 \log \frac{t_1}{2w_3}$$

The first numerical constant in the above expressions thus contains not only λ_{cb} but also λ_{ca}, except for the small innermost portion

The main fact is that with different numbers of slots, the relative proportions of λ_{ca} and λ_{cb} vary, yet their sum remains practically constant except for a small portion of the flux embracing the central slot or slots near to the centre where the slot opening w_3 has an appreciable effect, especially in the single-slot case. It has still to be recognized that when compared with the results of actual experiments on the stators of alternators when in air, the calculated estimates of the inductance are usually too low, and better agreement with measured values is obtained if the constant is raised to say 11 for 2 slots falling to 9 with 8 slots per pole. It is difficult to establish such high values on theoretical grounds, but to some extent it is due to lateral extension of the area through which the flux passes across the bore beyond the planes of the core width.

The calculations being necessarily only approximate, comparison of the several expressions shows that no great violence is done by adopting as a general expression for a small number of slots per pole per phase

$$\lambda_c = 10 + \frac{9{\cdot}2}{(g')^2} \log \frac{t_1}{2w_3}$$

(iv) *The 3-phase alternator under load*—So far only a single phase of a 3-phase alternator has been considered, and when we pass to the inductance of the same coil-side in 3-phase working, the case is altered. The mutual inductance of the other phases has then to be taken into account as increasing the apparent inductance of each separate phase.[1] An approximate allowance for this is sometimes made by adding, say, 15 per cent. to the inductance calculated for a single phase But Rezelman[2] has shown more accurately that the total reactance of two phases in simple series should be determined, and one-half of this value be taken At one moment in each period a current of the same value flows through two phases in series in agreement as to direction, the current in the third phase being zero This moment is the best to choose as giving a mean value for the inductance which in reality pulsates.

The two phases in series are then equivalent to a winding with the same tooth-pitch, but extending over twice as much of the pole-pitch. It is obvious that the slot inductance per phase and also the small local flux capping the opening of each slot as assumed in the previous calculations, are unaltered. But the inductance due to bands of symmetrical flux, linked with groups of slots but not with the whole, and the inductance due to the joint flux beyond the limits of the $2g'$ slots, will be increased, although the total will by no means be double the value in single-phase work.

The total number of slots in the two phases is always even, except when g' is fractional and the two phases partially overlap one another in the same slot. With a single slot per pole per phase, so that each slot is situated at 30 electrical degrees from the centre, their great distance and the curvature of the core increases the effects already mentioned under 2 slots per pole per phase in single-phase work The dividing line of the local flux in a 2-pole machine now falls at 43° $= 0{\cdot}716\,t_1$, and $a' = \pi/2pg = 0{\cdot}216\,t_1$ so that the local permeance is

$$\frac{2{\cdot}3}{\pi} \log \left(0{\cdot}432 \frac{t_1}{w_3}\right)$$

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[1] *Cp.* Parshall and Hobart, *Engineering*, Vol. 70, p. 819.

[2] *Electrician*, Vol. 63, p. 742, and Vol. 65, p. 100.

The value of λ_{cb} for the joint flux as read off from the curves of Fig. 547 must in every case be doubled in relation to the t turns of a single phase when in 3-phase work; for the joint permeance is acted upon by $2t$ turns, and the flux is doubled. So far as this flux is concerned, the inductance of the two phases is 4 times that of a single phase with t turns in the same slots, but the inductance of one only out of the two phases is doubled by the mutual induction. Using the intermediate curve of Fig. 547 the value of λ_{cb} at 43° is 4·6, and in 3-phase work λ_{cb} is 9·2. The total coefficient is, therefore,

$$\begin{aligned}\lambda_{c3} &= \lambda_{cb3} + \lambda_{ca3}\\ &= 9{\cdot}2 + 9{\cdot}2 \log\left(0{\cdot}432\,\frac{t_1}{w_3}\right)\\ &= 8{\cdot}62 + 9{\cdot}2 \log\frac{t_1}{2w_3}\end{aligned}$$

Analogously, with $g' = 2$ or more, so that in 3-phase work there are bands of flux linked with groups of slots, the values of λ_{cb} for each successive tooth-pitch reckoning from the joint flux end must be doubled and multiplied successively by $\left(\frac{g'-1}{g'}\right)^2$, $\left(\frac{g'-2}{g'}\right)^2$, etc, up to $\left(\frac{1}{g'}\right)^2$ for the slot nearest to the centre.

As might be expected, the increase by mutual inductance in 3-phase work is but small for a single slot per pole per phase, but for 2 slots per pole per phase is about 28 per cent. and but little over 30 per cent. for higher numbers of slots. But again the outstanding fact is that while λ_{cb3} sinks, λ_{ca3} rises from about 1·8 for 2 slots to 4 for many slots and, therefore, the sum $\lambda_{ca3} + \lambda_{cb3}$ is remarkably constant. If increased analogously to the preceding expression for the single-phase case[1] it may be taken that

$$\lambda_{c3} = 13 + \frac{9{\cdot}2}{(g')^2}\log\frac{t_1}{2w_3}$$

In both cases the value of the last term rapidly diminishes with a number of slots, and it is only retained in order in the general expressions to meet the case of a single or at most 2 slots per pole per phase.

(v) *The special case of ampere-conductors sinusoidally distributed over the whole pole-pitch.*—In the special case when the distribution of the ampere-conductors round the pole-pitch follows a sine law, M. Schenkel[2] has shown mathematically that the total flux crossing a radius and forming one section of the field[3] is per cm. length of core 1·257 times the amplitude of the curve of ampere-conductors, whatever the number of poles and whatever the radius of the bore. He has also shown that under the above assumption of a sinusoidally distributed set of ampere-conductors the flux passes across the bore of the two-pole machine in straight lines and at a uniform density inversely proportional to the radius; in the 4-pole machine the paths from pole to pole are hyperbolæ, and the density on the centre line increases directly in proportion to the distance from the centre. With more than 4 poles the density increases rapidly as the inner surface of the bore is approached.

Now at the moment which has been above considered when two phases are in series in 3-phase work, if each phase has several slots per pole, it follows from Chapter XXVII, § 12, Fig. 514 (ii) that if the current is 1 C.G.S. unit, the curve of M.M.F. rises at a uniform rate from zero to $4\pi t$ at 60°, remains constant at this value for 60°, and then sinks again to zero at the end of the pole-pitch. The fundamental sine-wave of such a trapezium has an amplitude which only exceeds the height of the trapezium in the ratio of $\frac{18}{\pi^2}$ to $\sqrt{3}$ or 1·05 to 1;[4] such a wave, therefore, fairly well represents the trapezium, merely rounding off its corners, and the total flux is then $4\pi t \times 1{\cdot}05$.

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[1] The tests of Rezelman (*Electrician, loc. cit.*) indicate even higher figures for λ_{cb}, viz., 12 for a single phase and 15 in 3-phase work.

[2] *Elektrotechnik und Maschinenbau*, Vol. 27, p. 201, abstracted in *Electrician*, 23rd July, 1909.

[3] The flux corresponding to a pole-pitch is twice this amount.

[4] Schenkel, *loc. cit.*

But to determine the inductance, the linkages of the flux with varying numbers of turns have also to be considered.

In the particular case of the two-pole machine the density on the curved bore for any small arc dx is $\frac{4\pi t \times 1{\cdot}05}{R}$ $\sin\alpha \, . \, dx = 4\pi t \times 1{\cdot}05 \sin\alpha \, . \, d\,\alpha$, and this is linked with $t \sin\alpha$ turns *of one phase*, the hypothesis being that the turns are sinusoidally distributed. The inductance of the one phase is, therefore,

$$\begin{aligned} &4\pi \times 1{\cdot}05\, t^2 \int_0^{\frac{\pi}{2}} \sin^2\alpha \, . \, d\,\alpha \\ &= 10{\cdot}35\, t^2 \end{aligned}$$

which bears out the general order of the figures already deduced for the inductance of a single phase in 3-phase work

The true linkages are, however, not strictly reproduced by the above assumption, and this is still more the case when the same method of calculation is applied to the multipolar machine

(vi) *The values to be taken for use during design* --Now though the general expressions above obtained yield good results when tested by comparison with experiment with the rotor removed, the question still remains as to how far it is legitimate to include the $\lambda_{ca} + \lambda_{cb}$ term when the rotor is inserted and excited, and the cross and direct ampere-turns are duly taken into account. Undoubtedly the values deduced for λ_a from experimental measurement of the reactance when the field-magnet is removed, are somewhat in excess of the true value that is required for the determination of the fall of potential.[1] Schenkel has advocated the subtraction of the whole term due to the surface-of-the-bore permeance so far as the flux crosses the bore in a 2-pole, or passes from pole to pole in a multipolar machine [2] But this appears to reduce the amount too greatly When it is remembered that under working conditions the main flux is also present, part of the fall of potential which is attributed to inductance may in reality be due to a secondary magnetic cause, viz , a virtual contraction of the iron area of every tooth on the side where the main and supposed self-induced fluxes are opposed, and increased saturation where they are in the same direction Such a local cross effect which would not appear as a genuine shifting of the entire field would increase the ampere-turns required under load or cause an apparent fall of potential, although in one sense not arising from a true electrical inductance As a compromise, therefore, may be taken the single-phase value of equation (255*a*) for use in 3-phase work with 2 or 3 slots per pole per phase In the design of a machine it is above all things necessary to err on the side of too high rather than of too low a value for the calculated inductance But with more slots, the value of the constant falls, and for turbo-alternators with, say, 6 slots per pole per phase and long cores, it may be reduced to 5 or 6

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[1] *Cp* Hobart and Punga, *Elektrische Kraftbetriebe u Bahnen*, Vol. 5, p 613.

[2] The subject-matter of the present Note derives further importance from the fact that by deducting the calculated value of the surface-of-the-bore inductance from the total value measured with rotor removed, an estimate can be obtained of the inductance of the end-connexions, self and mutual, upon which the initial amplitude of the short-circuit current mainly depends ; see especially Dr R. Pohl, *E.T.Z*, Vol 42, p 1057

CHAPTER XXVIII

CONSTRUCTION OF ALTERNATOR ARMATURES AND FIELD-MAGNETS

§ 1. The iron armature core.—The iron core of the alternator armature must be laminated similarly to that of the continuous-current dynamo, and owing to its large size is usually built up of segmental discs, 0·02 in. thick, or in special cases even thinner (0·014 in.). The radial depth of iron is given more lavish proportions in the alternator, in order to reduce the flux-density and thereby keep within reasonable limits the loss from hysteresis, which would otherwise be high owing to the greater frequency. Usual values for the density B_c in the iron below the teeth of heteropolar armatures do not exceed 12,000 lines per sq. cm. for 50 cycles to 13,000 for 25 cycles.

The apparent density in the teeth of the external stator for 50 cycles is not usually taken quite so high as in continuous-current machines, but may be 18,000 to 19,000, although at 25 cycles it may reach 21,000. A fairly high density reduces the necessary depth of slot and the weight of iron, and with it the total loss by hysteresis and eddy-currents, although the loss per unit volume may be increased. In all cases the discs require to be very tightly compressed by cross bolts and securely held, in order to prevent the humming noise due to the rapid variations of the magnetic field from becoming disagreeably loud. Ventilating ducts $\frac{3}{8}$ in. to $\frac{1}{2}$ in. wide are arranged every 2 or 3 inches along the length of the core, and it is even more important in the alternator than in the continuous-current dynamo that the separation of the laminæ should be effective, so as to minimize eddy-currents.

§ 2. Drum-winding.—In the *drum armature* the two classes of wire and bar-winding—each with its subdivisions according as it is hand- or former-wound—may again be found as in the continuous-current dynamo, but not so sharply divided from each other.

The coils are in either case embedded in the slots of the *stator*, so that they are relieved from any centrifugal force ; the field coils of the rotating magnet can better withstand centrifugal force, and the greater moment of inertia of the field-magnet as the rotor is of advantage as assisting to secure a uniform angular velocity.

For fairly high voltages and moderate outputs, round wire is usually employed. When a many-turn coil winding is embedded in closed tunnels stamped out round the edge of the armature discs, and the numerous turns are threaded through by hand, the holes or tunnels are first completely lined with seamless tubes of micanite or compressed paper projecting well out beyond the ends of the armature core, and the wires must be well insulated and sufficiently tight in the holes to prevent chafing. The method is, however, expensive in labour and somewhat wasteful of space, since plenty of room must be allowed to the winder. It has, therefore, been abandoned in favour of made-up coil-sides, which can be pushed through the tunnels and afterwards joined up by separate end-connectors at one (Fig. 552 (i)) or both ends. Still more commonly the slots are half-closed or open. With open slots the advantages of former-wound coils in that they are easy to wind, are more regular and stiffer, and can be thoroughly insulated before being placed in the slot, are all obtained. The coils are thickly wrapped round with oiled cloth, canvas, or micanite paper, taped over, and impregnated with insulating varnish (Figs. 551 and 552); they are then driven tightly into the slots and fastened with shallow wooden wedges, without further protection. Or the slot may itself be lined with an open insulating trough, but in such cases the junction of trough and retaining wedge is a weak spot under high pressure. The claim that a former-wound coil inserted in an open slot is easy to remove and replace if damaged is somewhat delusive, as after prolonged use the heating of the varnish causes the coils to adhere very tightly to the slots; considerable force is then required to detach them, and during the

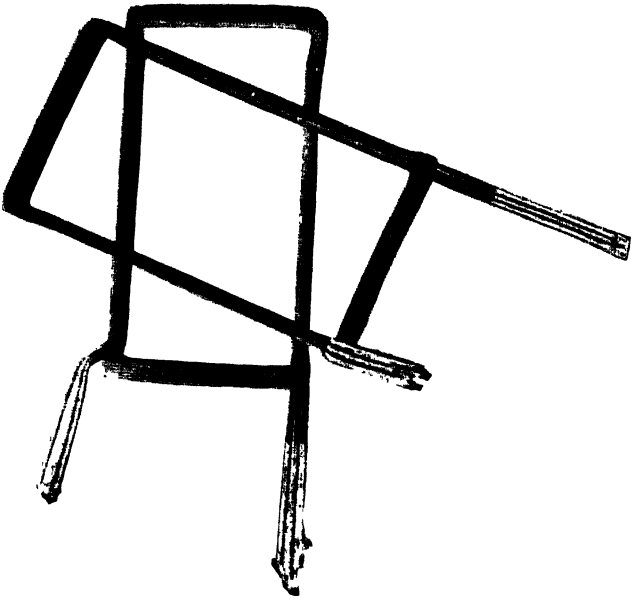
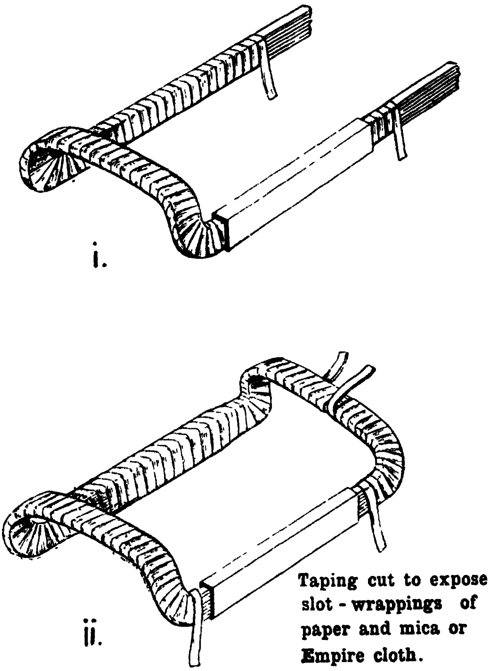


FIG. 551.—Armature coils of alternator.

process other coils which in polyphase machines overlap may become damaged.

To minimize the voltage between adjacent turns when there are two or more turns abreast in each layer of a slot the winding should work across each layer and so gradually proceed from layer to layer, rather than work down and up the slot. If the required section of copper in a conductor becomes too great for a single round wire, a number of wires in parallel can be adopted, or a stranded cable, but with loss of mechanical strength in the end-connexions. Rectangular wire or band copper can also be wound on shapers and used with open slots. With only one layer of coil-sides in the slots, if the slot portion is long as compared with the end-connexions, the rectangular strip is wound round a former edgewise if it is to lie flat in the slot, or flatwise if it is to be on edge in the slot, and the ends are then bent up on another former, the turns being stepped so that after bending they lie abreast. If on the other hand the end-connexions are longer than the slot portion, the reverse holds, and it is then the slot portions that are bent up.



Taping cut to expose slot - wrappings of paper and mica or Empire cloth.

FIG 552 —(i) Coil to push through slot ; (ii) former-wound coil for open slot.

The ends of the tubes or insulation of the slot portion of the coils should be sealed by being taped up with oiled cambric, in order to prevent " breathing," *i.e.* alternate expulsion and drawing-in of air due to heating and cooling. Effective sealing is difficult to secure on coils hand-wound in place and is only secured on former-wound coils by carrying the taping of the ends of the coils right over the slot insulation. Fig. 552 (i) shows a coil with slot insulation ironed on over the coil insulation, which may be thrust through closed or semi-closed slots, but only one end can then be completely sealed when the coil is constructed ; the coil of Fig. 552 (ii) can only be used with open slots, since it must be driven down into them, but in it both ends can be sealed. Rectangular or oval-shaped tubes of micanite with a long scarfed opening down one side have also been used, which enable the straight sides of the coils to be inserted into their insulating envelope before they are placed within the slots. With armatures of narrow width and well-exposed ends to the coils the current-density in the armature wires is usually about 2500 to 3000 amperes per square inch.[1]

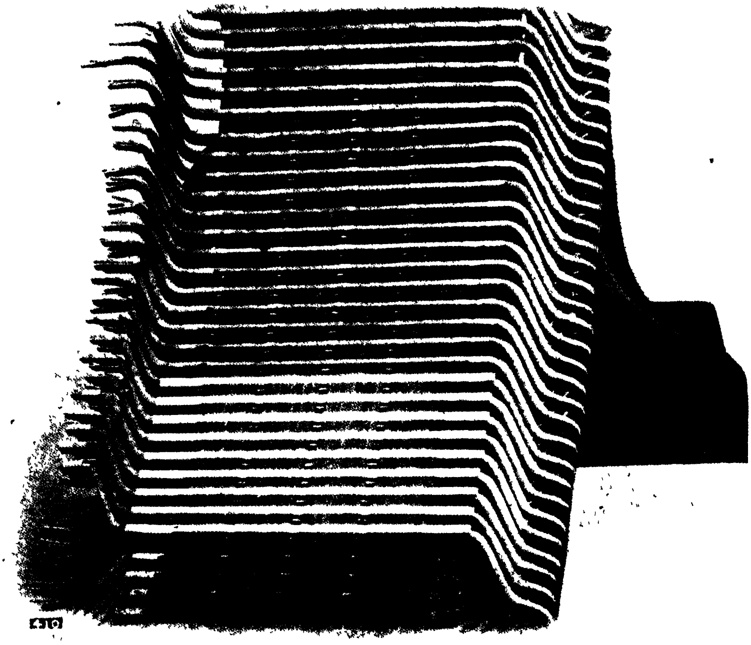


FIG. 553.—Portion of stator of alternator showing a double-layer winding with V end-connexions.

Fig. 553 shows a barrel-winding in two layers like that of a continuous-current armature with V end-connexions. For low voltages or large outputs at high voltages the transition is gradually made to the simple hand-wound bar armature with U-shaped loops pressed into the slots or finally to solid bars of rectangular or circular section, alternately long and short, threaded through insulated tunnels and bolted or riveted and soldered to end-connectors of flat copper strip bent to a double involute or forked shape.

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[1] For measurement of the temperature of the stationary armature coils in an alternator when running, an alcohol thermometer is to be preferred, if there is much stray field, as a mercury thermometer may give false readings if eddy-currents are set up in it.

The ends of the coils are lapped with several layers of insulating tape, with a final wrapping of cotton tape on the outside.

§ 3. Number, proportions, and type of slots.—The number of slots per pole per phase varies from two to twelve ; three or four are common, but beyond this latter number the loss of space in insulation becomes too pronounced with high pressures of, say, 10,000 volts. Turbo-alternators have higher numbers of slots per pole per phase, but in all cases the slot-pitch should not much exceed $2\frac{3}{4}$ in., a suitable slot width being then 1 in. This gives a mass of copper, the heat from which will be as much as the conducting surface of the surrounding slot can deal with. With a given number of active conductors the effect of the number of slots on the reactance is dependent upon a variety of complex conditions, chief among which is the width of the opening and the ratio of the width of the slot to its depth. Various shapes are in use, but in general they approximate to rectangles with a depth three times their width, and preferably the corners are rounded off (Fig. 554). It will be found from § 6 that a limit to the depth of slot is in many cases set by unequal current-distribution in large solid conductors.

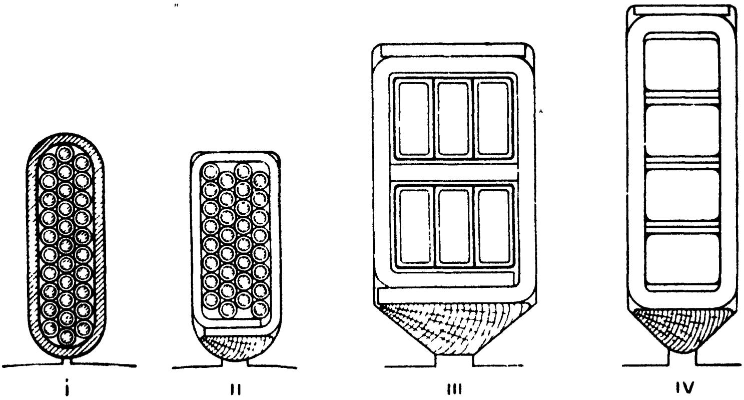

FIG. 554 —Types of slots and windings

The open slot (Fig. 580) necessitates the lamination of the pole-shoes, and has the disadvantage that has already been discussed in Chapter XXVI of introducing tooth-ripples into the E.M.F. wave. The closed slot is free from both these objections, and further it slightly reduces the reluctance of the air-gap ; yet apart from the expense of winding, it may be an objection that, however thin the iron bridge across the mouth of the slot, it considerably increases the inductance of the coil. This disadvantage is largely overcome by employing a half-closed slot such as Fig. 554 (ii) or (iii) ; this still involves more labour than the open slot with former-wound coils, but with it it is often possible to employ former-wound coils if the opening be not very small, and with a normal length of air-gap and a slot opening less than half its width solid poles may still be retained. The separate wires of a coil-side are fed by hand, one by one, through the opening of the slot and through the jaws of the micanite sheath which project out of the slot. Another design is shown in Fig. 555.

§ 4. Chemical action in high-pressure windings.—If there are air interstices between the turns of a coil and the slot insulation or between the slot insulation and the iron laminations, the potential stress is largely concentrated across the small air-gaps. For the voltage across any layer forming part of a composite dielectric will vary directly as its thickness and inversely as its specific inductive capacity, and since the specific inductive capacity of, say, mica is to that of air as 6 : 1, the potential gradient (*i.e.* the volts per mil of radial thickness) across the air is much steeper than across the mica. For a given total thickness of air and mica and a given voltage, this becomes more marked as the air-gap is reduced in thickness,[1] but though the gradient thus becomes very steep and the stress very great in the case of very thin gaps, this must not be pressed to imply that disruptive discharge would necessarily occur, since other conditions would then have to be taken into account to describe truly the complexities of the real case.

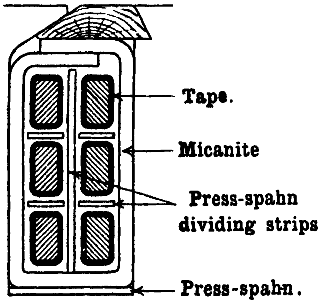

FIG 555.—Half closed slot.

If, then, there are air interstices and the potential to earth is high enough, an electrostatic discharge may take place across the air interstices, ozone and oxides of nitrogen, and in the presence of moisture, nitric and nitrous acids being formed. These products attack the oils and gums used in the varnish covering the cotton braid or taping, and oxidize them, so that marked corrosion results. The exact oxidizing agent is not clearly known ; it is not always nitric acid, but, whether ozone or a high oxide of nitrogen, the gases can spread beyond the region within which they are formed and so extend the area of damage. Linseed oil is especially affected by the strong oxidation, the insulation becoming sticky, and even the cement used in building up micanite tubes suffers ; but failure usually originates primarily from corrosion of the braiding or taping on the armature wires and consequent short-circuiting of adjacent turns.

Experience shows that with ordinary insulation if the voltage does not exceed about 3500 R.M.S. volts to earth (*e.g.* as in a 6000-volt 3-phase star-connected alternator with neutral point earthed), chemical action need not be feared. The reason is simple; the necessary clearances allowed to the winder being more or less a constant, the slot insulation in low-voltage machines is relatively to the voltage made so thick for mechanical reasons that it absorbs enough of the potential stress to prevent any silent discharge across the enclosed air-spaces.

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[1] See especially "Chemical Action in the Windings of High-voltage Machines," by A. P. M. Fleming and R. Johnson, *Journ. I.E.E.*, Vol. 47, p. 531, which deals fully with the matter, and is here followed.

In the case of voltages above 3500 to earth, it is not possible to estimate the potential gradient which occurs in practical cases across the small air-pockets and which has actually caused failure. We must, therefore, fall back as a guide on the total R.M.S. volts to earth per mil of radial thickness of insulation. As an approximate upper limit, above which special precautions in insulating must be observed to prevent failure due to chemical action, Messrs. Fleming and Johnson give 35 volts per mil of total slot insulation.

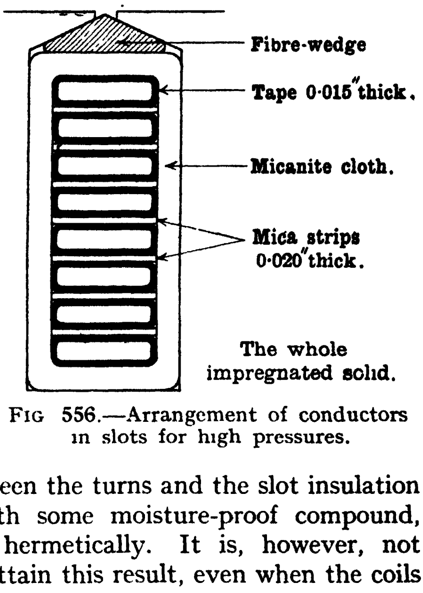

FIG 556.—Arrangement of conductors in slots for high pressures.

The ideal alternator coil is one in which throughout the slot portion all interstices between the turns and the slot insulation have been filled up solid with some moisture-proof compound, and the ends finally sealed hermetically. It is, however, not commercially practicable[1] to attain this result, even when the coils are dried in a vacuum chamber and impregnated with varnish or molten compound under pressure. For very high pressures it therefore becomes advisable to reduce the air-spaces to a minimum by employing rectangular conductors (Fig. 556) to the complete exclusion of round wires, by dividing each conductor into as small a number of conductors in parallel as possible, by placing separating strips of mica between adjacent turns to prevent short-circuits, and by solidly impregnating the coils with a molten asphaltum compound rather than with varnishes of the linseed oil type. Finally, therefore, for high pressures the former-wound coil for use with open slots, in which the slot insulation can be applied as a wrapping, hot-ironed on, and finally sealed by carrying the taping of the end-portions of the coils right over it (Fig. 552 (ii)), is the best.

§ 5. Thickness of slot insulation.—Apart from abnormal pressure rises due to lightning strokes, the effect of an intermittent arc on the external network, or electrostatic surges due to sudden switching on or short-circuiting of portions of the network, an accidental earth may always occur close to one terminal of an ungrounded alternator, in which case the maximum potential difference to earth thrown on to the winding near the other terminals, will be (assuming a sinusoidal E.M.F.) $\sqrt{2}$ times the virtual line voltage. From the point of view of durability, however, it is also justifiable to consider the maximum or the R.M.S. volts to earth in *normal* working, when we are entitled to take into account the fact which may be known at the time of design that the 3-phase alternator, if star-connected, is to be worked with its neutral point earthed. The maximum voltage to earth is then, of course, only $\frac{\sqrt{2}}{\sqrt{3}}$ times the line voltage.

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[1] For reasons well explained by Messrs. A. P. M. Fleming and R. Johnson in the above-quoted paper.

On a basis then of 35 R.M.S. volts per mil, the approximate figures given in the following Table for the two cases are obtained. A minimum thickness of insulation is necessitated by the requirements of mechanical strength to withstand handling during construction, as shown by the lower volts per mil at the upper end of the scale. Further, with proper precautions, many high-voltage machines have proved durable with a stress of 60 R.M.S. volts per mil of radial insulation ; the figures in brackets which are not unusual are, therefore, added, although it is evident that the same factor of safety is not obtained with them as with machines of lower voltage.

TABLE XXXVI

Radial thickness of slot insulation		Permissible R M S volts of rated output	R M S volts per mil	Permissible R.M S. volts of rated output if three phase star connected with neutral grounded	R.M.S volts per mil
Inches.	Mm				
0·04	1	500	12·5	1000	14·5
0·055	1·4	1000	18·2	2000	21
0·07	1·8	2000	28·6	4000	33
0·09	2·3	3000	33·3	5000	32
0·115	2·9	4000	35	7000	33
0·165	4·2	6000 (7000)	36·4 (42·5)	10,000	35
0·2	5·1	7000 (9000)	35 (45)	12,000	35
0·23	5·85	8000 (12,000)	35 (52·5)	14,000	35
0·27	6·85	10,000 (16,000)	37 (60)	16,000	34

It may be added that in calculating the necessary depth and height of slot the margin that must be allowed for " slack " should be increased slightly with increased length of core, since the conductors cannot be mathematically straight along their length.

It must not from the above be concluded that stationary armatures of large size cannot be successfully wound directly for very high voltages; 11,000 volts is not uncommon, while the 1800 kilowatt alternators for the Paderno installation were built by Messrs. Brown, Boveri & Co. for 15,000 volts, and the 1500 kilowatt 15-cycle 150 revolutions per minute alternators built by Messrs. Schuckert & Co. for the Valtellina 3-phase railway have star-connected tunnel-wound armatures for 22,000 volts. Messrs. Ganz & Co., who were pioneers in high-pressure machines, have built several 5200 to 6600 kVA alternators for 30,000 volts pressure at the armature terminals.[1] The various considerations affecting the use of extra high-pressure generators in large sizes, especially when driven by steam-turbines, their increased cost having to be set against the saving in transformers, are brought out in Mr. B. A. Behrend's paper[2] on "The Practicability of Large Generators wound for 22,000 Volts," and the discussion thereon.

The slot insulation should project beyond the slot ½ in. + an additional ½ in. for every 1000 R.M.S. volts between phases, or, say,

R.M.S volts between phases.	*Inches.*	*Cm.*
1,100	1	2·54
2,200	1½	4
3,300	2	5
5,000	3	7·5
6,600	3¾	9·5
11,000	6	15
15,000	8	20

Each coil after completion should be tested in place in its slots with double the working pressure between copper and iron preparatory to the final pressure test, and the application of a very high pressure should not be prolonged for more than a few minutes for fear of setting up heating of the dielectric which may cause incipient charring and subsequent break-down under a much lower voltage. On the completed machine the British Standard Specification calls for a test by the application for one minute of 1000 R.M.S. volts *plus* twice the rated pressure, with a minimum of 2000 volts.

§ 6. Additional loss under load due to non-uniform current-distribution.—Bearing directly not only on the heating of the stator winding but also on the dimensions of its conductors is the important question of the non-uniform distribution of the current over the section of solid conductors and even of laminated conductors when they carry an alternating current. An additional loss over and above the normal $I^2 r_a$ loss as calculated from uniform distribution of the current thence arises, which may be regarded as an eddy-current loss. The calculation of its amount relatively to the normal ohmic loss was first investigated by Mr. A. B. Field,[1] and subsequent writers[2] have further explained and elaborated the implications already contained in Mr. Field's original theory and mathematical treatment.

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[1] For Manojlovak in Dalmatia at 420 revs. per min. (*Electr. Eng.*, Vol. 42, p. 691); for Arci and Subiaco at 225 and 450 revs respectively. Two machines of 7500 kVA at 345 revs per min. were in 1914 in course of construction for Castelmadama (*Electr.*, Vol. 73, p. 893).

[2] *Trans. Amer. I.E.E.*, Vol. 26, part I, p. 351; *cp.* also Mr. J. S. Highfield, *Journ. I.E.E.*, Vol. 47, p 558, for difficulty of repairs in turbo-alternators as limiting the pressure.

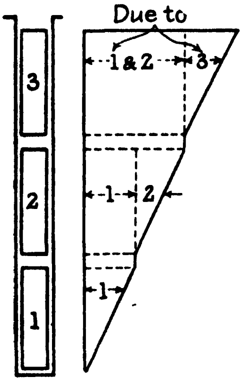

FIG. 557.
M.M.F. diagram for solid conductors in a slot.

The unequal current-distribution in a conductor embedded in a slot and carrying an alternating current is due to the self-induced flux which passes through it from side to side of the slot. In a solid circular conductor in air, carrying an alternating current, the well-known " skin effect " is due to the self-induced flux so far as it is wholly internal, but in the embedded conductor the flux causing the analogous effect is only partially internal and completes its circuit through the teeth and round the bottom of the slot ; the iron here traversed may be regarded as of negligible reluctance, as in the calculation of the inductance of coils.

The self-induced and mutually induced flux which is entirely outside the conductor throughout the whole of its path being only concerned in the property of inductance which the conductor possesses as a whole, the flux through any intervening insulation between layers of conductors in a slot gives rise to inductance, but is not a cause of unequal current-distribution. Consequently if as a first approximation a uniform current distribution is assumed throughout all the conductors in a slot, and they belong to the same phase, the M.M.F. diagram of Fig. 557 may be replaced by Fig. 558 (*a*), so far as unequal current distribution is concerned. In the latter the intervening spaces between the solid conductors occupied by insulation are imagined to be removed ; in these the M.M.F. due to the conductor or conductors below remains constant, but when each conductor is regarded as made up of a number of infinitely thin laminæ carrying the same infinitesimal current, it will be seen that the M.M.F. rises at a uniform rate in each as we proceed from the bottom to the top. The flux-density across the slot increases similarly, but the total flux, internal to a conductor, as linked with any given lamina in it increases towards the bottom of the conductor, and gives rise to a flux-linkage diagram (*b*) (contrast Fig. 358, where the *sum* of the flux-linkages with the wires when added up increases towards the top of the *coil*). Diagram (*b*)

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[1] *Trans. Amer. I.E.E.*, Vol. 24, p. 761, " Eddy-currents in Large Slot-wound Conductors."

[2] See especially M. B. Field, *Journ. I.E.E.*, Vol. 37, p. 83, " Idle Currents " ; Prof. Miles Walker, *The Specification and Design of Dynamo-Electric Machinery*, pp. 145–150 ; W. Rogowski, *Archiv. f. Elektrot.*, Vol. 2, p. 81, 1913 ; R. E. Gilman, *Trans. Amer. I.E.E.*, Vol. 39, Part I, p. 997, *Journ. Amer. I.E.E.*, Vol. 43, p. 194, S. L. Henderson, *Electr. J.*, Vol. 17, p. 418, and Waldo V. Lyon, *Trans. Amer. I.E.E.*, Vol. 40, 1921, p. 1361, and Vol. 41, 1922, p. 199 ; the last-mentioned writer deals with the problem by means of hyperbolic functions of *complex* angles.

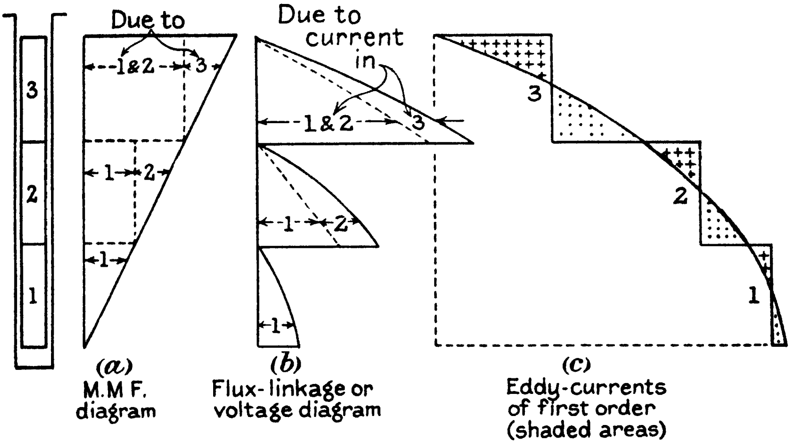

FIG. 558.—Eddy-current losses in solid conductors in a slot.

measures also to some scale the self- and mutually-induced voltage at every level, the measurements both for linkages and voltage being made horizontally from the vertical line. Thence, by moving the vertical across the voltage curve for each conductor, until it yields two equal areas, one on either side, a figure is obtained (Fig. 558 (*c*)) which is proportional to the spacial density of a local or " eddy " current of the first order, such that when superimposed on the original uniformly distributed current it fulfils the requirements of the alternating flux as due to the latter current. This eddy current is everywhere 90° out of phase in time with the original current.

But in each conductor the eddy current of the first order will set up its own magnetic field, and, as the flux threads through the laminæ, will affect analogously the current-distribution, *i.e.* it will call for an eddy-current of the second order 90° in time phase behind the first eddy-current or directly opposed to the original uniformly distributed current. The requirements of the alternating flux set up by the eddy current of the first order will be thereby fulfilled. The second eddy current will again in turn call for an eddy-current of the third order, and so on. Thus, by successive calculations of higher orders, the final current distribution can be approached indefinitely closely.

But the merit of the above simple treatment[1] is that investigation shows that in practical cases it is unnecessary to push the calculations beyond that for the eddy-current of the first order ; if the conductors are so deep that the eddy-current of the second order appreciably affects the non-uniform current-distribution deduced from the eddy-current of the first order and the calculated additional ohmic loss due thereto, the total loss in such a conductor is so high that it could not be used in practice.

The curves followed by the outer edges of 1, 2, 3 in the voltage diagram of the first order when pieced together, fall into a parabola, and the areas of the diagram (*c*) marked with dots and crosses well show that the eddy-currents become smaller as we descend from the top to the bottom of the slot, *i.e.* as the trace of the parabola becomes more vertical. But the greater the radial dimension of any one conductor, the greater its eddy-current.

(*a*) *The case of a single solid conductor* In a single solid conductor of thickness t cm. and height h cm carrying a uniformly distributed alternating current of amplitude **I**, at the bottom of a slot of width w_s cm , the M M F. at the moment of maximum current rises uniformly from zero to $\frac{4\pi \mathbf{I}}{10}$ at its top, and the transverse self-induced flux-density similarly from zero to $4\pi\mathbf{I}/10w_s$ (Fig 558*a*) ; when there are n_a such conductors abreast in a layer, all carrying currents of the same phase and magnitude, the values will be the above multiplied by n_a Let the solid conductor be imagined to be divided up into a number of very thin horizontal laminæ, and consider the thin lamina of thickness dx at the height x cm from the bottom of the slot It can be shown that the final distribution of the current is " lamellar " throughout the slot portion, *i e* that the density at any moment is throughout the same at any given level,[2] although the phase of the current will vary from layer to layer. But beyond the slot at each end of the core the current will quickly redistribute itself, and become of uniform density, save in so far as there is a weaker but analogous effect from self-induced flux traversing the substance of the end-connexions With a solid conductor, therefore, it is unnecessary to distinguish between the length of the core l and the length l_1 between the two planes where the current again becomes uniformly distributed. All statements in the present case can, therefore, be made per cm. length of slot or core

The total flux traversing the solid conductor is per cm. length of core

$$\frac{4\pi n_a}{10\, w_s}\mathbf{I} \times \frac{h}{2} = \frac{2\pi n_a}{10\, w_s}\mathbf{I}\, h$$

since the average M.M F acting on its whole slot depth h is $\frac{1}{2}\,\frac{4\pi n_a}{10\, w_s}\mathbf{I}$

The flux linked with the lamina, so far as the present effect is concerned, is that which crosses the conductor *above* the level of x, *i.e.* the difference between the total flux crossing the conductor and the flux

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[1] Due to H. W Taylor in his paper on " Eddy Currents in Stator Windings " (*Journ. I.E E.*, Vol. 58, p. 279), which is here followed. *Cp.* also Dr. R. Pohl, *E.T Z* , Vol. 41, p 908.

[2] M. B. Field, *Journ. I.E E.*, Vol. 37, p. 94.

$$\frac{4\pi n_a}{10\,w_s}\mathbf{I}\int_0^x \frac{x}{h}\,dx = \frac{2\pi n_a}{10\,w_s}\mathbf{I}\frac{x^2}{h}$$

which crosses beneath it. The difference is, therefore, $\frac{2\pi n_a}{10\,w_s}\mathbf{I}\left(h-\frac{x^2}{h}\right)$, and, this being the maximum value of an alternating flux linked with the lamina, the maximum voltage induced in it per cm. length is $-\frac{d\Phi_l}{dt}\times 10^{-8} = 2\pi f\Phi_l\times 10^{-8}$

$$= \frac{4\pi^2 n_a f}{w_s\times 10^9}\mathbf{I}\left(h-\frac{x^2}{h}\right) = k\,\mathbf{I}\left(h-\frac{x^2}{h}\right) \text{ where } k = \frac{4\pi^2 n_a f}{w_s\times 10^9}$$

Thus the self-induced voltage of the first order is a maximum in the lamina at the bottom of the conductor, and sinks to zero at its top (Fig. 558*b*). It is, however, 90° out of phase in time with the original uniformly distributed current, but finally after re-distribution the resultant self-induced E.M.F. follows the same law, viz., that it is a maximum at the bottom, and since it ends by being of the nature of a back E M F., the larger part of the current is compelled to seek the upper laminæ of the conductor with a density increasing up to the top.

Returning to the first self-induced voltage, the average of all its values in the different laminæ at the same moment of their maxima is

$$k\,\mathbf{I}\frac{1}{h}\int_0^h\left(h-\frac{x^2}{h}\right)dx = k\,\mathbf{I}\times\frac{2}{3}h$$

and the height from the bottom at which this average value holds is that when $h - x^2/h = \frac{2}{3}h$, i.e. when $x = h/\sqrt{3} = 0{\cdot}577h$, or a little above the central layer. At this level no eddy-current of the first order flows: in the laminæ above it the current induced by the greater E.M.F. of the lower laminæ finds its return path. The resistance of a cm. length of any lamina is ρ/tdx, where t is its width or the thickness of conductor and ρ is the specific resistance of the copper in ohms per cm^3 at the temperature which it finally reaches. The maximum eddy-current of the first order in any lamina at height x is, therefore,

$$\frac{k\,\mathbf{I}\left(h-\frac{x^2}{h}-\frac{2}{3}h\right)}{\rho/t\,dx} \qquad \frac{kt}{\rho}\mathbf{I}\left(\frac{h}{3}-\frac{x^2}{h}\right)dx$$

The integrated currents from 0 to 0·577h and from 0·577h to h respectively are equal and their sum is zero. Hence, if the vertical of the lowest conductor in Fig. 558*b* is moved across the voltage diagram to the right until it cuts the parabola at the height 0·577h, a current diagram to some scale is obtained as in Fig. 558*c*.

In every lamina the maximum value of the uniformly distributed current is $\mathbf{I}dx/h$. But the eddy-current of the first order lags 90° in time-phase behind the uniformly distributed current. The resultant current of the two is, therefore, the square root of the sum of their squares. Correspondingly the rate of loss in watts in a cm. length of any lamina is equal to the product of the sum of the squares of the component currents with the resistance of the cm. length; deducting the normal watts, the extra loss in watts is that due to the eddy-current alone. In other words, the two component currents being at 90° in time-phase, each acts independently of the other, so far as the ohmic watts are concerned. The extra loss factor in a lamina, being the ratio of the extra watts to the watts for uniform distribution, is, therefore, over the whole length of a slot of the stator winding

$$\frac{\frac{k^2t^2}{\rho^2}\,l\left(\frac{h}{3}-\frac{x^2}{h}\right)^2}{l/h^2} = \frac{k^2t^2}{\rho^2}h^2\left(\frac{h}{3}-\frac{x^2}{h}\right)^2$$

and this integrated and averaged over the full height of the conductor so as to include all the laminæ is

$$\frac{k^2t^2}{\rho^2} h^2 \times \frac{1}{h}\int_0^h \left(\frac{h}{3} - \frac{x^2}{h}\right)^2 dx = \frac{k^2t^2}{\rho^2} h^4 \times \frac{4}{45}$$

Let $m^2 = \frac{kt}{\rho} = \frac{4\pi^2 n_a tf}{w_s\rho \times 10^9}$; then in the present case of a single solid conductor in the depth of the slot the extra loss factor is cm^4h^4, where the constant $c = 4/45$. The total loss factor for the slot portion is then $(1 + c\,m^4h^4) = K$, and the total loss factor for the whole of the winding is

$$\frac{Kl + l_e}{l_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (261)$$

where l_e is the length of the end-connexions of a half turn and l_1 is the length of a half turn.

(*b*) *n_l layers of solid conductors of the same phase per slot.* When there are n_l similar layers, and taking into consideration the pth layer, to the self-induced transverse flux there must also be added the mutually induced flux. from $(p - 1)$ layers below it. This has the uniform density $4\pi \mathrm{I} n_a(p - 1)/10\, w_s$, and the amount crossing the pth conductor *above* the level of a considered lamina at height x reckoned from the bottom of the conductor per cm. length of core is $4\pi n_a \mathrm{I}(p - 1)\,(h - x)/10\, w_s = 2\pi n_a \mathrm{I} \times 2(h - x)\,(p - 1)/10\, w_s$. Adding this to the self-induced flux within the conductor and above the lamina, the several steps become

$$2\pi f\, \Phi_l \times 10^{-8} = k\mathrm{I}\left\{2\,(h - x)\,(p - 1) + h - \frac{x^2}{h}\right\}$$

$$\text{Average value} = k\mathrm{I}\frac{1}{h}\int_0^h \left\{2\,(h - x)\,(p - 1) + h - \frac{x^2}{h}\right\} dx$$

$$= k\mathrm{I}\left\{h\,(p - 1) + \frac{2}{3}h\right\} = k\mathrm{I}\,h\left(p - \frac{1}{3}\right)$$

Maximum value of eddy-current of the first order in a lamina at height x

$$= \frac{kt}{\rho}\mathrm{I}\left\{(h - 2x)\,(p - 1) + \frac{h}{3} - \frac{x^2}{h}\right\} dx$$

Extra loss factor in a lamina

$$= \frac{k^2t^2}{\rho^2} h^2 \left\{(h - 2x)\,(p - 1) + \frac{h}{3} - \frac{x^2}{h}\right\}^2$$

and over the conductor as a whole

$$= \frac{k^2t^2}{\rho^2} \times h^4\left(\frac{4}{45} + \frac{p^2 - p}{3}\right) = m^4h^4\left(\frac{4}{45} + \frac{p^2 - p}{3}\right) \quad . \quad . \quad (262)$$

while finally the average extra loss factor for n_l layers is

$$m^4h^4\frac{1}{n_l}\left(\frac{4n_l}{45} + \frac{\lfloor n_l^2 - \lfloor n_l}{3}\right) = m^4h^4\left(\frac{4}{45} + \frac{n_l^2 - 1}{9}\right)$$

$$= m^4h^4\frac{1}{9}\left(n_l^2 - \frac{1}{5}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (263)$$

It will be observed that the extra loss factor is proportional to the 4th power of the height of the conductor. As the parabola of Fig. 558 (*c*) turns over to the horizontal, so does the eddy-current area and the loss increase. For example, the change from three deep conductors abreast in the slot in one layer to three layers each of one-third the height would roughly speaking reduce the extra loss in the latter case to 13½ per cent. of that in the former case.

The total loss factors, $K = 1 + cm^4h^4$, for the 1st, 2nd, 3rd . . . layer out of n_l layers, or its average value for the whole of the n_l layers are conveniently plotted as ordinates against the value of

mh as abscissæ. In Figs. 559 and 560 this has been done, use being made of the true values as derived by Mr. A. B. Field. Approximately when mh is $< 0{\cdot}7$, the loss ratio in the pth layer is

$$K = 1 + \frac{p^2 - p}{3} (mh)^4$$

and the average loss ratio in n_l layers is

$$K_a = 1 + \frac{n_l{}^2 - 1}{9} (mh)^4$$

In the expression for m, $\rho = 1{\cdot}589$ microhms at 0° C., and at temperature $t°$ C. $= 1{\cdot}589 \ (234{\cdot}5 + t°)/234{\cdot}5$. Hence

$$m = 2\pi\sqrt{\frac{n_a t}{w_s} \times \frac{f}{\rho \times 10^9}} = 2{\cdot}42\sqrt{\frac{n_a t}{w_s} \times \frac{f}{(234{\cdot}5 + t°)}} \quad . \quad (264)$$

It will be seen that $n_a t/w_s$ is the ratio of the total copper width across the slot to the width of the slot. In order to be able to decide upon a reasonable value for h by use of the curves, some assumption must be made as to the temperature $t°$ of the copper. At 45° C. $\rho = 1{\cdot}890 \times 10^{-6}$ ohms, so that then $2\pi/\sqrt{1890} = 0{\cdot}145$ and $m = 0{\cdot}145\sqrt{\frac{n_a t}{w_s} f}$ as assumed in the original papers of Mr. A. B. Field and Mr. M. B. Field. But in actual work the temperature of stator windings will usually be higher; if 100° C. is assumed, $\rho = 2{\cdot}26$ microhms, and $m = 0{\cdot}132\sqrt{\frac{n_a t}{w_s} f}$. If h'' is the height of conductor in inches, a convenient approximate expression is obtained[1] as

$$mh = 0{\cdot}132\sqrt{\frac{n_a t}{w_s} f} \times h = \frac{1}{3}\sqrt{\frac{n_a t}{w_s} f} \times h''$$

The total slot loss, $\frac{\mathrm{I}^2 \rho l}{ht}(1 + cm^4h^4)$, in relation to h is proportional to $\left(\frac{1}{h} + cm^4h^3\right)$. The loss for any individual layer or for the average of all layers is then a minimum when $h^4 = 1/3cm^4$. Inserting this value for h, it follows that the minimum loss is obtained when the total loss factor is 1·33, or $cm^4h^4 = 0{\cdot}33$. Thence the critical depth of a conductor for minimum loss

$$h_{crit} = \frac{1}{(3c)^{\frac{1}{4}}\, m}$$

with the approximate value $\frac{n_l{}^2 - \frac{1}{5}}{9}$ for c is in centimetres

$$= \sqrt{\frac{\sqrt{3}\, w_s \rho \times 10^9}{4\pi^2 n_a t f \sqrt{n_l{}^2 - 1)}}}$$

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[1] H. W. Taylor, *Journ. I.E.E.*, Vol. 58, *loc. cit.*

If $\rho \times 10^9 = 1890$, this becomes approximately[1]

$$h_{crit} = \sqrt{\frac{\sqrt{3}}{n_l} \times \frac{w_s}{n_a t} \times \frac{50}{f}} = \sqrt{\frac{\sqrt{3}\, w_s}{zt} \times \frac{50}{f}} \quad . \quad . \quad (265)$$

But this conclusion by no means holds when with the same section of copper maintained throughout the whole length of a half-turn account is taken of l_e; yet owing to the fact that in the end-connexion lengths there is a similar, though smaller, extra loss, due to their own inductance, the critical depth of conductor for the whole winding is brought more nearly into agreement with the critical depth as determined from the slot portion only.

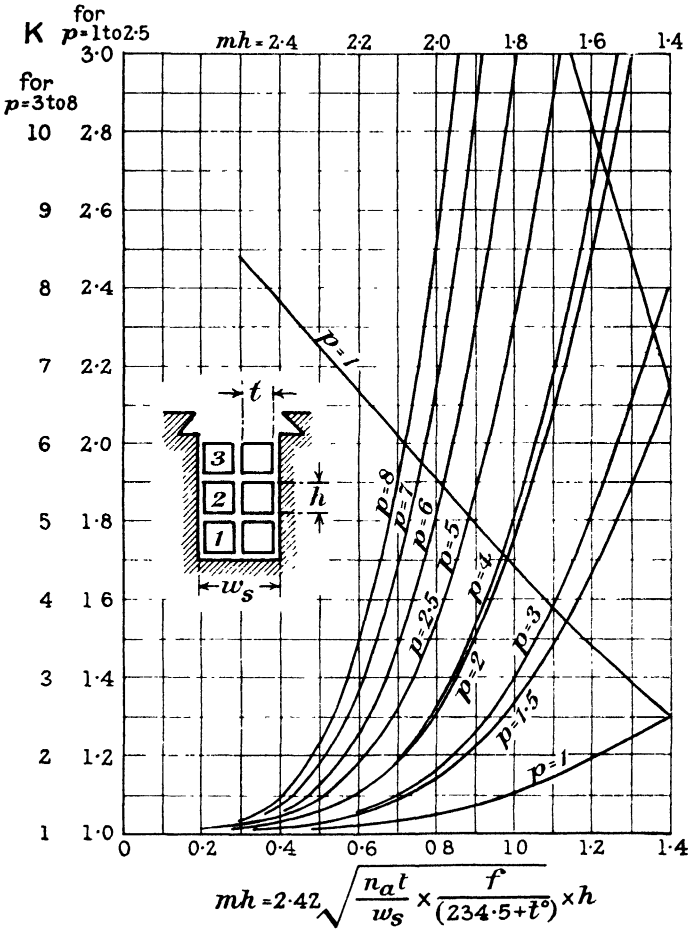

FIG 559—Ratio of total loss with eddy-current to loss with uniformly distributed current, for the *p*th layer.

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[1] As given by R. Rudenberg, *Archiv. fur Elektrotechnik*, Vol. 2, p. 207.

When either the critical depth or the average loss factor for all layers has been decided, it still remains necessary for the designer to calculate the loss separately for the uppermost conductor, so

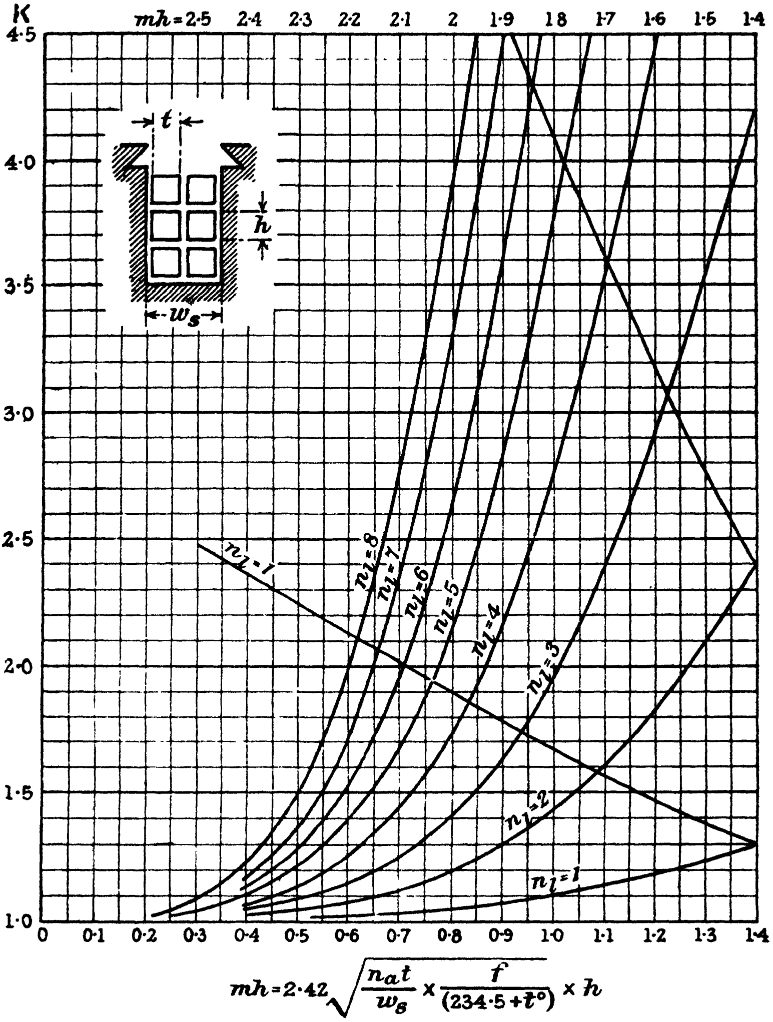

FIG. 560.—Ratio of average total loss with eddy-current to loss with uniformly distributed current for n_l layers.

as to ensure that the heating of this conductor does not exceed a permissible limit.

For high voltage machines $n_a t/w_s$ may be 0·6, so that at 50 frequency and 100° C. $m = 2{\cdot}42\sqrt{0{\cdot}09} = 0{\cdot}725$. Hence, with a depth of copper $h = \frac{1}{2}$ in. $= 1{\cdot}27$ cm., mh is less than 1, and such is usually now the case in modern machines.

(*c*) *Double-layer winding with coils of fractional pitch.* So far it has been assumed that the currents in all the conductors of a slot are of the same phase. But if in a two-layer winding the pitch of the coils is fractional, in some of the slots the $n_l/2$ component turns of the upper principal layer are acted upon by the M.M.F. and flux from the lower principal layer carrying a current which differs in phase. This leads to a reduction in the extra loss factor of the upper layer and in the average extra loss factor of both layers in the slots in question.[1]

(*d*) *Simple subdivision or lamination of the conductor.* If the conductor is simply divided into a number of thinner laminæ transverse to its height, and these are joined up solid immediately outside the core length, no benefit is gained from the lamination. But the insulated laminæ may be extended to cover practically the full length of a half turn or of a whole turn or of a coil having two or more turns before they are soldered together, and thereby placed in metallic connexion. In this case some portion at least of the total eddy-current in a lamina, say, at the bottom of a conductor is compelled to traverse the full laminated length before it can return *via* an upper lamina, again through the full laminated length.

Thus in a single layer bar-wound armature let the insulated laminæ extend over the length l_1 of a half-turn, being soldered together at the bar junctions. If the number of laminæ were infinite, each being infinitely thin, the *whole* of the eddy-current would be compelled to follow the longer path. The total self-induced E.M.F. of a lamination, proportional to the slot length l, would then be expended over a resistance which is proportional to l_1. The eddy current density would thereby be reduced from its previous value in the proportion l/l_1, and the extra loss factor would be reduced in proportion to the square of this ratio. To take account of this ideal case or of very thin laminations, m must be given the value

$$m = 2{\cdot}42\sqrt{\frac{n_a t}{w_s} \times \frac{f}{(234{\cdot}5 + t^\circ)} \times \frac{l}{l_1}} \quad . \quad . \quad . \quad (266)$$

so that in m^4 there will appear the term $(l/l_1)^2$ as required.

Still maintaining infinitely thin laminations, if the insulation between them is maintained continuously over the whole length of the turns of n_l layers per slot, the value of K for the whole coil of n_l layers is the same as for the pth layer in Fig. 559, when p has the value $(n_l + 1)/2$, and for use in this connexion are given the curves for the fractional values of $p = 1{\cdot}5$, and $2{\cdot}5$.

But when the above is applied to the actual case of a finite number of laminations of appreciable thickness, the total loss factor is under-estimated, and its true value lies between that for the infinitely laminated conductor and for the solid conductor. The explanation is that, so far as the lamination itself has thickness, it acts as a solid conductor of height h/n, where n is the number of laminations; there then remains a difference of potential available to circulate a uniformly distributed current between laminations, which has to traverse the longer path $2l_1$ as compared with the path $2l$ traversed by the local eddy-current of a lamination. It is only, therefore, the current circulating between laminations that is subject to the reducing factor $(l/l_1)^2$. The calculation must, therefore, proceed in two stages; assuming division of the conductor of copper height h and of total height h' into n equal laminæ, the layer number of the (p')th lamina in the pth conductor in terms of the thinner laminæ is $(p-1)n+p'$. Substituting this expression for the simple value p in eq. (262), the local extra loss factor within each lamina is

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[1] For its mathematical expression and its deduction, see H. W. Taylor *loc. cit.*

$$m^4\left(\frac{h}{n}\right)^4\left[\frac{4}{45}+\frac{\{(p-1)n+p'\}^2-\{(p-1)n+p'\}}{3}\right]$$

and the average value of this for the whole of the laminæ of the pth conductor is

$$m^4\left(\frac{h}{n}\right)^4\frac{1}{n}\left\{\frac{4n}{45}+\frac{1}{3}\left(n^3p^2-2n^3p+n^3-n^2p+n^2\right.\right.$$
$$\left.\left.+(p-1)2n\lfloor\underline{n}+\lfloor\underline{n^2}-\lfloor\underline{n}\right)\right\}$$

which reduces to

$$m^4\left(\frac{h}{n}\right)^4\left[n^2\left\{\frac{(p^2-p)}{3}+\frac{1}{9}\right\}-\frac{1}{45}\right]=m^4h^4\left(\frac{p^2-p}{3n^2}+\frac{1}{9n^2}-\frac{1}{45n^4}\right)$$
$$=m^4h^4\,A$$

The extra loss factor for the pth conductor, *if solid*, would be $m^4h^4\left(\frac{4}{45}+\frac{p^2-p}{3}\right)=m^4h^4\,B$. The difference $B-A$ reduced in the proportion $(l/l_1)^2$ but increased in the proportion $(h'/h)^2$ is then the additional extra loss factor due to the current circulating from lamina to lamina over the length l_1. This is increased by any flux passing through the insulation between laminæ and to take this into account the factor $(h'/h)^2$ is introduced. The total extra loss factor of the pth conductor is thus

$$m^4h^4\left\{A+(B-A)\left(\frac{lh'}{l_1h}\right)^2\right\}$$

At this point the reader is referred to Mr. H. W. Taylor's original paper where the effect of equal and unequal subdivisions of the conductor is fully investigated and quantitative formulæ deduced. Approximately for equal subdivisions, the laminated conductor can be replaced by an equivalent solid conductor of height $= h\left\{\frac{1}{n^2} + \frac{n^2 - 1}{n^2} \times \left(\frac{l}{l_1}\right)^2\right\}$. With a solid bar $n = 1$ and $n^2 - 1$ is zero; with an infinitely laminated bar, the height is reduced in the proportion $(l/l_1)^2$.

(*e*) *Laminated and twisted conductors.* Further elimination of the eddy-current effect is frequently necessary in large alternators and is obtained by twisting the composite conductor or crossing its insulated laminations at the centre of the slot (Fig. 561)[1], or, when the laminations are continuously insulated, at the centre of the end-connexions between slot and slot, at one end or at both ends. In either case the laminations reckoned as 1, 2, 3 . . . . in numerical sequence from the lowest layer in one part of the conductor appear in the reverse order in another part of the conductor when also reckoned from the bottom.

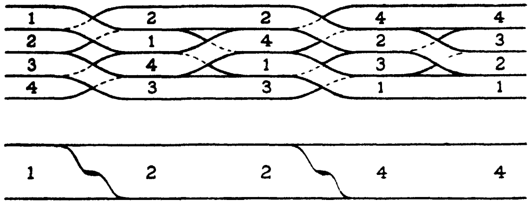

FIG. 561.—Laminated and twisted conductor viewed from the side and in plan from above. (British Pat. No 129,788, 1918)

Thus in a single-layer single-turn coil twisted at its centre between the slots in which its sides are embedded the extra loss from the circulating current is equal to that of a single layer ($p = 1$, Fig. 559) of a solid conductor of height $h/2$.

In a double-layer single-turn coil, laminated and twisted, the extra loss from the circulating current is for the coil as a whole the same as for a conductor of the same height with $p = 1$ in Fig. 559. Or more generally for a double-layer multiple-turn coil, similarly treated, the curve for $p = (n_l + 2)/4$ in Fig. 559 applies.

Many other arrangements have been devised, patented,[2] and used; thus the stator bars have been divided radially into a number of copper strips, each separately insulated and joined up with a twist to another strip in another position in the next slot, the change of position proceeding gradually until it becomes complete and a strip traced through a whole phase from star point to terminal has occupied all positions.[3]

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[1] See L. Fleischmann, *Journ I E.E.*, Vol 43, p. 1062, and Dr. R. Pohl, *Archiv. f. Elektrot.* Vol 14, p. 129

[2] For a list of British patents which includes many for different methods of laminating and twisting stator bars or coils, see J. Shepherd, *Journ. I.E E.*, Vol. 58, p. 141. For German patents, see F Punga and H. Roos, *E.u.M.*, Vol. 39, p. 485.

[3] See H. W. Taylor, and also R. E. Gilman and W. V. Lyon, as cited on p. 226.

But it must be added that a recurrent difficulty lies in the fact that in general the better the elimination of eddy-currents, the less the mechanical strength of the winding to withstand sudden short-circuits—the latter a subject which will be treated in § 8.

§ 7. Eddy-current loss in end-connexions.—The end-connexions in air have not as yet been considered as themselves the seat of an additional extra loss due to their own self-induced flux so far as it traverses the solid copper. So various are the arrangements of the end-connexions and so complicated the conditions that a satisfactory general treatment is hardly possible. But for the simple case of a number of bars forming a group running side by side in a double-layer barrel winding (Fig 562, *cp.* Fig. 553), Dr. R Pohl has suggested[1] an approximate method of dealing with the problem which is here outlined.

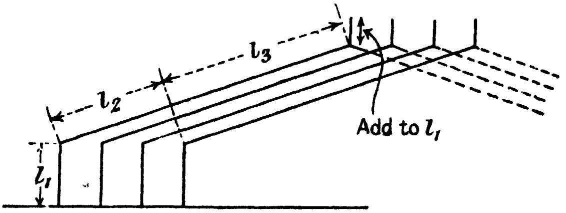

FIG 562.—Portion of double-layer end-connexions of barrel winding.

When the field encircling the conductors is split up into two component fields at right angles, the one parallel with the height h of the solid bars and the other a cross field traversing their thickness t, the latter is the more important source of eddy-current loss. The paths to complete the circuit of either component field are assumed to be semicircles, and for the actual cross field there may be assumed a constant length of path $b + \frac{3}{4} \times \frac{h}{2}\pi$ = say $b + 1{\cdot}2h$ where b is the width of any group under consideration (*cp.* Fig. 391) as in the calculation for coil reactance. The principle is then to substitute an open slot of this width (Fig. 563) and make use of the curve of Fig. 559 for $p = 1$ and for a copper height of $h/2$ cm., *i e.*

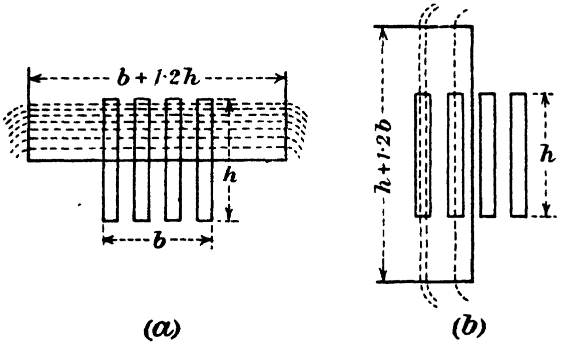

FIG 563 —Equivalent slots corresponding to (*a*) transverse flux, and (*b*) parallel flux

$$mh = 2{\cdot}42\sqrt{\frac{n_a t}{(b + 1{\cdot}2h)} \times \frac{f}{(234{\cdot}5 + t^\circ)}} \times \frac{h}{2} \qquad (267)$$

The straight projections from the slot over the length l_1 (Fig. 562) are so far apart that their fields are practically independent; n_a is, therefore, to be reckoned as 1 and b becomes $= t$.

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[1] *E.T.Z.*, Vol. 41, p. 997.

For the length l_2, averaged values of, say, $\frac{2}{3}n_a$ and $\frac{2}{3}b$ may be taken, and for the length l_3 we have the full values of n_a and b

$$\text{Thence } K = \frac{K_1 l_1 + K_2 l_2 + K_3 l_3}{l_1 + l_2 + l_3} \quad . \quad . \quad . \quad . \quad (268)$$

The parallel flux need only be considered in relation to the lengths l_2 and l_3. Over these lengths in place of $n_a t/(b + 1{\cdot}2h)$ we have by analogy $h/(h + 1{\cdot}2 \times \frac{2}{3}b)$ and $h/(h + 1{\cdot}2b)$, and t replaces h. The average extra loss factor is then obtained from the curve for $n_l = n_a/2$ in Fig. 560 after deducting 1. The simple addition of this to K_2 or K_3 over-estimates the loss, but on the other hand no account of the effect of other phases has been taken which tends to make the calculated loss too small.

§ 8. Ability of stator winding to withstand short-circuits.—The consideration of the dimensions of the stator conductor, and of its lamination or stranding, leads us to the important question of the rigid support of the end-connexions of the armature coils, so as to enable them to withstand the effects of an accidental short-circuit. When a large alternator is suddenly short-circuited under full excitation there arises for a few seconds a heavy current, which may exceed the permanent short-circuit current under steady conditions by 5 times or more, and the normal full-load current by 15 to 20 times. The reason for this is that for such momentary conditions the apparent impedance of the armature including the effect of armature reaction is reduced almost to that corresponding to the true reactance of the end-windings and from local flux in the slots, *i.e.* practically to but little more than x_a. The effect of a short-circuit under no-load or full-load excitation being, broadly speaking, a reduction of the main flux which initially exists linked with the field-turns and armature turns, when the short-circuit is sudden, the first initial decline of the doubly linked flux sets up in the field coils which are closed through the exciter a forward E.M.F. which greatly increases the exciting current; more than the initial excitation is thus temporarily provided, and thereby the value of the flux is maintained as nearly as possible at its original value. The two opposing sets of ampere-turns, on armature and field respectively, thus rise initially together *pari passu*, but to unequal maxima. In the latter must also be included the secondary ampere-turns due to eddy currents in the poles or any solid parts of the magnetic circuit; such currents are also caused by the diminishing flux and also assist in partially balancing the back ampere-turns of the armature.

After quickly reaching a maximum, both the armature and field current gradually settle down, the former to the current corresponding to permanent short-circuit under the original excitation, and the latter to its original value. The process is illustrated in

Fig. 564,[1] which shows the gradual decay of the rotor flux towards its value for sustained short-circuit, and the sudden rise of the exciting current accompanying the sudden rise of the armature current. The *maximum* momentary value of the short-circuit current will, therefore, be proportionate to a value of the flux in the rotor not far short of the normal value, and so proportional to the full E.M.F. of a phase acting on the transient impedance of the phase. If the exciting current rises, say, to 3½ times its steady value, the sudden short-circuit current will be something more than 3½ times its permanent value, *i.e.*, it will exceed 3½ I_{psc} owing to the fact that the eddy-currents in solid pole-pieces assist the field current in balancing its rise.[2]

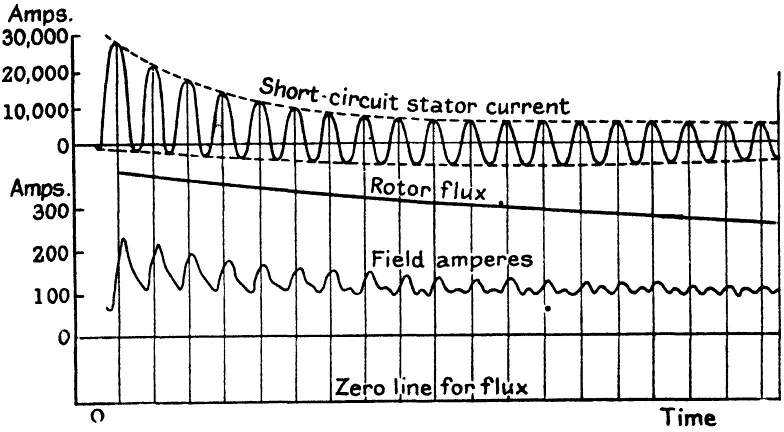

FIG. 564.—Change of exciting current and of rotor flux after sudden short-circuit of stator winding when voltage was near zero.

Between neighbouring bars of a phase and between the coils of different phases, a sudden short-circuit can thus cause an enormous mechanical force proportional to the square of the current, the effect of which can only be likened to that from the explosion of a charge of dynamite. Thus consequent upon a high-power surge on the distribution system of the Manhattan Railway, New York, a 5000-kW. 3-phase alternator was short-circuited and the armature bars 2 ins. deep by ⅛ in. thick, three in each slot, were bent edgewise for over six inches, showing that the force bending them amounted to at least 3000 to 4000 lb.[1] As a general rule with slow-speed machines, the span of the coils across the pole-pitch is short, and their projection is not great, so that the end-connexions have sufficient mechanical strength to prevent displacement or distortion. But the stresses increase roughly as the square of the pole-pitch, and when this reaches such figures as 100 inches, as it may in the case of turbo-alternators owing to their high speed, the end-connexions require strong clamps to support them from the stator frame.[2] The same necessity arises also in the case of large high-speed single-phase alternators for low frequencies, such as 25 cycles per second for heavy railway work, where there is a likelihood of sudden short-circuits; it has been calculated[3] that in such cases the mechanical stress on one end of a coil may be from 2 to 10 tons, and this comes as a sudden shock.[4]

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[1] Based on Fig. 5 of Mr. A. B. Field's paper on "Operating Characteristics of Large Turbo-Generators," *Trans. Amer. I.E.E.*, Vol. 31, Part II, p. 1645, to which the reader should refer.

[2] *Cp.* F. Punga, "The Sudden Short-circuiting of 3-phase Alternators," abstracted in *Electr. Review*, Vol. 59, p. 1015. To reduce the momentary short-circuit current in very large turbo-alternators, external reactance coils are now sometimes added. See "The Transient Reactions of Alternators," by W. A. Durgin and R. H. Whitehead, *Trans. Amer. I.E.E.*, Vol. 31, Part II, p. 1657.

§ 9. The influence of the moment in the period at which short-circuit occurs.—The maximum value reached by the armature short-circuit current is, however, also dependent on the moment in the period at which the short-circuit occurs.

To illustrate this, let it be assumed that the balancing action of the additional field and eddy-current ampere-turns is so nearly perfect that at least for a few cycles the main flux can be regarded as held constant. The case of an alternator short-circuited when running on no-load is then very similar to that of an electrical circuit of resistance r_a and inductance l_a, which is suddenly closed by a switch at some moment in a period, while an alternating E.M.F. of constant virtual value E_o is impressed on it. As is well known, the current in this case would be composed of the permanent or settled alternating current, $i_1 = \mathbf{I}_1 \sin(2\pi ft - \phi)$ where $\mathbf{I}_1 = \mathbf{E}_o/(\sqrt{r_a^2 + 4\pi^2 f^2 l_a^2})$, and $\phi = \tan^{-1} 2\pi f\, l_a/r_a$, and a unidirectional transient current, following an exponential law, of value

$$-i_2 = -\varepsilon^{-a_a(t-t_1)}\,\mathbf{I}_1 \sin(2\pi f t_1 - \phi)$$

where a_a is the attenuation factor $= r_a/l_a$, t_1 is the fixed time when the switch is closed reckoned in seconds from a moment when the E.M.F. is zero and becoming positive, and t is the increasing time reckoned from the same zero as that from which t_1 is reckoned. Or

$$i = \frac{\mathbf{E}_o}{z_a}\left\{\sin(2\pi ft - \phi) - \varepsilon^{-a_a(t-t_1)} \sin(2\pi f t_1 - \phi)\right\}$$

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[1] *Trans. Amer. I.E.E.*, Vol. 24, p. 301.

[2] *Cp.* Figs. 656, 657, 664, and Prof. Miles Walker, " Short-circuiting of Large Electric Generators," *Journ. I.E.E.*, Vol. 45, p. 295.

[3] W. L. Waters, " Single-phase Generators," *Trans. Amer. I.E.E.*, Vol. 27, Part II, p 1075.

[4] For other mechanical effects, see " The Mechanical Effect of Electrical Short-circuits," by S. H. Weaver, *Gen. Electr. Review*, Vol. 18, p. 1066.

t_1 being a constant, the second term gives a component current which is always on the same side of the zero line, but dying away as t increases. At the moment of closing the switch, $t = t_1$, and $t - t_1 = 0$, so that the two terms at that moment always cancel out, and the current starts from zero, as must necessarily be the case.

The two extreme possibilities as regards the moment when the switch is closed are then as follows—

(i) that it is closed when the E.M.F. is passing through zero.
(ii) „ „ „ „ a maximum.

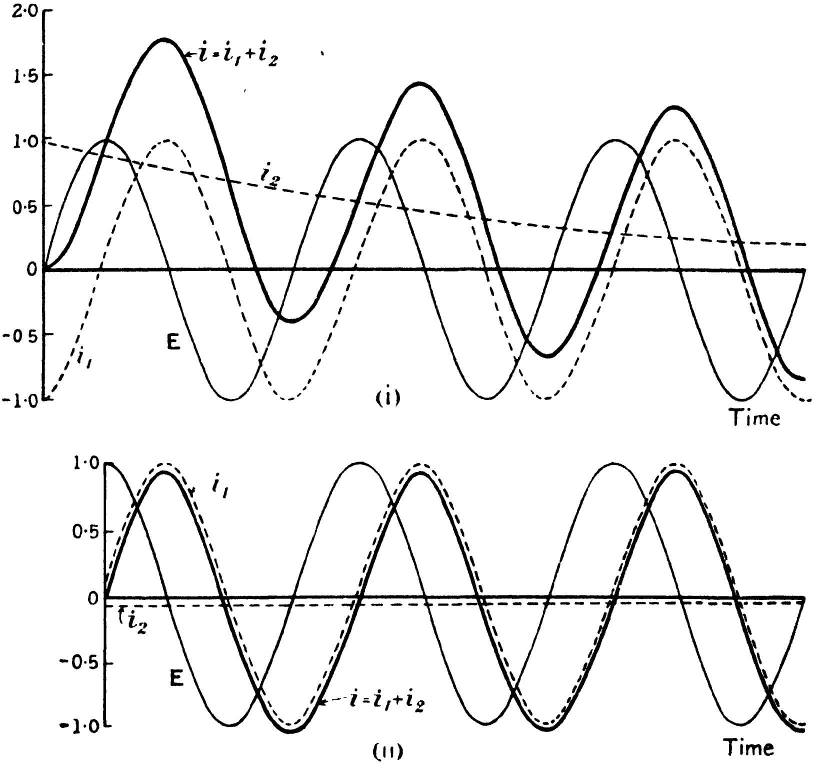

FIG. 565.—Current-curves with constant impressed E.M.F and switch closed (i) at zero E.M.F., (ii) at maximum E M F.

Now in the former case both E.M.F. and current begin to grow up together, and the initial rate of rise of the current is at its lowest. In the latter case, the current begins to rise at its maximum rate under the action of the maximum E.M.F. But the absolute maximum value that the current reaches in the two cases depends on the time during which the impressed E.M.F. *acts in the same direction.* In the former case this is one-half, in the latter case only a quarter, of a period. In consequence when $\mathscr{L}_a$ is large and ϕ great, the absolute maximum in the former case may be nearly twice as great as in the latter case (Fig. 565).

The exponential term, which is required owing to the phenomenon being transient, when $\mathscr{L}_a$ is large as compared with r_a displaces the initial waves to one or other side of the zero line. As the extreme limit when $\mathscr{L}_a$ is very large as compared with r_a and a_a is correspondingly small, ϕ = nearly 90°, and the preceding equation approaches the value

$$i = \frac{\mathbf{E}_0}{z_a}(-\cos 2\pi ft + \cos 2\pi ft_1)$$

When $t_1 = 0$ or $T_p/2$ (case i), this becomes $i = \pm\frac{\mathbf{E}_0}{z_a}(-\cos 2\pi ft + 1)$, which at its maximum when $t = T_p/2$ or T_p gives

$$\mathbf{I} = \pm 2\frac{\mathbf{E}_0}{z_a}$$

When $t_1 = T_p/4$, or $3T_p/4$ (case ii), $i = \pm\frac{\mathbf{E}_0}{z_a}(-\cos 2\pi ft + 0)$, which at its maximum when $t = T_p/2$ or T_p gives

$$\mathbf{I} = \pm\frac{\mathbf{E}_0}{z_a}$$

On the above imaginary supposition which forms the ideal limit when in fact there would be no dissipation of energy over the negligible resistance, the absolute maximum of case (i) would be *exactly* double that of case (ii) ; in case (i) the current-wave is symmetrical about an exponentially-decaying curve which finally becomes tangent to the time axis, in case (ii) it is symmetrical about the time axis, and in practice the maximum of the former will be about 1·8 times that of the latter. When the closing of the switch or the short-circuit occurs at any intermediate instant, the absolute maximum is not reached with the current in its first initial direction ; before the maximum current is reached, there occurs a smaller undulation (Fig. 567) lasting for a portion only of a half period, by which the passage is made from the one case to the other.

The greatest possible short-circuit current will always result with a position of the poles when the flux-linkages with the short-circuited coils are at their maximum, in agreement with the fact that the E.M.F. at the moment of short-circuit is then passing through zero. Thus let a short-circuit occur at no-load between two terminals of a star-connected three-phase alternator ; then Fig. 566 (i) shows that the two phases which are concerned (the third phase is not shown) together link the maximum possible amount of flux. Both stator and rotor coils and eddy currents oppose any change in the flux-linkages. But in the first half cycle after

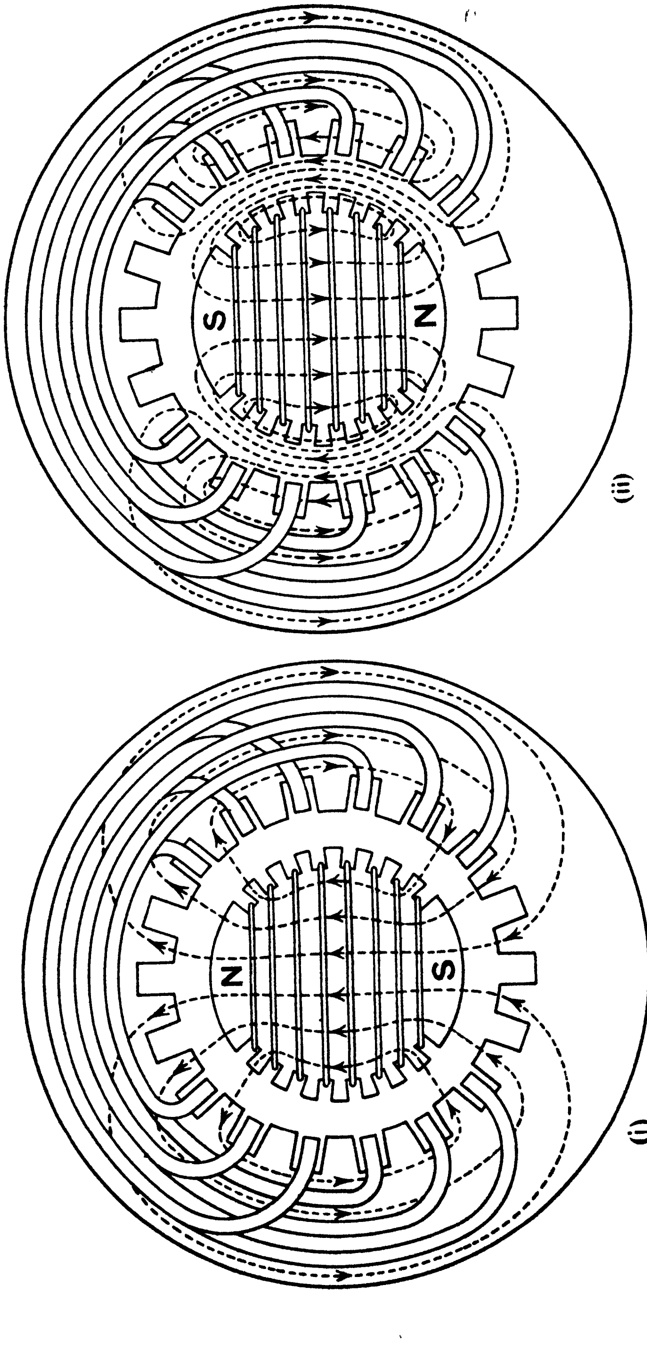

FIG. 566.—Two phases of a 3-phase machine (i) at moment of short-circuiting when their E.M.F. is zero and flux-linkages are greatest, (ii) half a cycle later.

short-circuit the rotor moves forward a pole-pitch (Fig. 566 (ii)). The two sets of ampere-turns on stator and rotor respectively are then in exact opposition across the air-gaps under the pole-centres, but unite to drive flux transversely through the slots and air-gaps on the interpolar line (*cp.* also Fig. 568*a*). The end-connexion fluxes round the over-hang of the stator coils, and the leakage fluxes of the rotor increase, and thus, accompanied by a considerable redistribution of the amounts at different parts, the number of flux linkages severally of the stator and rotor is maintained almost unaltered for the first cycle. Fig. 566 (ii) then shows the position of the greatest possible instantaneous current for two phases short-circuited. There is almost complete separation of the fluxes of stator and rotor, and large increases of current are necessary to drive enough flux through their respective leakage paths to keep the linkages nearly constant.

§ 10. The gradual decline of the main flux.—But the impressed E.M.F. is not a constant alternating quantity, as provisionally assumed in the last section ; the main rotor and stator flux is falling from its initial value to the much smaller value which holds under permanent short-circuit. The additional field current and eddy currents, though maintained by their inductance, are being killed by the resistance with which they meet, and their decay must be taken into account by means of a second expression involving an exponential term $\varepsilon^{-a_f(t-t_1)}$, where a_f is the attenuation factor of the field $= R_f/\mathcal{L}_{fdr}$, where R_f and $\mathcal{L}_{fdr}$ are the joint resistance and inductance of the field coils and secondary eddy circuits, if assumed constant. Let the full E.M.F. of the machine E_o be mentally divided at the moment of short-circuit into a part E_{psc} which will finally persist and drive the permanent short-circuit current through the stator phase against its resistance and inductance, and a remainder $E_o - E_{psc}$ which gradually vanishes as the eddy currents die away and the main flux declines. Then the current at any instant after short-circuit becomes

$$i = \frac{(\mathbf{E}_o - \mathbf{E}_{psc})\varepsilon^{-a_f(t-t_1)} + \mathbf{E}_{psc}}{z_a} \sin(2\pi ft - \phi)$$

$$- \frac{\mathbf{E}_o\,\varepsilon^{a_a(t-t_1)}}{z_a} \sin(2\pi ft_1 - \phi)$$

$$= \quad i_1 \text{ (alternating)} \quad + i_2 \text{ (direct)}$$

The only difficulty lies in the indeterminateness of the resistance and inductance of the eddy-current paths, especially in solid poles, which change as the poles move under the armature ampere-turns.[1]

§ 11. The two components of the short-circuit current.—The instantaneous value of the short-circuit current is thus at any moment completely represented by an alternating component i_1 superposed on a direct-current component i_2 which decays after an exponential law. The amplitude of the first-named also decays as an exponential function in dependence on the decay of the flux in the field-magnet.[1] If the curves formed by joining respectively the positive and the negative crests of the current wave of a phase as recorded on an oscillogram are produced backwards to cut the vertical axis of zero time (Fig. 567), let the intercept between them on that axis[2] be symbolized as $2\hat{I}_0$. This quantity remains practically the same, whatever the moment in the period at which short-circuit occurs, although the proportions of it above and below the zero line depend on the moment of short-circuit; it can thus be obtained from an oscillogram taken at any point on the voltage wave.

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[1] See Prof. Miles Walker, *Journ. I.E.E.*, Vol. 45, p 295, and *Specification and Design of Dynamo-Electric Machinery*, pp. 123–132.

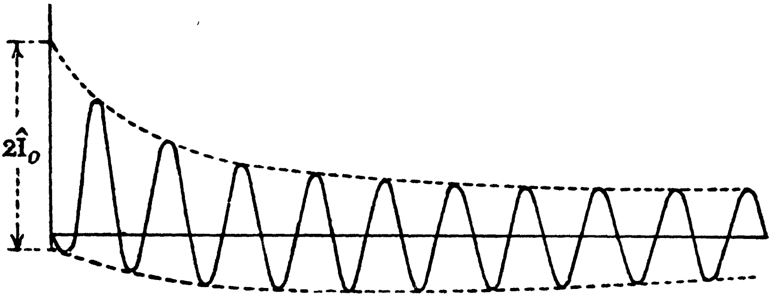

FIG 567 —Short-circuit occurring shortly before moment of zero E.M.F. in one phase

Even assuming short-circuit to occur at zero voltage so that the curve of total current starts entirely above the horizontal axis of time, its instantaneous crest never actually quite reaches $2\hat{I}_0$, and the alternating component on either side of a line *a b c* drawn centrally through the alternations (Fig. 568) is never quite $\hat{I}_0$, but they closely approach these hypothetical values in the first cycle. The alternating component is then at the outset equal to the direct-current component, and the curve of the positive crests in accordance with the preceding equation is given by the expression

$$\hat{I}_{ssc} = (\hat{I}_o - \hat{I}_{psc})\,\varepsilon^{a_f t} + \hat{I}_{psc} \quad + \hat{I}_o\varepsilon^{-a_d t}$$

$$= \underbrace{\hat{I}_t \qquad + \hat{I}_{psc}}_{\text{alternating}} \quad + \underset{\text{direct}}{i_2}$$

$$= (\text{amplitude of } i_1) + i_2$$

---

[1] See N. S. Diamant, "Calculation of Sudden Short-circuit Phenomena of Alternators," *Trans. Amer. I.E.E.*, Vol. 34, Part II, p. 2237.

[2] The treatment here suggested follows in the main that of R. E. Doherty and O. E. Shirley, "Reactance of Synchronous Machines and its Applications," *Trans. Amer. I.E.E.*, Vol. 37, Part II, p. 1248 ff., to which the reader is especially referred for fuller discussion.

where $\hat{I}_{psc}$ is the crest value of the permanent short-circuit current
a_f is the attenuation factor of the field $= R_f/\mathcal{L}_{fdr}$
a_a ,, ,, ,, armature $= r_a/\mathcal{L}_{ao}$
and $t =$ seconds after short-circuit.

For salient-pole machines, a_a is much larger than a_f, so that i_2 dies out much faster than $\hat{I}_t$, say, after 5 to 7 cycles. As soon as i_2 has practically died out at *d c e* the wave has become symmetrical about the horizontal axis, but its amplitude does not reach its final value $\hat{I}_{psc}$ until the difference between $\hat{I}_o$ and $\hat{I}_{psc}$ or $\hat{I}_t$ has died out, and after perhaps two or more seconds the alternating component has reached its final steady value.

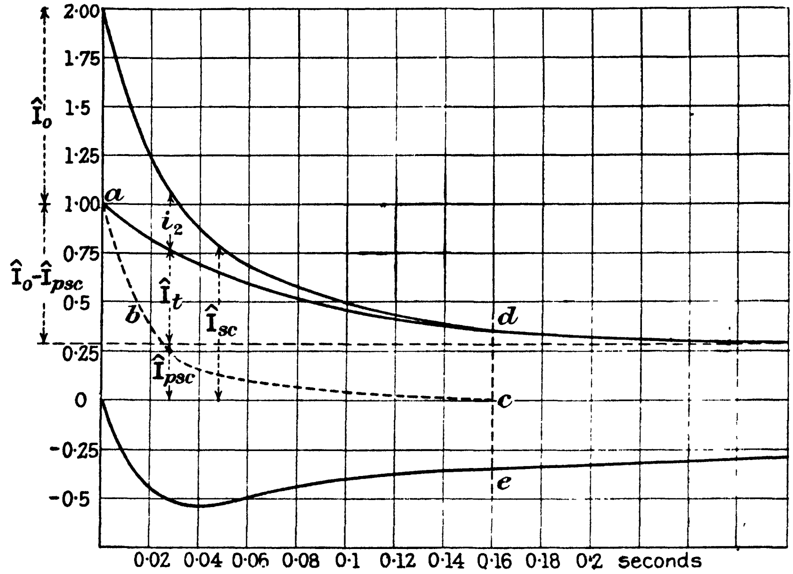

FIG. 568.—Outline of crests of armature current when short-circuit occurs at zero E.M.F.

Fig. 568[1] shows diagrammatically the curves of positive and negative crest values for the data that $\hat{I}_o - \hat{I}_{psc} = \sqrt{2}\ (0{\cdot}707 - 0{\cdot}2)$, that $a_f = 2{\cdot}6$ and $a_a = 8{\cdot}7$. The curve of negative crests is simply obtained by plotting $\hat{I}_1 = \hat{I}_t + \hat{I}_{psc}$ negatively from the dotted curve *a b c* which repeats i_2, used as a zero line.

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[1] After a similar diagram in the paper of Messrs. Doherty and Shirley, who give convenient expressions for plotting such curves, viz.,

$$2{\cdot}3\, a_f t = \log_{10} \frac{\hat{I}_o - \hat{I}_{psc}}{\hat{I}_t}$$

$$2{\cdot}3\, a_a t = \log_{10} \frac{\hat{I}_o}{i_2}$$

§ 12. The maximum instantaneous armature and field currents on short-circuit.—The crest value $\mathbf{I}_{ssc}$ of the current in an armature when suddenly short-circuited always reaches its maximum within the first half cycle, or after a lesser transitional portion of a wave within the following half cycle, and thence gradually decreases down to its permanent value on steady short-circuit at a rate depending partly on the resistances involved. But for the purpose of calculating the *maximum* crest value $\mathbf{I}_{ssc\ max}$ which the initial rush of current can ever reach under the most unfavourable conditions, an approximate estimate erring on the high side is obtained by considering the first half cycle only, upon the assumptions of zero resistance in both armature and field circuits, in which case there can be no variation in the flux-linkages of either circuit.[1]

For simplicity let a single-phase two-pole generator, having T_f turns on the rotor and T_a turns on the stator distributed over some arc of its inner periphery, be considered in the first place, and let it be suddenly short-circuited when running on no-load.

The total flux is always divisible into the lines that pass through both stator and rotor, and leakage lines; in the case of the rotor and stator systems considered separately, but with the iron parts having their final resultant reluctivities, if the unit amounts for 1 ampere flowing are indicated by small ϕ's, and the amounts for some particular value of amperes by capital Φ's, we have—

	Due to 1 ampere		Due to normal current I_f in field.	Due to current i_{ssc} in armature.
	in rotor.	in stator		
Total flux . . .	ϕ_{tf}	ϕ_{ta}	$\Phi_{tf} = I_f\phi_{tf}$	$\Phi_{ta} = i_{ssc}\phi_{ta}$
Mutual flux linked with both rotor and stator	ϕ_{Mf}	ϕ_{Ma}	$\Phi_{Mf} = I_f\phi_{Mf}$	$\Phi_{Ma} = i_{ssc}\phi_{Ma}$
Leakage flux . .	ϕ_{lf}	ϕ_{la}	$\Phi_{lf} = I_f\phi_{lf}$	$\Phi_{la} = i_{ssc}\phi_{la}$

The total self-inductance of either rotor or stator is similarly divisible. Let the co-efficients less than unity which take into account the fact that all the flux in each case is not linked with all the turns be represented by the letter K.

When one ampere flows in the *rotor* winding,

$K_1 T_f \phi_{tf} \times 10^{-8} = \mathscr{L}_{tf}$, total self-inductance of field in henrys,

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[1] See W. Rogowski, *Archiv f. Elektrot,* Vol. 11, p. 147; R. E. Doherty, "A Simplified Method of Analyzing Short-circuit Problems," *Trans. Amer, I.E.E.*, Vol. 42, p. 841; and C. M. Laffoon, "Short Circuits of Alternating-current Generators," *Journ. Amer. I.E.E.*, Vol. 43, p. 736.

$K_1' T_f \phi_{Mf} \times 10^{-8} = \mathscr{L}_{Mf}$, self-inductance of field so far as it is due to lines which also pass through the stator,

$K_1'' T_f \phi_{lf} \times 10^{-8} = \mathscr{L}_{lf}$, self-inductance of field due to its leakage lines.

When one ampere flows in the *stator* winding,

$K_2 T_a \phi_{ta} \times 10^{-8} = \mathscr{L}_{ta}$, total self-inductance of armature phase,

$K_2' T_a \phi_{Ma} \times 10^{-8} = \mathscr{L}_{Ma}$, self-inductance of armature due to its own lines which also pass through the rotor,

$K_2'' T_a \phi_{la} \times 10^{-8} = \mathscr{L}_{la}$, self-inductance due to its own leakage lines.

When 1 ampere flows in the field-circuit, the linkages between ϕ_{Mf} and T_a must be equal to the linkages between ϕ_{Ma} and T_f when 1 ampere flows in the armature (*assuming saturation to be negligible* and the magnetic circuit to have a constant reluctivity) *i.e.*,

$$K_{Ma} T_a \phi_{Mf} \times 10^{-8} = K_{Mf} T_f \phi_{Ma} \times 10^{-8} = \mathscr{M},$$

the mutual inductance between armature and rotor.

If Φ_{tf} is the total normal field-flux of the rotor, the flux-linkages to be maintained constant are at any instant

in the rotor, $K_1 T_f \Phi_{tf} = N_1 = I_f \mathscr{L}_{tf} \times 10^8$.

The flux-linkages to be maintained constant in the stator phase depend upon the instant of short-circuit. When the axes of field-winding and armature phase coincide, the mutual flux that enters the stator $\Phi_{Mf} = \Phi_{tf} - \Phi_{lf}$ is all linked with stator turns if the mean pitch of the coil is 180 electrical degrees, and the flux-linkages are $K_{Ma} T_a \Phi_{Mf} = N_2 = I_f \cdot \mathscr{M} \times 10^8$. But reckoning time from this instant, the stator flux-linkages are $N_2 \cos \omega t$, and if t_1 is the time at which short-circuit begins, the flux-linkages to be maintained constant from this moment are

in the stator, $K_{Ma} T_a \Phi_{Mf} \cos \omega t_1 = N_2 \cos \omega t_1 = I_f \cdot \mathscr{M} \cos \omega t_1 \times 10^8$

At any moment after short-circuit, the flux-linkages in the rotor will be those due to its own lines and to the mutual flux from the stator winding. And similarly the flux-linkages in the stator will be those due to its own self-induced lines and to the mutual flux from the rotor linked with it.

If, therefore, i_{ssc} and I_f' are the instantaneous values of the armature and field-current, I_f being the original value of the latter, the required conditions are fulfilled when

$$I_f' \mathscr{L}_{tf} + i_{ssc} \cdot \mathscr{M} \cos \omega t = I_f \mathscr{L}_{tf}$$

and

$$i_{ssc} \mathscr{L}_{ta} + I_f' \cdot \mathscr{M} \cos \omega t = I_f \cdot \mathscr{M} \cos \omega t_1,$$

and t is given values increasing from t_1 upwards.

By substitution, it is found that

$$i_{ssc} = I_f \frac{\mathcal{M}}{\mathcal{L}_{ta}} \times \frac{(\cos \omega t_1 - \cos \omega t)}{\left(1 - \frac{\mathcal{M}^2}{\mathcal{L}_{ta}\,\mathcal{L}_{tf}} \cos^2 \omega t\right)} \quad . \quad . \quad . \qquad \text{(i)}$$

and

$$I_f' = I_f \frac{1 - \frac{\mathcal{M}^2}{\mathcal{L}_{ta}\,\mathcal{L}_{tf}} \cos \omega t_1 \cos \omega t}{1 - \frac{\mathcal{M}^2}{\mathcal{L}_{ta}\,\mathcal{L}_{tf}} \cos^2 \omega t} \quad . \quad . \quad . \quad . \qquad \text{(ii)}$$

To give the largest value of i_{ssc} for any one value of $t > t_1$, it is evident that $\cos \omega t_1$ must be 1, *i.e.*, $t_1 = 0$, or short-circuit must occur when the axes of field-winding and stator coil coincide and the stator coil embraces the maximum possible flux, and that the maximum crest[1] value $\mathbf{I}_{ssc\ max}$ which it can then reach occurs when $\omega t = 180°$, *i.e.*, after half a cycle. At this moment

$$\mathbf{I}_{ssc\ max} = \frac{2\, I_f \cdot \mathcal{M}}{\mathcal{L}_{ta} - \frac{\mathcal{M}^2}{\mathcal{L}_{tf}}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(iii)}$$

and under the given condition the instantaneous value i_{ssc} remains throughout of the same sign, *i.e.*, entirely above or below the zero line. When $\omega t_1 = 90°$, and no flux is embraced by the stator coils, $\mathbf{I}_{ssc}$ is only half the above value, and i_{ssc} rises and falls on either side of the zero line by equal amounts. For a value of ωt_1 between 90° and 180°, a smaller portion of a wave precedes the rise to $\mathbf{I}_{ssc}$.

Next, if I_f' is expressed as $I_f + i_f$, where i_f is the additional secondary current induced in the field winding,

$$i_f = I_f \frac{\frac{\mathcal{M}^2}{\mathcal{L}_{ta}\,\mathcal{L}_{tf}} \cos \omega t\,(\cos \omega t - \cos \omega t_1)}{1 - \frac{\mathcal{M}^2}{\mathcal{L}_{ta}\,\mathcal{L}_{tf}} \cos^2 \omega t} \quad . \quad . \quad . \qquad \text{(iv)}$$

But

$$I_f \frac{(\cos \omega t - \cos \omega t_1)}{1 - \frac{\mathcal{M}^2}{\mathcal{L}_{ta}\,\mathcal{L}_{tf}} \cos^2 \omega t} = - i_{ssc} \frac{\mathcal{L}_{ta}}{\mathcal{M}}$$

Therefore

$$i_f = - i_{ssc} \frac{\mathcal{M}}{\mathcal{L}_{tf}} \cos \omega t \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(v)}$$

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[1] For the crest value of the true alternating current when ωt appears in the originating equation, we return to $\mathbf{I}_{ssc}$ in place of $\hat{I}_{ssc}$.

It is thus an alternating component *added* to I_f, and its maximum value after half a cycle when $t_1 = 0$, $\omega t = 180°$, and $i_{ssc} = \mathbf{I}_{ssc\ max}$, is

$$\hat{\imath}_{f\ max} = \mathbf{I}_{ssc\ max} \frac{\mathscr{M}}{\mathscr{L}_{tf}} \quad . \quad . \quad . \quad . \quad . \qquad \text{(vi)}$$

The physical reason for these relations at the end of the first half-cycle when $\omega t_1 = 0$ may be traced if the effects on the stator and rotor of the superposed fluxes are considered separately in successive steps. The three component systems are—

(*a*) the original field flux due to the normal exciting current, I_f,

(*b*) the added field flux due to the maximum secondary current $\hat{\imath}_{f\ max}$,

(*c*) the armature flux due to the maximum armature current, $\mathbf{I}_{ssc\ max}$, in the stator phase.

(1) Starting then with N_2 flux-linkages in the stator from the original field, the turning of the rotor through the first 90 electrical degrees would cause them to disappear, since there would then be no lines of the original field linked with the armature turns. Further rotation through another 90°, making half a period in all, would re-insert in the stator the same flux in the opposite direction, and give the same linkages reversed in sense. So far, therefore, the change in the armature flux-linkages is $N_2 - (-N_2) = 2N_2$, and in order to compensate for this in its two stages, it would appear that the armature current must rise first to such a value as to give total flux-linkages numerically equal to N_2 and then to such a value as to give *double* this amount so as to counterbalance the new reversed linkages from the original field. But this would not allow for the effect of the secondary current $\hat{\imath}_{f\ max}$ that has arisen in the field circuit.

(2) The exciting-coil linkages are not affected merely by rotation of the rotor with its own original field, but whatever the value to which $\mathbf{I}_{ssc\ max}$ actually rises, some portion of the new armature flux, viz. Φ_{Ma}, opposed to the reversed original field-flux and therefore in its original direction, crosses the rotor and is linked with its turns, giving $\mathbf{I}_{ssc\ max} \cdot \mathscr{M} \times 10^8$ flux-linkages. To compensate for this change, the additional field-current must rise to such a value that its total linkages, $K_1\, T_f\, \Phi_{ti} = K_1\, T_f\, \hat{\imath}_{f\ max}\, \phi_{tf} = \hat{\imath}_{f\ max}\, \mathscr{L}_{tf} \times 10^8$ are numerically equal to the mutual flux linkages from the armature, $\mathbf{I}_{ssc\ max} \cdot \mathscr{M} \times 10^8$. The original field-linkages N_1 are thus left unaltered. Hence

$$\hat{\imath}_{f\ max}\, \mathscr{L}_{tf} \times 10^8 = \mathbf{I}_{ssc\ max} \cdot \mathscr{M} \times 10^8$$

$$\hat{\imath}_{f\ max} = \mathbf{I}_{ssc\ max} \frac{\mathscr{M}}{\mathscr{L}_{tf}} \quad . \quad . \quad . \quad . \quad . \qquad \text{(vi)}$$

(3) Of the total flux-linkages in the stator arising from $\mathbf{I}_{ssc\ max}$, part are due to Φ_{Ma} and part to Φ_{la}, *i.e.*,

$$K_2 T_a \mathbf{I}_{ssc} \phi_{ta} = K_2' T_a \mathbf{I}_{ssc} \phi_{Ma} + K_2'' T_a \mathbf{I}_{ssc} \phi_{la}$$

$$\mathbf{I}_{ssc} \mathscr{L}_{ta} \times 10^8 = \mathbf{I}_{ssc} \mathscr{L}_{Ma} \times 10^8 + \mathbf{I}_{ssc} \mathscr{L}_{la} \times 10^8$$

Now some of the flux Φ_{ti}, viz. the amount Φ_{Mi}, will pass through the stator coils and by its linkages therewith amounting to $\hat{i}_f \mathscr{M} \times 10^8$ will neutralize a part of $\mathbf{I}_{ssc} \mathscr{L}_{Ma} \times 10^8$. Out of this latter term, therefore, it is only the difference $(\mathbf{I}_{ssc} \mathscr{L}_{Ma} - \hat{i}_f \mathscr{M}) \times 10^8$, or the proportion $\mathbf{I}_{ssc} \mathscr{L}_{Ma} \left(1 - \dfrac{\mathscr{M}^2}{\mathscr{L}_{tf} \mathscr{L}_{Ma}}\right)$ which is available to counterbalance the change described under (1). Hence

$$\mathbf{I}_{ssc} \mathscr{L}_{ta} \times 10^8 - \hat{i}_f \mathscr{M} \times 10^8 = 2 N_2 = 2 I_f \mathscr{M} \times 10^8 \quad . \quad \text{(vii)}$$

Or, expressed differently, the total self-induced linkages from the armature current must be equal to the linkages of mutual flux from $2 I_f$ and $\hat{i}_{f\ max}$, since

$$\mathbf{I}_{ssc\ max} \mathscr{L}_{ta} = (2 I_f + \hat{i}_{f\ max}) \mathscr{M}$$

Inserting the value previously found for $\hat{i}_{f\ max}$

$$\mathbf{I}_{ssc\ max} \left(\mathscr{L}_{ta} - \frac{\mathscr{M}^2}{\mathscr{L}_{tf}}\right) = 2 I_f \mathscr{M}$$

$$\mathbf{I}_{ssc\ max} = \frac{2 I_f \mathscr{M}}{\mathscr{L}_{ta} - \dfrac{\mathscr{M}^2}{\mathscr{L}_{tf}}} \quad . \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

If the armature phase has resistance, the armature linkages instead of remaining constant must at every instant vary by an amount $d N_2/dt = i_{ssc} r_a \times 10^8$. If $k =$ the ratio of the average i_{ssc} to $\mathbf{I}_{ssc\ max}$ during the first half-cycle or time $T_p/2$, the change of armature flux linkages required by resistance will be $\int_0^{T_p/2} N_2\, dt = k \dfrac{T_p}{2} \mathbf{I}_{ssc\ max} r_a \times 10^8$. Consequently the total change under (1) becomes $2 N_2 - k \dfrac{T_p}{2} \mathbf{I}_{ssc\ max} r_a \times 10^8$ on the right-hand side of the equation (vii), and

$$\mathbf{I}_{ssc\ max} = \frac{2 I_f \mathscr{M}}{\mathscr{L}_{ta} - \dfrac{\mathscr{M}^2}{\mathscr{L}_{tf}} + k \dfrac{T_p}{2} r_a}$$

so that both armature and field currents are reduced.

When the field circuit also has resistance, the original field flux-linkages must alter at the end of the first half cycle by the amount $\int_0^{T_p/2} i_f r_f \times 10^8$, and the proportion of these linkages due to flux entering the stator is $\mathcal{M}/\mathcal{L}_{tf}$. Consequently the total change under (1) becomes

$$2 I_f \cdot \mathcal{M} \times 10^8 - k \frac{T_p}{2} \mathbf{I}_{ssc\ max} r_a \times 10^8 - \frac{\mathcal{M}}{\mathcal{L}_{tf}} \int_0^{T_p/2} i_f r_f\, dt \times 10^8$$

and finally

$$\mathbf{I}_{ssc\ max} = \frac{2 I_f \cdot \mathcal{M} - \dfrac{\mathcal{M}}{\mathcal{L}_{tf}} \displaystyle\int_0^{T_p/2} i_f r_f\, dt}{\mathcal{L}_{ta} - \dfrac{\mathcal{M}^2}{\mathcal{L}_{tf}} + k \dfrac{T_p}{2} r_a}$$

For a constant value of the armature conductors per inch of periphery, the mutual interlinkages with the flux of the field or $\mathcal{M}$ is practically proportional to the sine of half the angle χ covered by the width of the belt (*cp.* eq. 32*a*). Hence since $\mathcal{L}_{tf}$ is a constant, $\mathcal{M}^2/\mathcal{L}_{tf}$ is proportional to $\sin^2(\chi/2)$. Calculating $\mathcal{L}_{ta}$ for different widths of belt and deducting $\mathcal{M}^2/\mathcal{L}_{tf}$, the difference is found to increase when the width of belt is increased So far only a single phase has been considered.[1] In a 3-phase alternator with star-connected armature a short-circuit on no-load between one terminal and the neutral line to the star point is the short-circuit of a single phase of width 33 per cent. of the pole-pitch, and a short-circuit between two terminals is virtually the short-circuit of a single phase of width 66 per cent. of the pole-pitch (Fig. 566) The latter, therefore, gives a much smaller value of $\mathbf{I}_{ssc\ max}$.

While the above treatment illustrates the physical causes at work, the numerical calculation of $\mathbf{I}_{ssc\ max}$ by it requires a calculation of the values of the several *K's* with the iron in its final state of saturation. It would, therefore, be as simple to deal only with the final resultant flux distribution after the half cycle; this resolves itself into two separate systems of leakage lines linked respectively with armature and field, and in each case maintaining the initial numbers of flux linkages constant (as indicated diagrammatically in Fig. 568*a*). Allowance must be made in the armature for the linkages self-induced in the end-connexions of the winding, and in the field-poles for the flank leakages, which do not appear in the diagram. Something less, therefore, than twice the normal flux must be forced across the leakage paths by the M.M.F.'s of field and armature *AT*, while at the end of one cycle the flux is again re-established in its normal path. Across the face of the pole-shoe there is no transverse difference of magnetic potential to pass field flux transversely through the air-gap. But in the interpolar gap the determination of the exact line of demarcation between armature and field flux remains a matter of trial and error, since in the first instance some relative values of the armature and field AT must be assumed, and this may need afterwards to be corrected. Simpler approximate methods have therefore been devised.[1]

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[1] For the effect of the addition to a single-phase generator of a damper winding, see C. M. Laffoon, *Journ. Amer. I.E.E.*, Vol. 43, p. 740. See also F. Kade, *Archiv. f. Elektrot*, Vol. 12, p. 345.

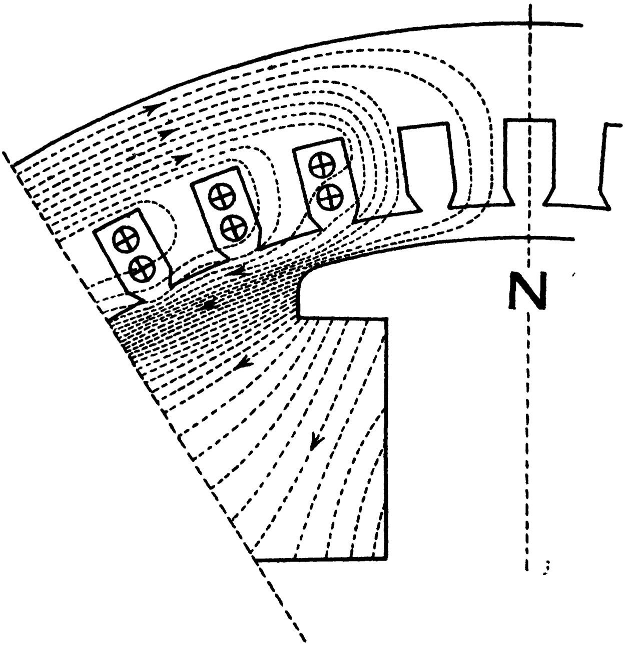

FIG. 568*a*.—Diagram of resultant fluxes half a cycle after short-circuit at a moment when phases A and B in series had maximum linkages.

A 3-phase short-circuit at no-load, *i.e.*, all three terminals short-circuited, differs only in that there is not the possibility of all the short-circuited coils enclosing zero flux which attaches to a single-phase short-circuit, terminal to neutral, or a short-circuit of two phases only from terminal to terminal. When the coil-sides of phase C stand over the poles and are not effectively linked with any flux, phases A and B in series are linked as a whole with as large an amount of flux as at any time (Fig. 566 (i)). When phase C embraces the pole and has maximum linkage, phases A and B are each linked with some flux, and by intermediate stages the cases would pass into each other. The result is that the direct armature reaction corresponding to the stationary poles formed on the stator must be counterbalanced by the M.M.F. of an alternating current in the field which rises as a maximum to the same value as the direct M.M.F. of the armature. If with all 3 phases simultaneously short-circuited at zero voltage in phase A, the factor of asymmetry is 1·8 in A, it will be about 1·35 in phases B and C.

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[1] For one such method, see Dr. A. Fraenckel, *Electr.*, Vol. 92, p. 225.

§ 13a. The magnetic effect of the alternate-current component.—Each of the two components of the short-circuit current exerts its appropriate magnetizing effect, not indeed independently, but sufficiently so to enable them to be distinguished for purposes of analysis. A three-phase short-circuit on a salient-pole machine with laminated poles running on no-load is here assumed.[1]

Considering the first cycle upon the assumption that within it there is no decrement of the current, the alternating component of virtual value I_0 in each phase calls forth the demagnetizing ampere-turns of the three phases amounting in combination to $k_D \frac{I_0 Z}{4pq}$ per pole, moving synchronously with the poles and acting as a scalar quantity in opposition to the field ampere-turns per pole. As they initially grow up, the flux linked with field and armature must thence fall from the normal Φ_{an} to some smaller new value Φ_a', this fall in turn calling forth a forward E.M.F. in the field turns, a rise in the field exciting current, and an increase in the field ampere-turns to balance partially $k_D \frac{I_0 Z}{4pq}$. If the coupling were perfect and there were no field-magnet leakage, the direct-current component $I_{f_{dr}}$ of the immediate rise of the field current I_{f_r} would with the magnet turns per pole T_f yield additional ampere-turns sufficient to balance $k_D \frac{I_0 Z}{4pq}$, save for the fall of flux necessarily required to yield the voltage to overcome the increased ohmic drop over the field resistance. But actually there is leakage and the function of the additional field ampere-turns is to maintain not Φ_a, but Φ_m, the original value of the flux in the magnet pole, as nearly constant as the physical conditions permit. As the flux Φ_a is decreased and the field ampere-turns rise, the proportion of the new leakage flux to the new Φ_a' increases. Assuming the action to be perfect,

$$\Phi_m = \Phi_{an} + \phi_l = \Phi_a' + \phi_l + \phi_{lr}$$

where ϕ_l is the normal leakage for the interpolar excitation AT_{P_0} given by AT_{f_0}, and ϕ_{lr} is the *rise* of the leakage flux due to the addition of $AT_{f_{dr}}$ to AT_{f_0}. Thence $\Phi_{an} - \Phi_a' = \phi_{lr}$, and it will be seen that the initial action of the two opposing sets of ampere-turns on field and armature respectively is a deflection of some portion of the original flux into the leakage paths of the field, so that they no longer cross the air-gap. The further question as to what proportion y of the armature ampere-turns must be balanced by the direct-current rise of the field ampere-turns if the flux Φ_m were actually maintained constant is also answered by the same equation. If $\mathfrak{S}_g =$ the permeance of a single air-gap, teeth and core, and $2\mathfrak{S}_l =$ the permeance of the field leakage paths in relation to AT_P,

$$\Phi_{an} = 1{\cdot}257\, AT_P \times \mathfrak{S}_g$$

$$\Phi_a' = 1{\cdot}257\left(AT_P + k_D \frac{I_0 Z}{4pq}\, y - k_D \frac{I_0 Z}{4pq}\right)\mathfrak{S}_g$$

$$\text{and } \phi_{lr} = 1{\cdot}257\, k_D \frac{I_0 Z}{4pq}\, y \times 2\mathfrak{S}_l$$

Thence, since $\Phi_{an} - \Phi_a' = \phi_{lr}$,

$$y = \frac{\mathfrak{S}_g}{\mathfrak{S}_g + 2\mathfrak{S}_l} = \frac{\Phi_{an}}{\Phi_{an} + \phi_l} = \frac{\Phi_{an}}{\Phi_m} = \frac{1}{\nu_1}$$

where ν_1 is the ratio of the total to the useful flux at no-load.

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[1] As in Messrs. Doherty and Shirley's paper, *loc. cit.*

§ 13b. The magnetic effect of the direct-current component.—So far, only the magnetic effect of the maximum alternating component of the armature current calling for a direct current in the field to partially balance it has been considered. But this is not all. There remains the *direct-current component* of the armature current, which in turn imposes an alternating current on the field. The direct-current component, also of value $\hat{I}_0$ at its maximum, produces (by Chap. XXVII, § 29) a set of ampere-turns of value $k_D \frac{I_0 Z}{4pq}$ per pole-pitch. These ampere-turns, as due to a direct current, are *stationary* with respect to the poles, and their M.M.F. by itself would cause stationary polar regions of alternate polarities round the armature. As a moving field-pole passes under them, its flux is first decreased by an opposing M.M.F. and then increased by an assisting M.M.F. Their action is, therefore, virtually to superpose on a lessened normal value an alternating flux through the pole which causes an alternating E.M.F. and an alternating current of machine frequency imposed on the direct-current rise of the field current. The magnitude of the field alternating current is limited by the resistance of the circuit and by the field leakage reactance. The true inductance of the field circuit per pole is reduced by the mutual inductance of the armature on the field to the much smaller value corresponding to the leakage, and the apparent inductance per pole is only $\mathcal{L}_f/2p = T_f^2 \times 12{\cdot}57 \times 2\mathcal{P}_l' \times 10^{-9}$. The reactance from $2p$ poles is then

$$x_f = 2\pi f \mathcal{L}_f = 2\pi f T_f^2 \times 1{\cdot}257 \times 2\mathcal{P}_l' \times 10^{-8} \times 2p$$

Here $\mathcal{P}_l'$ is an equivalent permeance which when acted upon by $2AT_f$ gives a number of linkages of flux with all the T_f turns of a pole equal to the number due to the actual leakage flux linked with a varying number of turns.

Assuming a sinusoidal current in the first cycle of amplitude I_{fdr} = half of the total maximum rise I_{fr}, the virtual value of the alternating field component initially is $I_{fdr}/\sqrt{2}$, and its virtual reactance voltage is $I_{fdr}\,x_f/\sqrt{2}$. To give this voltage, the alternating flux required per pole is

$$\phi = \sqrt{2}\,\frac{I_{fdr}\,x_f}{2p\,\sqrt{2}} \times \frac{1}{2\pi f T_f \times 10^{-8}} = I_{fdr}\,T_f \times 1{\cdot}257 \times 2\mathcal{P}_l'$$

This implies that the flux in a pole-pitch sinks during the first initial rise of the field current by twice this amount, and varies between a maximum Φ_m and a minimum $\Phi_m - 2\phi$ about a mean of $\Phi_m - \phi$. As a steady effect therefore over the first cycle, the ampere-turns called for by ϕ are saved out of AT_{fo}, and are available as an additional item to assist in balancing the initial combined effect of the armature ampere-turns. Their amount is $0{\cdot}8\,\phi\,\mathcal{R}_{do} = I_{fdr}\,T_f\,(1{\cdot}257 \times 2\mathcal{P}_l')\,\frac{\mathcal{R}_{do}}{1{\cdot}257}$, and multiplying numerator and denominator by AT_{fo}, this is

$$= I_{fdr}\,T_f\,\frac{1{\cdot}257\,AT_{fo} \times 2\mathcal{P}_l'}{1{\cdot}257\,AT_{fo}/\mathcal{R}_{do}} = I_{fdr}\,T_f\,\frac{\phi_l'}{\Phi_{an}}$$

Thus finally the initial direct-current rise of the field ampere-turns, so far from equalling the initial scalar value of the armature ampere-turns, is checked in two ways by the field leakage. Instead of

$$I_{fdr}\,T_f = \frac{k_D I_0 Z}{4pq} \times \frac{\Phi_{an}}{\Phi_{an} + \phi_l} - I_{fdr}\,T_f\,\frac{\phi_l'}{\Phi_{an}}$$

$$I_{fdr}\,T_f\left(1 + \frac{\phi_l'}{\Phi_{an}}\right) = \frac{k_D I_0 Z}{4pq\,\nu_1}$$

it need only be

$$I_{fdr}\,T_f = \frac{k_D I_0 Z}{4pq\nu_1\nu_2}$$

where the ratio $\nu_2 = (\Phi_{an} + \phi_l')/\Phi_{an}$ is a calculated quantity somewhat smaller than ν_1, but of the same order.

§ 14. The value of the linked leakage permeance, $\mathfrak{S}_l'$.—In § 23 of this Chap. an equivalent permeance is calculated, which when acted upon by the magnetomotive force $1{\cdot}257 \times 2AT_{\mathrm{P}}$ yields the effective leakage flux that for the determination of the field excitation may be regarded as passing through the whole length of the magnet-core. For the calculation of the field leakage inductance, the same items, $\mathfrak{S}_1$, $\mathfrak{S}_2$, and $\mathfrak{S}_3$, are immediately available, since the leakage fluxes corresponding thereto are, in fact, linked with all the field ampere-turns per pole, T_f. But when use is made of AT_f for the magnetomotive force, the reluctance of a share of the section of the magnet pole, proportionate to the leakage, must be added in series with the reluctance of the leakage paths. Or to allow for the ampere-turns expended over the magnet-core and yoke, it may approximately be assumed that AT_{P} equals, say, $0{\cdot}8\,AT_f$; when one C G.S. absolute unit of current flows through T_f, the M.M.F. acting on the leakage path will then be $12{\cdot}57 \times 0{\cdot}8\,T_f = 10\,T_f$, although, in fact, the proportion will increase as the flux declines and saturation decreases. That is, the inductance is not strictly a constant.

For the remaining items, the equivalent permeances $\mathfrak{S}_4'$, $\mathfrak{S}_5'$, $\mathfrak{S}_6'$ as calculated in § 23, giving the same number of linkages with the whole of the T_f, turns as the actual number, will be used. The effective permeance for linkages is then, say,

$$\mathfrak{S}_l' = 0{\cdot}8\,(\mathfrak{S}_1 + \mathfrak{S}_2 + \mathfrak{S}_3 + \mathfrak{S}_4' + \mathfrak{S}_5' + \mathfrak{S}_6')$$

if the assumption of $AT_{\mathrm{P}} = 0{\cdot}8\,AT_f$ is retained. The field inductance of all poles is

$$\mathcal{L}_f = 2p\,T_f^2 \times 1{\cdot}257 \times 2\,\mathfrak{S}_l' \times 10^{-8} \text{ henrys,}$$

and $x_f = 2\pi f\,\mathcal{L}_f$. Only when ϕ_l', the equivalent leakage from the viewpoint of inductance, is roughly identified with ϕ_l as calculated for the determination of field ampere-turns, is it the case that

$$\frac{I_{fdr}\,T_f}{k_{\mathrm{D}}\dfrac{I_0 Z}{4pq}} = \left(\frac{\Phi_{an}}{\Phi_{an} + \phi_l}\right)^2 = \frac{1}{\nu^2}$$

§ 15. The prediction of I_0 from the equivalent reactance.—The higher the leakage permeance, and the reactance voltage from the field, the greater is the fall of the flux through the armature coils that is required to overcome it, and the less the remainder left to cause the initial rise of the armature current upon short-circuit. If field and armature ampere-turns at the outset rose equally and there were no leakage, Φ_{an} would remain constant, and the armature current per phase would initially rise until the self-induced E.M.F. from the end-connexion leakage lines called for the whole of Φ_{an} to overcome it. Neglecting the resistance of a phase, the maximum short-circuit current in the first cycle would then only be limited by the armature reactance, x_a. But the difference between the magnitude of the two sets of opposing ampere-turns corresponding to ϕ may be looked upon either as reducing Φ_{an} to $\Phi_{an} - \phi$, or as causing a reactance voltage of value x_f' as referred to an armature phase, which absorbs part of the normal no-load flux. That is,

$$I_0(x_a + x_f') = m(\Phi_a' + \phi) = m\Phi_{an} = E_0$$

Thus if the resistances are negligible, the maximum possible value of I_0 for a three-phase short-circuit is $I_0 = E_0/x_0$ and the maximum possible current is $2\sqrt{2}\,I_0$, where E_0 is the voltage on open circuit per phase and $x_0 = x_a + x_f'$ It is limited by the armature leakage reactance, and the field leakage reactance as referred to the armature, *i.e.* by the " equivalent reactance " of the two in combination, just as in a transformer. The value of x_f' is

$$\begin{aligned} x_f' &= \frac{m\phi}{I_0} = \frac{m}{I_0}(I_{fdr}\,T_f \times 1{\cdot}257 \times 2\mathfrak{S}_l') \\ &= \frac{m}{I_0}\frac{k_{\mathrm{D}}\,I_0 Z}{4pq\,\nu_1\nu_2} \times 1{\cdot}257 \times 2\mathfrak{S}_l' \\ &= m\,\frac{k_{\mathrm{D}} Z}{4pq} \times \frac{1{\cdot}257 \times 2\mathfrak{S}_l'}{\nu_1\nu_2} \ (cp.\ \text{expression (248)}) \end{aligned}$$

This may also be checked as follows: The ratio of transformation from field reactance voltage to armature voltage per phase is $\dfrac{m/f}{\dfrac{2\pi}{\sqrt{2}}2p\,T_f \times 10^{-8}}$, $= \dfrac{k_{s1}'\,k_{s1}''\dfrac{Z}{2N_{ph}q}}{2p\,T_f} = k_{s1}'\,k_{s1}''\dfrac{Z}{3 \times 4pq\,T_f}$. The ratio of transformation from field current to armature current per phase is not simply $\dfrac{T_f}{\dfrac{k_D Z}{4pq}}$, but from § 13*b* (end) in relation to equivalent magnetizing ampere-turns is $\dfrac{T_f}{\dfrac{k_D Z}{4pq\,\nu_1\nu_2}}$

Thence

$$x_f' = \frac{E_2'}{I_2'} = \frac{E_2}{\dfrac{I_{fdr}}{\sqrt{2}}} \times \frac{\dfrac{k_{s1}'\,k_{s1}''\,Z}{3 \times 4pq\,T_f}}{\dfrac{\sqrt{2}\,T_f \times 4pq\,\nu_1\nu_2}{k_D Z}} = x_f \times \frac{k_{s1}'\,k_{s1}''\,k_D Z^2}{3\sqrt{2}\,T_f^2\,(4pq)^2\,\nu_1\nu_2}$$

$$= 2\pi f\,\frac{k_{s1}'\,k_{s1}''}{2\sqrt{2}}\,\frac{Z}{3q} \times \frac{k_D Z}{4pq} \times \frac{1{\cdot}257 \times 2\mathfrak{S}_l'}{\nu_1\nu_2} \times 10^{-8}$$

Since $m = 2\pi f\dfrac{k_{s1}'\,k_{s1}''}{2\sqrt{2}} \times \dfrac{Z}{3q} \times 10^{-8}$, this reduces to the same expression as above. Or since $k_{s1}'\,k_{s1}''/\sqrt{2}$ may be identified with k_D, and identifying ν_1 and ν_2,

$$x_f' = x_f\,\frac{1}{3}\left(\frac{k_D Z}{4pq\,\nu}\right)^2 \times \frac{1}{T_f^2} = 2\pi f\left(\frac{k_D Z}{2q\nu}\right)^2 \times \frac{1{\cdot}257 \times 2\mathfrak{S}_l'}{3 \times 2p} \times 10^{-8}$$

In salient-pole machines the field leakage and the inductance therefrom are large, ν being of the order 1·1. The second term x_f' assisting to limit the maximum short-circuit current then becomes of importance, especially with laminated poles as above assumed without damper winding.

Low-resistance damper windings and any eddy currents in solid steel poles have the effect of lessening the change of flux at short-circuit; the effect of the field inductance or x_f' is thus reduced, and the short-circuit current is larger. With cylindrical solid rotors of turbo-alternators admitting of large eddy-currents, a gradual approach is made more nearly to the first term x_a only. Since the maximum possible current has chief interest as causing the very great force of attraction between neighbouring conductors of the same phase or of repulsion between the groups of end-connexions of different phases when carrying current in opposite directions, it may thus finally be said that even when the current wave is initially entirely offset in case (i), the maximum instantaneous value of the short-circuit current will be something less than *twice* the normal maximum voltage induced per phase divided by the reactance $2\pi f\,\mathfrak{L}_a$ per phase, *i.e.*

$$2\sqrt{2}I_o = \frac{2\pi f\,k_{s1}'\,k_{s1}''\,\Phi_1\,Z \times 10^{-8}}{2\sqrt{2}\,N_{ph}\,q \times 2\pi f\left(\dfrac{Z}{N_{ph}\,q}\right)^2\dfrac{1}{2p}\,\Sigma l\lambda \times 10^{-9}} \times 2\sqrt{2}$$

$$= \frac{k_{s1}'\,k_{s1}''\,2p\,\Phi_1}{\dfrac{Z}{N_{ph}\,q}\,\Sigma l\lambda \times 10^{-1}} = \frac{k_{s1}'\,k_{s1}''\,\Phi_1}{\dfrac{Z}{2p\,.\,N_{ph}\,q}\,\Sigma l\lambda \times 10^{-1}}$$

where $\Sigma l\lambda = l\lambda_s + L\lambda_c + (l_1 - l)\lambda_p + l_s l_e$ as in equation (253), and

$$\frac{Z}{2p\,.\,N_{ph}\cdot q}\,\Sigma l\lambda \times 10^{-1}$$

is the leakage flux for one ampere flowing through the turns of a phase corresponding to one pole.

A short-circuit between two terminals only of a three-phase machine, *i.e.* of two phases, yields practically the same maximum current as a three-phase short-circuit, but a single-phase short-circuit between a terminal and the neutral point yields a current, perhaps 50 per cent. higher, the conditions being more complicated owing to the pulsation of the armature M.M.F. Under load the current is higher than on no-load owing to the greater flux involved.

§ 16. The attenuation factors, a_f and a_a.—(*a*) The field inductance $\mathcal{L}_{fdr}$ which enters into the attenuation factor a_f determining the rate of decay of the field flux and of the alternating component of the armature current depends on the linkages of the *transient* flux $\Phi_t = \Phi_{an} - \Phi_{psc}$ with the field turns; it is equal to the number of linkages per ampere rise of the field current, *i.e.*

$$\mathcal{L}_{fdr} = \frac{2p\,\Phi_t\,T_f}{I_{fdr}} \times 10^{-8}\,\text{henrys}$$

The permanent value Φ_{psc} of the flux under permanent short circuit is

$$\Phi_{psc} = I_{psc}\,x_a/m$$

Thence $$\mathcal{L}_{fdr} = \frac{2p\,T_f \times 10^{-8}}{I_{fdr}}\,(\Phi_{an} - \Phi_{psc})$$

Since $$I_{fdr} = \frac{k_D I_o Z}{4pq\,v_1 v_2 T_f} \text{ and } I_o = m\,\Phi_{an}/x_o$$

$$\mathcal{L}_{fdr} = \frac{2p\,T_f^2\,x_o \times 10^{-8}}{\dfrac{m\,k_D Z}{4pq\,v_1 v_2}}\left(1 - \frac{\Phi_{psc}}{\Phi_{an}}\right)$$

From equation (251*a*) under permanent conditions,

$$AT_{fo} = \frac{k_D\,I_{psc}\,Z}{4pq} + 0{\cdot}8\,\frac{I_{psc}\,x_a}{m}\,\mathfrak{R}_{do}$$

$$= \frac{m\,\Phi_{psc}}{x_a}\,\frac{k_D Z}{4pq} + 0{\cdot}8\,\Phi_{psc}\,\mathfrak{R}_{do}$$

whence $$\Phi_{psc} = \frac{AT_{fo}}{\dfrac{m\,k_D Z}{4pq\,x_a} + 0{\cdot}8\,\mathfrak{R}_{do}}$$

Φ_{an}, the normal flux $= \dfrac{AT_f}{0{\cdot}8\,\mathfrak{R}_{do}}$; therefore

$$\Phi_t = AT_{fo}\left(\frac{1}{0{\cdot}8\,\mathfrak{R}_{do}} - \frac{4pq\,x_a}{m\,k_D Z + 0{\cdot}8\,\mathfrak{R}_{do}\,4pq\,x_a}\right)$$

$$= \frac{1{\cdot}257\,AT_{fo}}{\mathfrak{R}_{do}} \times \frac{m\,k_D Z}{m\,k_D Z + 0{\cdot}8\,\mathfrak{R}_{do}\,4pq\,x_a}$$

$$= \frac{\Phi_{an}}{\left(1 + \dfrac{0{\cdot}8\,x_a\,\mathfrak{R}_{do}}{m} \times \dfrac{4pq}{k_D Z}\right)}$$

Thence $$\mathcal{L}_{fdr} = \frac{2p\,T_f\,\Phi_{an} \times 10^{-8}}{I_{fdr}\left(1 + \dfrac{0{\cdot}8\,x_a\,\mathfrak{R}_{do}}{m} \times \dfrac{4pq}{k_D Z}\right)}$$

Substituting $\dfrac{k_{\mathrm{D}}\,I_0 Z}{4pq\,v_1 v_2 T_f}$ for I_{fdr}, and since $I_0 = m\,\Phi_{an}/x_0$

$$\mathscr{L}_{fdr} = \frac{2p \times T_f^2 \times x_0 \times 10^{-8}}{\dfrac{k_{\mathrm{D}}\,mZ}{4pq\,v_1 v_2} + 0{\cdot}8\,\dfrac{x_a\,\mathfrak{R}_{do}}{v_1 v_2}} = \frac{2p\,T_f^2\,x_0 \times 10^{-8}}{0{\cdot}8\,\dfrac{x_f'}{2\,\mathfrak{P}_l'} + 0{\cdot}8\,\dfrac{x_a\,\mathfrak{R}_{do}}{v_1 v_2}}$$

$$= 2p\,T_f^2 \times 1{\cdot}257 \times 2\,\mathfrak{P}_l' \times 10^{-8} \times \frac{x_a + x_f'}{x_a\,\dfrac{\mathfrak{R}_{do} \times 2\,\mathfrak{P}_l'}{v_1 v_2} + x_f'}$$

$$= \mathscr{L}_f\,\frac{x_a + x_f'}{x_a\,\dfrac{2\,\mathfrak{P}_l'}{v_1 v_2\,\mathfrak{P}_{do}} + x_f'} = \mathscr{L}_f\,\frac{x_a + x_f'}{x_a\,\dfrac{v_2 - 1}{v_1 v_2} + x_f'} \text{ or larger than } \mathscr{L}_f$$

Oscillograms have shown that the field attenuation factor during the first few cycles is larger than subsequently[1]—an effect which may in part be due to saturation of part of the paths of leakage flux, but which is also traceable to eddy currents in the solid rotor rim reducing the effective field inductance.[2]

(*b*) The inductance in the armature attenuation factor a_a which determines the rate of decay of the direct-current component of the armature current rise depends on the linkages of the flux maintained through the leakage paths of both armature and field. It is thus

$$\mathscr{L}_{ao} = \frac{x_0}{2\pi f} = \frac{x_a + x_f'}{2\pi f}$$

and is greater than the armature leakage inductance $\mathscr{L}_a$.

§ 17. The air-gap.—The densities employed in the air-gap are generally comparable with those of continuous-current dynamos. Usual values of B_g are from 8000 to 10,000 in large and 6000 to 8000 in small sizes for 25 cycles, or 7000 to 9000 in large and 6000 to 7000 in small sizes for 50 cycles, the lower and upper limits being usually accompanied respectively by a high and a low voltage.

A small air-gap is advantageous on the score of economy in copper on the magnet, but from the mechanical point of view it necessitates very accurate workmanship and alignment of the bearings in order to avoid unequal magnetic pull. The mechanical and electrical advantages of a large air-gap are further reinforced by the consideration that it may render lamination of the pole-shoes unnecessary. Hence l_g seldom is less than 0·2 in. to 0·25 in., except in quite small machines. Means for equalizing the air-gap in case of wear of the bearings are usually provided.

In order to secure an air-gap length opening out from l_{gc} at the centre to l_{gt} at the pole-tip, the radius R_1 to which the pole-faces must be stamped or turned must be

$$R_1 = \frac{\{R - (l_{gt} + l_{gc})\}\,R\left(1 - \cos\dfrac{\beta\pi}{2p}\right) + \tfrac{1}{2}\,(l_{gt}^2 + l_{gc}^2) - l_{gt}\,l_{gc}\cos\dfrac{\beta\pi}{2p}}{R\left(1 - \cos\dfrac{\beta\pi}{2p}\right) + l_{gt}\cos\dfrac{\beta\pi}{2p} - l_{gc}}$$

where R = radius of stator bore

β = ratio of pole-arc to pole-pitch.

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[1] N S. Diamant, *loc. cit.*, p. 2266.

[2] R. E. Doherty and O. E. Shirley, *loc. cit.*, p. 1271.

The last two quantities in the numerator are small, and may be neglected as nearly cancelling out, so that approximately

$$R_1 = \frac{R - (l_{gt} + l_{gc})}{1 + \dfrac{l_{gt}\cos(\beta\pi/2p) - l_{gc}}{R\{1 - \cos(\beta\pi/2p)\}}}$$

A sine wave is approximately obtained[1] with $\beta = \frac{2}{3}$ when $l_{gt} =$ about $1{\cdot}8\ l_{gc}$.

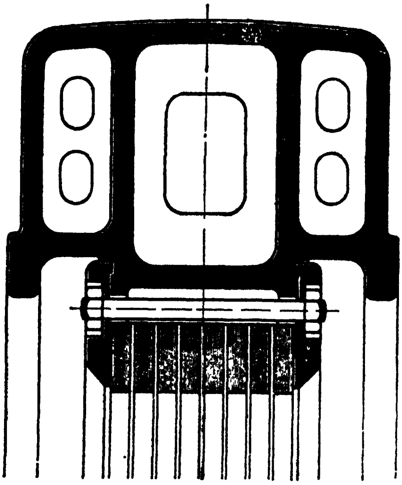

FIG. 569.—Section of stator of large slow-speed alternator.

With the large number of poles usual in slow-speed alternators, the expressions of equations (104) and (105) in Chapter XV, § 14, may be used to calculate approximately the magnetic pull in lb. on the shaft due to a displacement δ of the magnet wheel from its true position concentric with the armature. But the deflection δ of the stator frame must be approximately estimated in the same manner as in Chapter XV, § 16. For the weight of yoke-ring and poles must now be substituted the weight of the entire stator, discs, and frame, and the stiffening added by the internal ring of discs may be roughly taken into account by assuming the modulus of elasticity of the entire stator as that of cast-iron only. In difficult and important cases it may be worth while to make a small model on which the deflections can be measured when suitable forces are applied.[2]

§ 18. The external stationary armature.—Evidently an external stationary armature ring, although free from centrifugal strains, yet requires to be of great mechanical strength to resist deflection.

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[1] As suggested to the writer by Mr. R. G. Jakeman. See also B. Hague, *Journ. I.E.E.*, Vol. 62, p. 921.

[2] See especially H. de Pistoye, *Rev. Gén. d'Él*, Vol. 9, p. 307.

If the cast-iron case be round when bored out in a horizontal position, it naturally suffers a certain deformation when raised to a vertical position, and supported by brackets on either side; and this deformation then becomes further reinforced by very powerful stresses due to any unbalanced magnetic pull when its internal surface is not perfectly concentric with the magnet wheel, or by reason of any inequality in the strength of the poles. Up to diameters of 30 ft. the requirements of mechanical strength may be met by the employment of a deep cast-iron casing of inverted *U*-shape; round the inside of this the segmental laminations, assembled so as to break joint, are in the process of building threaded over numerous transverse bolts, and are clamped tightly together by plates engaging in a turned rim on the armature case. Or the segments are dovetailed into wedge-shaped grooves in the cast-iron stator frame, the transverse bolts passing through clearance spaces at the back of the discs (Fig. 569). The back or sides of the armature casing are usually pierced with a number of large openings to allow of the exit of the air driven through the ventilating ducts of the core.

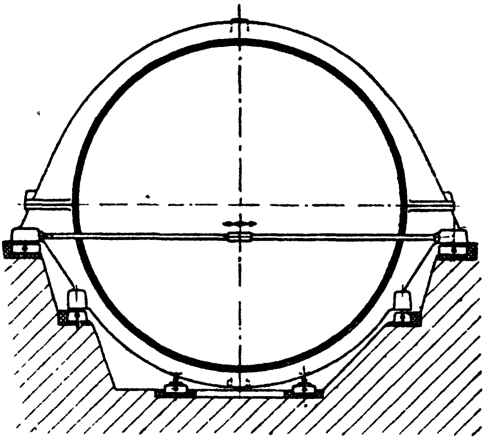

FIG. 570.

With very large diameters the prevention of the mechanical deformation of the armature ring becomes more difficult, and several supporting feet (Fig. 570), may be required. In some cases greater stiffness is imparted by radial tie-rods terminating in a ring, concentric with but free from the shaft (Fig. 571). A convenient and advantageous design employed by Messrs. Brown, Boveri & Co. mounts the armature case on a pair of star frames, one on either side, with massive arms which terminate in rings embracing trunnion journals on the plummer blocks that carry the revolving magnet (Fig. 572). The armature is thus maintained in accurate concentricity with the magnet wheel, without increase of its external diameter or of the dimensions of the foundations, while the further advantage is gained that when the bolts which fix the armature to the concrete foundation and prevent it from turning are withdrawn, the whole armature can be barred round on the trunnion bearings so as to bring any coil into a convenient position for inspection or cleaning.

Figs. 573–576 illustrate a slow-speed three-phase 32-pole alternator constructed by Messrs. the General Electric Co., Ltd., at their Witton works, Birmingham to give 1750 kVA with 80 per cent. power factor 50-cycles 5200 volts at 187½ revs. per min., in which owing to difficult transport conditions the stator has been specially divided into four quadrants; as will be seen, these are not only registered together by keys and bolted together at flanged joints, but also are fastened by two long bolts passing through the outer frame at each joint. The slots are open, and to eliminate harmonics in the E.M.F. wave, some are left empty, as can be seen in Fig. 574.

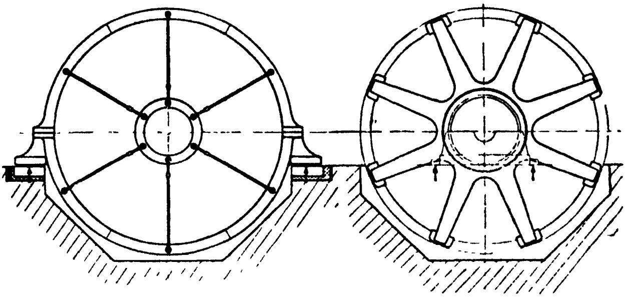

FIG. 571. FIG. 572.

Fig. 577 shows the completed stator of a small slow-speed engine-driven single-phase alternator constructed by Messrs. the Metropolitan-Vickers Electrical Co., Ltd., to give 425 kVA at 200 revs. per min., 50 periodicity, 2400–2700 volts; its rotor, shown in Fig. 578, is fitted with damper winding (Chapter XXXI, § 38).

§ 19. The magnet-system of heteropolar alternators.—When once the armature inductance has been reduced to the minimum that the circumstances of the design allow, the rise of volts when the full current is removed can only be brought to a small value by the methods described in Chapter XXVII, § 50; the field leakage must be kept as small as possible, and a high saturation must be employed in the poles—as high, in fact, as is compatible with reasonable certainty of obtaining material of the estimated permeability. A small coefficient of leakage is given by the use of very short poles, but on the other hand this must not be pushed to such an extreme that the thickness of the copper winding unduly increases the mean

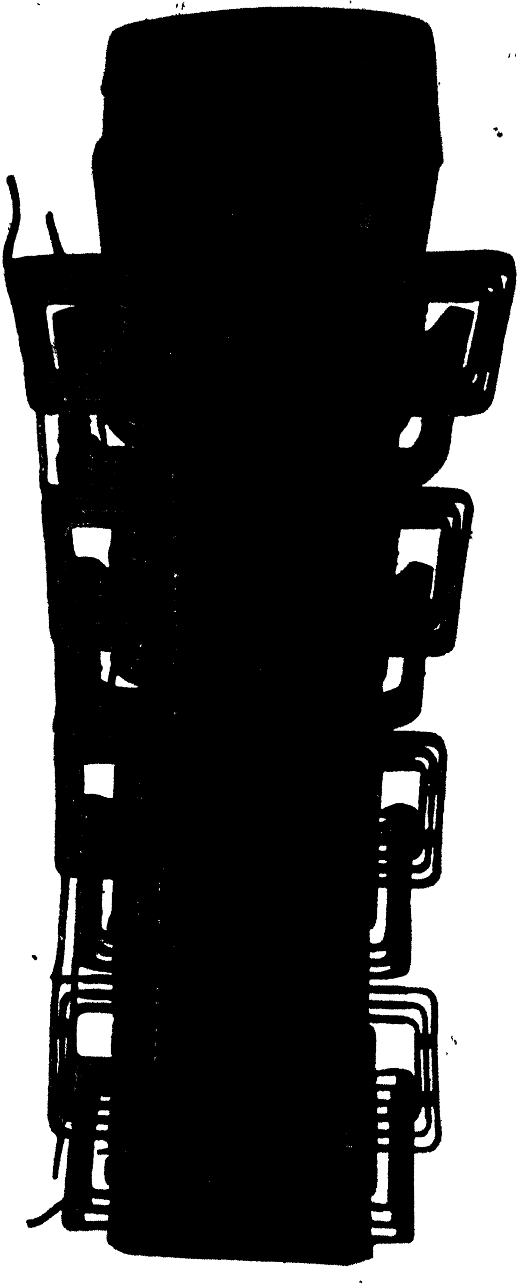

FIG. 574.—A quarter section of the stator of FIG. 573.

length of a turn and the consequent weight of copper, with a reduced cooling surface. Ingot iron or steel is more uniform in permeability than cast steel, gives greater exactness of section, and is free from the danger of blow-holes; yet with cast-steel poles, if carefully tested, the average flux-density is sometimes carried as high as $B_m = 18,000$ when a small rise of volts is specially required, or say B_m at the outer end of the bobbin $= 16,000$, and at the base $= 18,000$. With such high densities the yoke-ring must itself also be of cast-steel, since with cast-iron the lines would be throttled at the point of junction between pole and yoke; or the pole must be dovetailed into the ring. In any case it is safer to allow a certain small number of ampere-turns for the reluctance of the joint. Thus the advisability or otherwise of a high degree of saturation in the magnet turns upon the conditions of the design

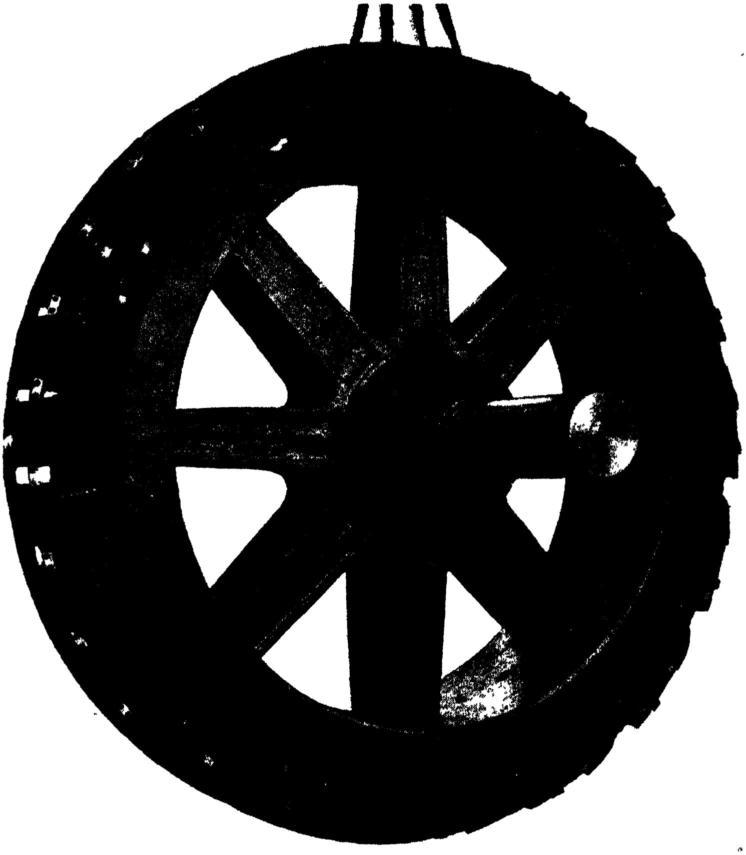

FIG. 575.—The rotor of Fig. 573.

and the nature of the service for which the alternator is intended. As a close degree of inherent regulation is always expensive to obtain, it is now more usual to adopt a cheaper design and to employ an automatic voltage regulator to obtain a constant potential. In generators of low frequency for the supply of rotary converters a moderate degree only of saturation is to be recommended.

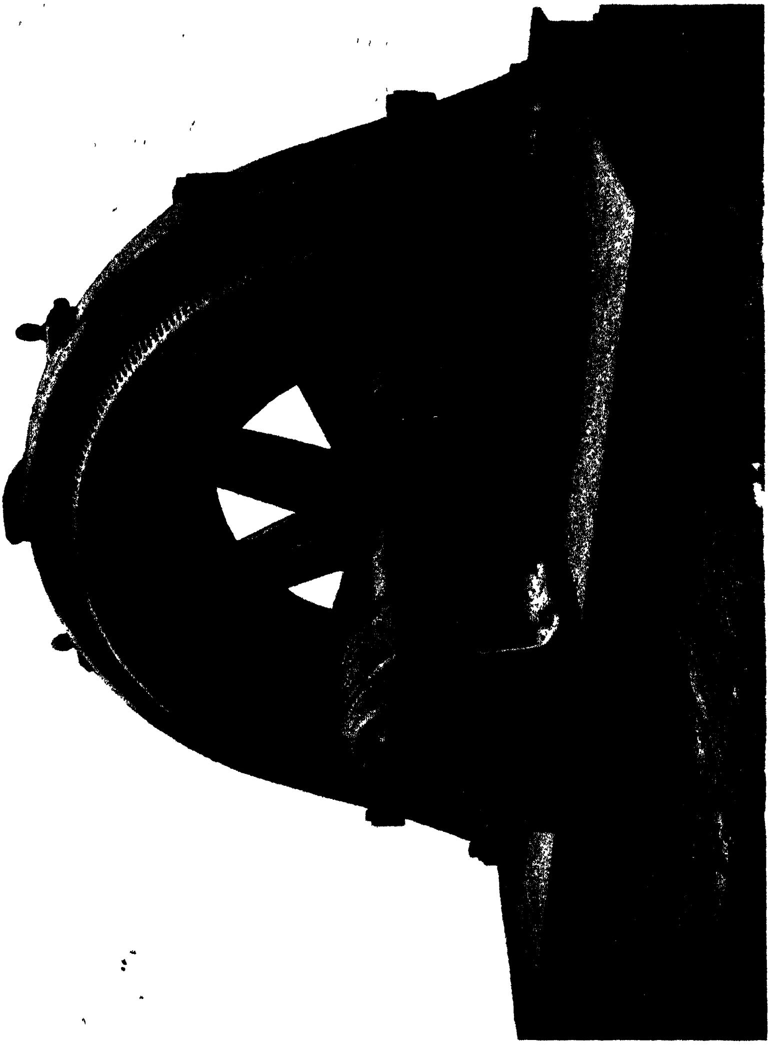

FIG. 576.—General view of 1750 kVA alternator of Fig. 573.

The yoke may be given considerable thickness and be worked at a lower density, since lavish proportions are here not disadvantageous, in so far as they increase the fly-wheel effect and the steadiness of the running.

The leakage coefficient of the heteropolar field-magnet varies with the ratio of the pole-width to the pitch, with the absolute value of the pitch as determined by the frequency, and with the length of the poles, and is also affected by the load. It averages in slow- and medium-speed alternators from 1·2 to 1·4, becoming larger with small poles, but, as in the case of continuous-current dynamos, unless it has been experimentally ascertained from machines of similar proportions and similarly saturated, even an approximate calculation of the leakage permeance $\mathcal{P}_l$ is to be preferred, and

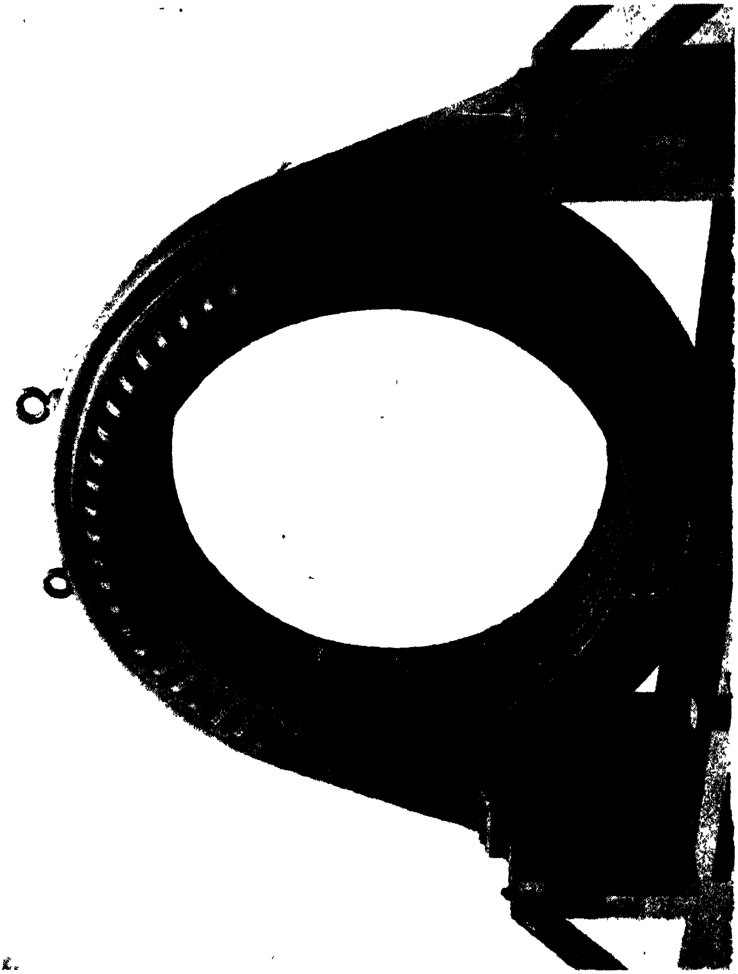

Fig. 577.—Stator of 425 kVA single-phase alternator, 200 r.p.m.
(Metropolitan-Vickers Electrical Co., Ltd.)

such calculation should be made with the greatest possible accuracy.

The use of the field-magnet as the sole fly-wheel in the case of alternators directly coupled to their steam engines is frequent, yet is only to be recommended when the most favourable diameter of armature and of fly-wheel naturally approximate to each other, and this is usually the case only with machines of large size. In other cases, if the rotor is given the dimensions which would best suit the alternator, an unduly large weight of cast-iron in the yoke-ring may be necessitated and a larger shaft to carry it, while if the diameter of the alternator be increased to suit the fly-wheel requirements the armature frame must be of greater depth and size to give it sufficient rigidity.[1]

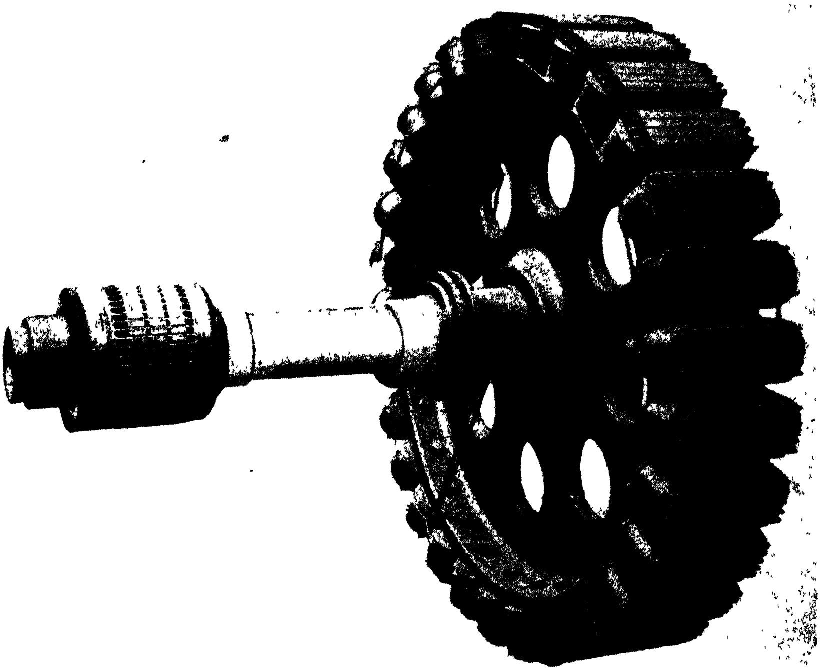

FIG 578.—Rotor of 425 kVA single-phase alternator with damper winding. (Metropolitan-Vickers Electrical Co., Ltd.)

With six or more half-closed slots per pole, *i.e.* two or more per phase in a 3-phase machine, solid pole-faces may be employed if the width of opening of the slots is not more than 1·5 l_g, but with wider openings it is advisable to laminate at least the pole-shoes (Fig. 579) in order to avoid the eddy-currents arising from the variation of the flux due to the slotted stator Although the eddies do not penetrate far into the mass of the pole, it is frequently the practice to laminate the entire pole. This construction is employed partly from its cheapness in manufacture, since it is not easy to dovetail or fasten the laminated pole-shoes into solid poles, and partly also because the connexion of the two usually involves a considerable radial depth of pole-shoe, so that the leakage

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[1] Rothert, *L'Éclairage Électrique*, Vol. 29, p. 313. In a brochure, "Der Aufbau und die planmässige Herstellung der Drehstrom-Dynamomaschinen," by O. Lasche (translated in *Engineering*, Vol. 72, p. 173 ff.), various arrangements of magnet and fly-wheel are shown diagrammatically and discussed.

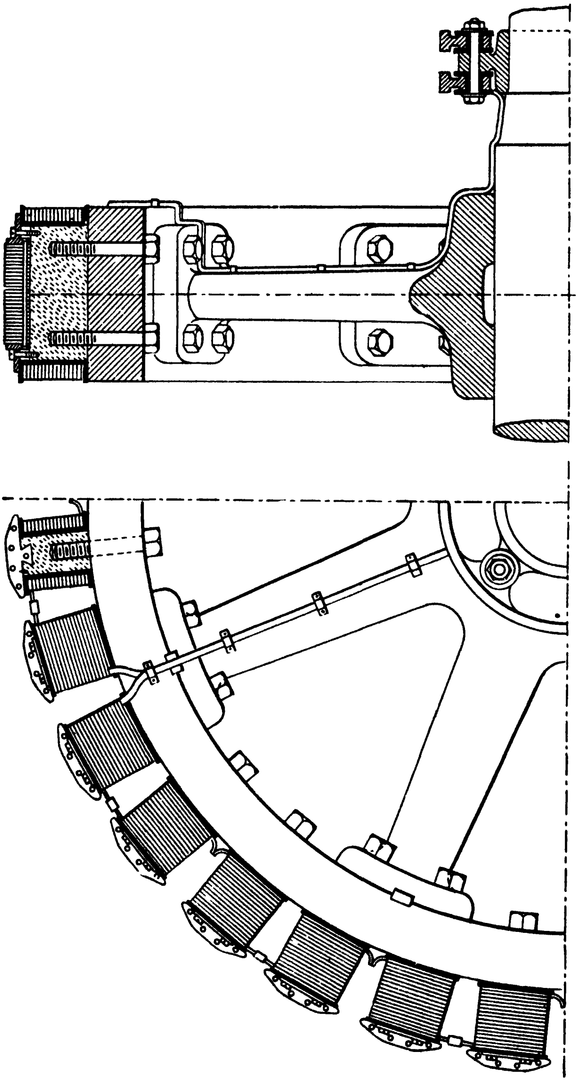

FIG 579 —Portion of magnet-wheel with solid poles and laminated pole-shoes.

between the pole tips is increased and the exciting coils do not approach close to the ends of the poles. An objection to the laminated pole-shoe as also to the laminated pole is that it does not assist towards easy parallel working; if there are periodic fluctuations in the flux or speed, the solid pole-face itself plays the part more or less of an amortisseur winding (Chapter XXXI, § 38) owing to the eddy-currents set up in it. Laminated poles of sheet steel stampings 0·025 in. thick, riveted between two stout steel end-plates, are bolted up to the field-ring, usually by studs screwing into an iron bar of rectangular or circular section which is driven through the laminations (Figs. 580 and 581). Or they are dovetailed into the rim of the field-ring (Figs. 573 and 582), a method which has the advantage that the pole may then be drawn out sideways for examination or removal of the bobbin without shifting the armature or magnet as a whole. The section of the pole if laminated must be rectangular, but with solid cast-steel poles, although it is seldom possible to employ the most economical section of a circle owing to the necessity of arranging a large number of poles round a wheel of moderate diameter, yet at least an approximation to the best form may be made by giving the poles an oval shape. A considerable saving of copper is thus obtained, and the bobbins are themselves easier to wind. For high peripheral speeds the sheet-steel pole has the advantage that the material is mechanically reliable, and can be tested in small packets. The same is also true of its magnetic qualities, as it is very constant in permeability,

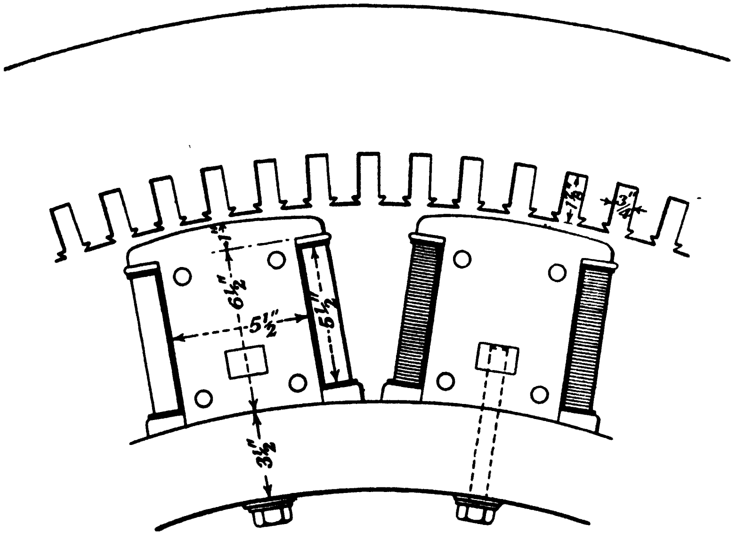

FIG 580 —Laminated pole with rectangular bar.

and hence the use of laminated poles enables a high density to be employed in the magnets with considerable confidence.

While a peripheral speed of 6000 ft. per min. is permissible in the case of a plain cast-iron fly-wheel with solid rim and arms well rounded into the rim, such a limiting speed must be considerably reduced to, say, 4500 ft. per min., if the poles are bolted to the wheel and the further centrifugal force due to their mass and to the exciting coils which they carry is added to that of the rim itself. Especially must this be the case when the rim of cast-iron is divided and has to be locked together. The employment of a cast-steel wheel is then necessary for greater safety at speeds up to 6000 ft. per min. A single casting in most cases can be employed, but with larger

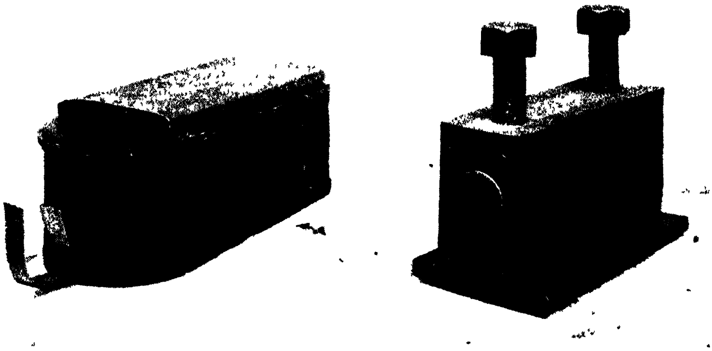

FIG. 581.—Laminated bolted-on pole with mild steel rod through stampings and coil of strap wound on edge.
(Metropolitan-Vickers Electrical Co., Ltd.)

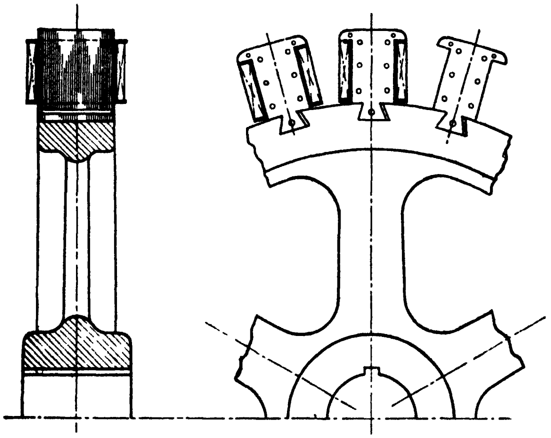

FIG. 582.—Laminated poles dove-tailed into yoke.

diameters, the hub and spokes may be cast separately, and fitted into the field-ring. Or where transport is difficult, the rotor can be divided into two halves in a plane perpendicular to the shaft, and afterwards bolted together.

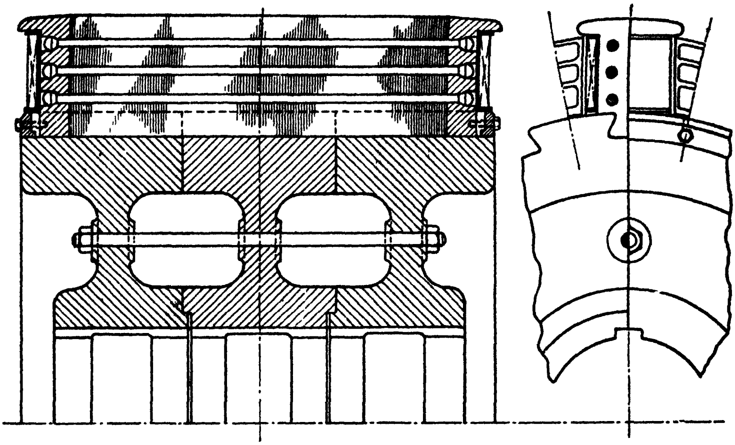

FIG. 583.—Three-part rotor with laminated dovetailed poles.
(English Electric Co.)

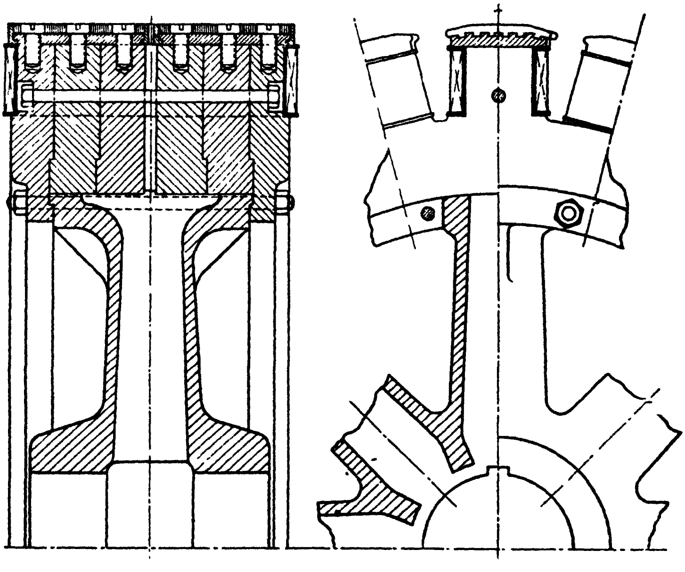

FIG. 584.—Combined poles and yoke with laminated and skewed pole-shoes dovetailed into pole.
(Swedish General Electric Co.)

A two- or three-part rotor in many forms is common (Fig. 583), and the division into two may even be carried up through the pole

(Figs. 584, 585).[1] At high speeds above, say, 8000 ft. per min. at the pole-face, both poles and rim may be laminated as shown in Figs. 586 and 587. In the latter construction which has been employed by the Allgemeine Elektricitäts Gesellschaft, the interlocked ring of laminated sheet-steel plates is wedged on to the arms of the cast-iron centre-wheel. Each packet of segmental plates is held by several bolts, and the whole ring is free to expand under the action of centrifugal force without undergoing deformation. The further advantage is gained that the tensile quality of the materials

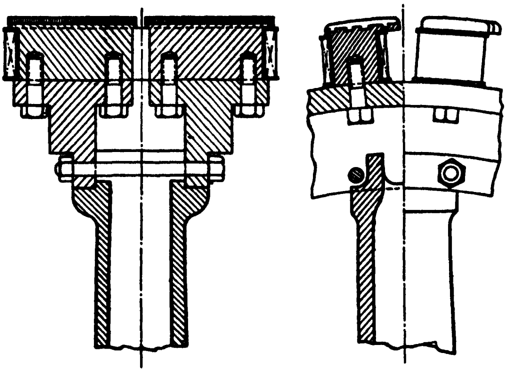

FIG. 585 —Two-part rotor with poles bolted on and skewed pole-shoes. (Swedish General Electric Co.)

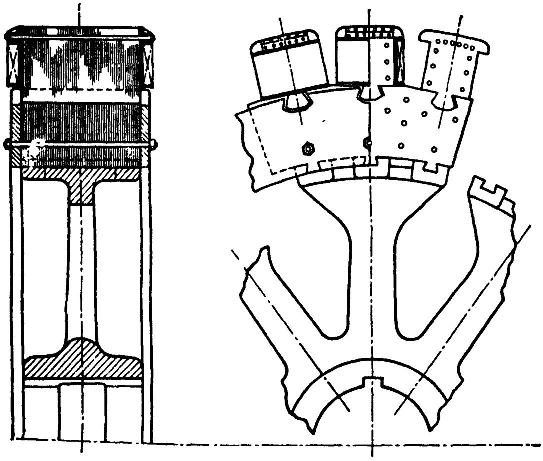

FIG 586.—Laminated poles and yoke ring dovetailed to hub. (Westinghouse Co.)

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[1] For these and other pole-construction diagrams the writer is indebted to Prof. S. P. Smith; see his paper on "Large Electric Units" (British Association Meeting, 1921).

of the plates and the shearing quality of the bolts can be experimentally ascertained, while the stresses in each case admit of close calculation. The pole-pieces are dovetailed into the yoke ring and are fastened by double wedges, these latter being themselves locked by a wire passing through them and bent over in opposite directions at the two ends.

§ 20. Water-driven alternators.—The construction of modern alternators driven by Pelton-wheels or water-turbines (with peripheral speeds up to 12,000 or even 15,000 ft. per min.) approximates in some respects to that of alternators driven by steam turbines. To reduce noise and windage losses and to provide protection against possible leaks from the turbine they are usually now totally enclosed with artificial ventilation, and their stator windings are braced to enable them to withstand sudden short-circuit.

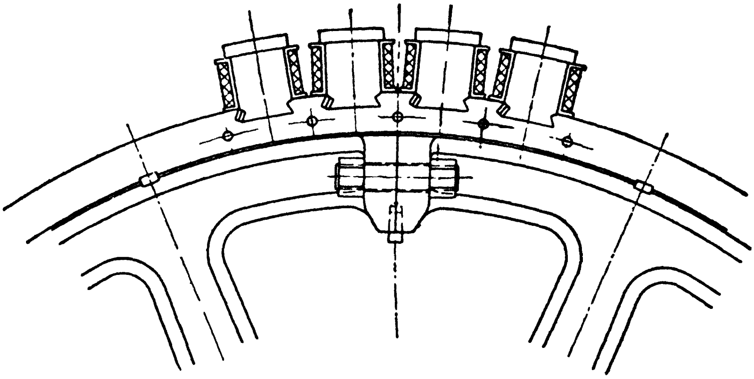

FIG 587 —Magnet-wheel construction with laminated poles and rim.

The speeds for given outputs, depending on the head of water and type of runner, cover a wide range. For example, outputs of 4000 kVA 17,000 volts 300 revs. per min. (Kanderwerke), of 8000 kVA 18,000 volts 345 r.p.m. (Schnalstalwerke in the Adige valley, Tyrol), 17,500 kVA 6600 volts 375 r.p.m. (Big Creek, Pacific Light and Power Co.), 28,000 kVA 428 r.p.m. 50-cycle (Southern California Edison Co.), 45,000 kVA and 55,000 kVA 12,000 volts 187½ r.p.m. (Queenston), 65,000 kVA 12,000 volts 107 r.p.m. 25-cycle (Niagara Falls Power Co.) are found.

The run-away speed which may exceed twice the normal speed and the speed regulation in general as dependent on the length of pipe line and the rise of pressure in it due to the inertia of the water when the gates are moved by the hydraulic governor, introduce difficulties which can only be overcome by the most careful design of the whole hydro-electric unit. A large fly-wheel effect is required to give time for the absorption of energy, and the value of

WR^2 must be accurately adjusted to suit the time element of the motion of the gates or nozzles.[1]

In the alternator of Fig. 573 which was driven by a water turbine, the required fly-wheel effect (GD^2) of 1420 ton-ft.^{2} was obtained by two additional cast-steel rings (Fig. 573, *cp.* also Fig. 605), spigotted one on either side and also fastened by set screws to the two inner rings of cast-steel which are themselves fastened to the arms of the cast-iron hub by set screws and by taper keys driven in from opposite sides, their dividing plane being radial. On the sides of poles are plates carrying brackets ; through these and also through the gunmetal clamping plates of each field coil there pass long set screws which screw into the rims of the outer fly-wheel rings.

To withstand the centrifugal stresses especially on the pole joint due to the run-away speed, many different forms of pole construction, some of which have already been given, are employed. For peripheral speeds from 10,000 to 12,000 ft. per min. Messrs. Ganz & Co. have adopted a construction in which circular pole-cores are cast in one with the field-ring, and over these are slipped steel rings carrying the coils and forming at the top a part of the pole-shoe. The steel rings are fastened to the pole-cores by a ring of studs (Fig. 588), screwing half into the pole-core and half into the steel sleeve.[2] The Oerlikon Co. employ a comb type of pole, the teeth of which are formed by steel plates projecting below the laminations of the pole proper. The projecting teeth pass through the shrunk-on steel rim forming the yoke, and are locked by double keys (Figs. 589–592). A totally-enclosed horizontal-type of alternator constructed by the Oerlikon Co. for the Amsteg power station of the St. Gotthard railway, is shown in Figs. 593, 594. The 6-pole machine, which is driven by a 15,000 h.p. double Pelton wheel under a head of 920 ft. at 333⅓ revs. per min., gives a single-phase continuous output of 10,000 kVA 15,000 volts at 75 per cent. power-factor and at the railway frequency of 16⅔ cycles per second, with an overload

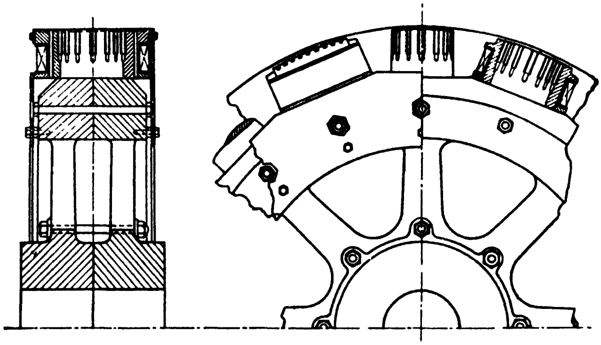

FIG. 588.—Pole construction of Messrs. Ganz & Co.

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[1] See especially *Journ. Amer. I.E.E.*, Vol. 43, pp. 843–4.

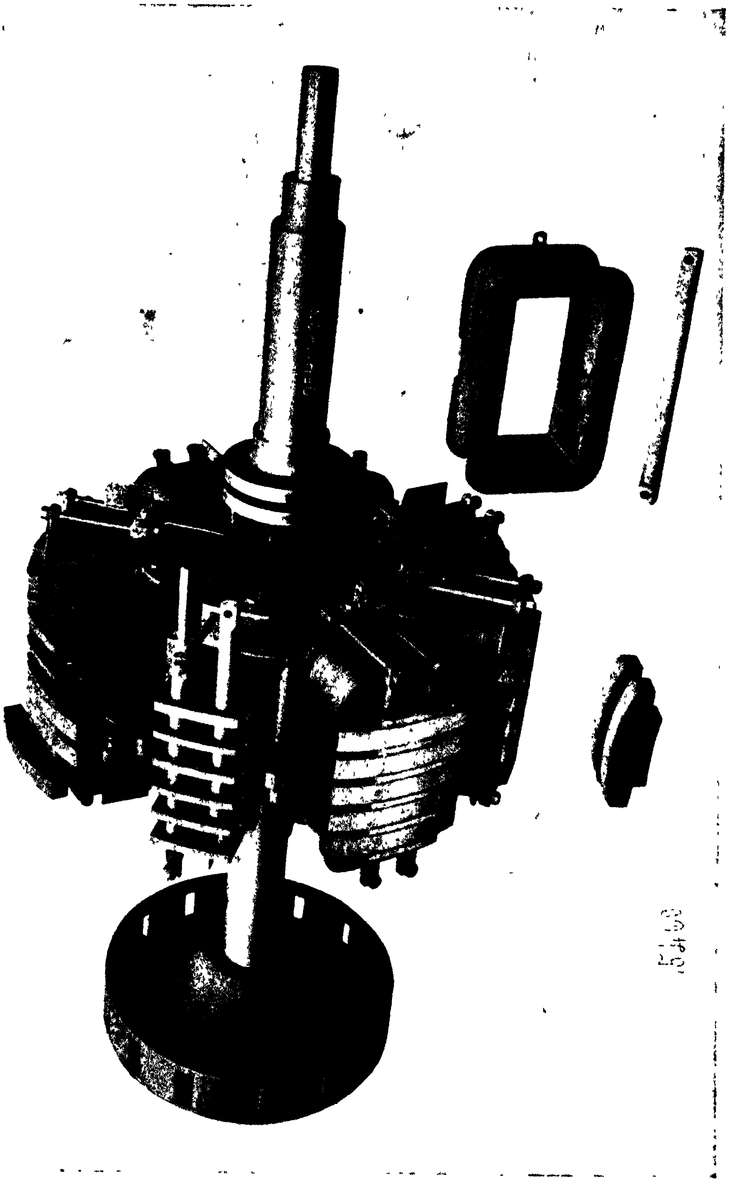

FIG. 589.—Comb method of fixing pole-shoe.

(Maschinenfabrik Oerlikon.)

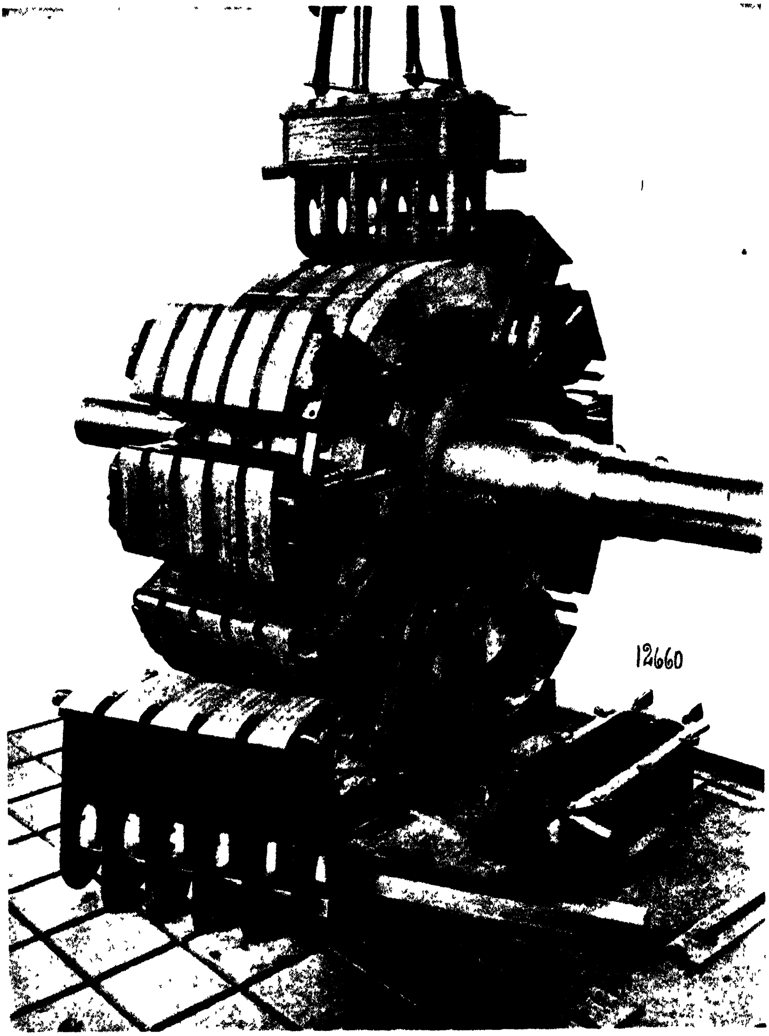

FIG. 590.—Rotor with steel rim shrunk on and steel plates and keys for fixing poles.
(Maschinenfabrik Oerlikon.)

capacity of 25 per cent. for 1½ hours and a pressure variation of ± 10 per cent. obtainable at full-load and maximum load. The stator bars of circular section are placed in half-closed slots, and are insulated for a working pressure of 16,500 R.M.S. volts, and a test pressure of 55,000 volts (crest value) between conductors and earth; one pole of the alternators is earthed. Comb-like projections from the base of each laminated pole-shoe here fit between corresponding solid projections from the pole, the two being locked together by two large pins. An automatic pressure regulator acts on the excitation of the exciter. Fans on either side of the rotor drive the cooling air through the core, and temperature detectors indicate at the switchboard the temperature at two points of the stator winding.

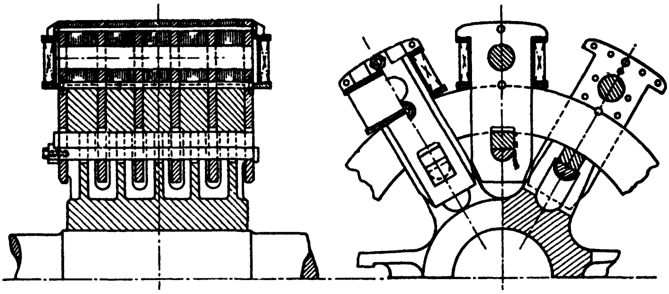

FIG. 591.—Section of pole of Fig. 590.

Figs. 595 and 596 show the stator of a water-wheel alternator, constructed by the Metropolitan-Vickers Electrical Co., Ltd., to give 3000 kVA, 11,000 volts, at 375 r.p.m., with concentric coil winding and 4 slots per pole per phase; the former Figure is taken from the terminal end, and in the latter Figure from the opposite end is seen the air-inlet trunk in the lower half of the sheet-steel end guard.

Figs 597-600 illustrate details of a 6000 kVA water-wheel alternator, also constructed by the Metropolitan-Vickers Electrical Co., Ltd., for 375 r.p.m. Into the dovetailed grooves of the cast-steel magnet wheels, two of which are shown in Fig 597, are forced in under hydraulic pressure the laminated poles of Fig 598 with cast-steel semicircular end-plates, without keys or wedges The unwound stator of Fig 599 has cast-iron segmental end-plates, clamped by bolts passing outside the core; the malleable cast-iron tooth supports, cast in groups and riveted to the end-stampings, are well seen, and the air-ducts in the core, formed by rolled steel H-section separators welded to one of the thicker stampings which are placed on either side of an air-duct. The concentric coil-winding (Fig. 600) for 11,000 volts has alternately 4 and 5 slots per pole per phase; the end-connexions are divided for good ventilation.[1]

The larger the runner of a turbine, the greater the difficulty in placing it horizontally, so that modern units for large outputs or low heads are arranged with the shaft vertical. This leads to the so-called "umbrella" type of machine with a stator ring placed

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[1] For other details and drawings of water-driven alternators, see especially Chapters I—IV contributed by S. Neville to Volume II of *Hydro-Electric Engineering*, edited by Prof. A. H. Gibson. (Blackie & Son, Ltd.)

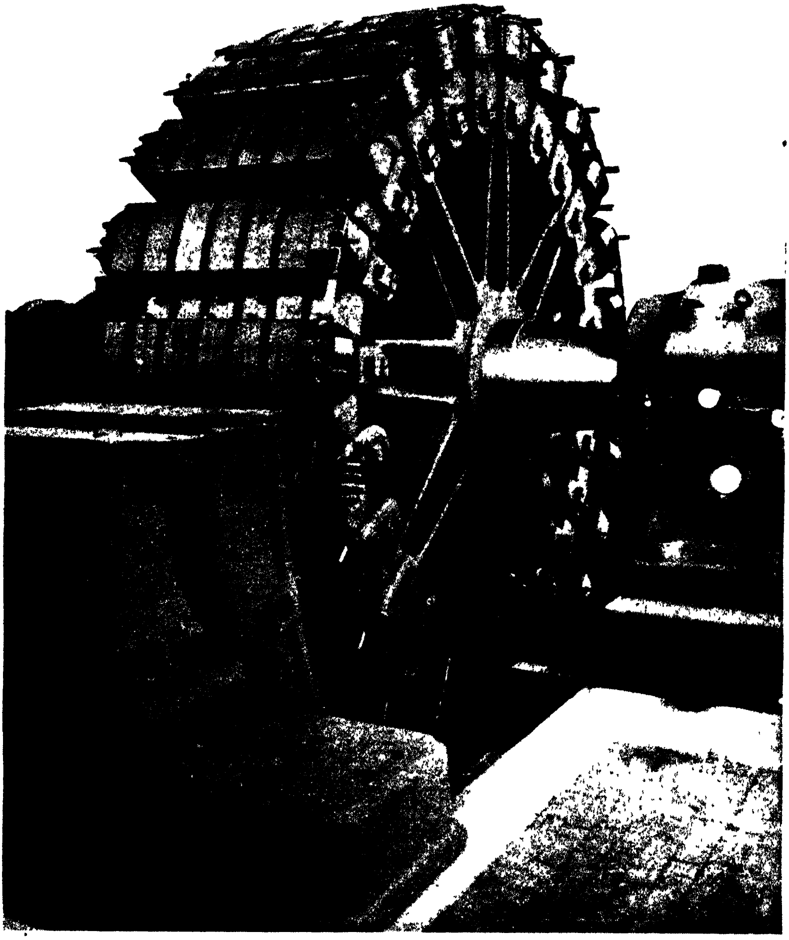

FIG. 592.—17,000 kVA generator with top half of stator removed.
(Maschinenfabrik Oerlikon.)

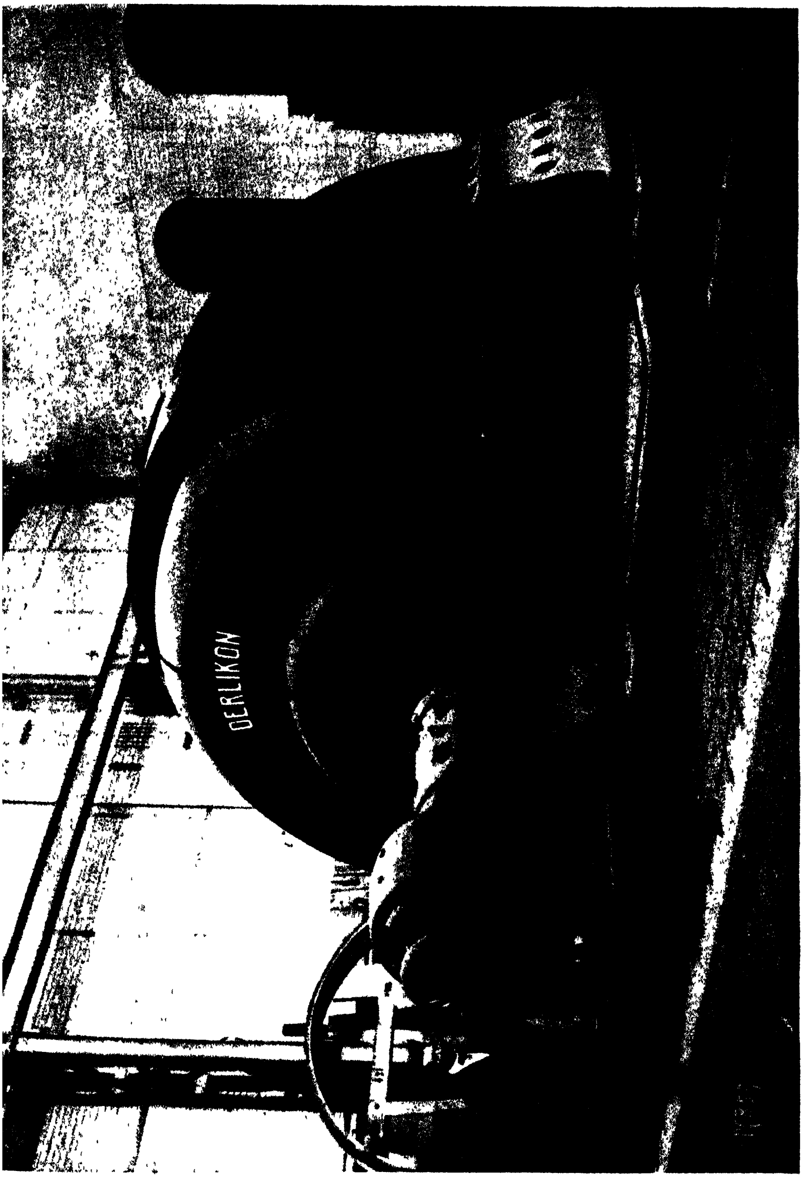

FIG. 593.—10,000 kVA single-phase alternator, constructed by the Oerlikon Co. for Amsteg power station.

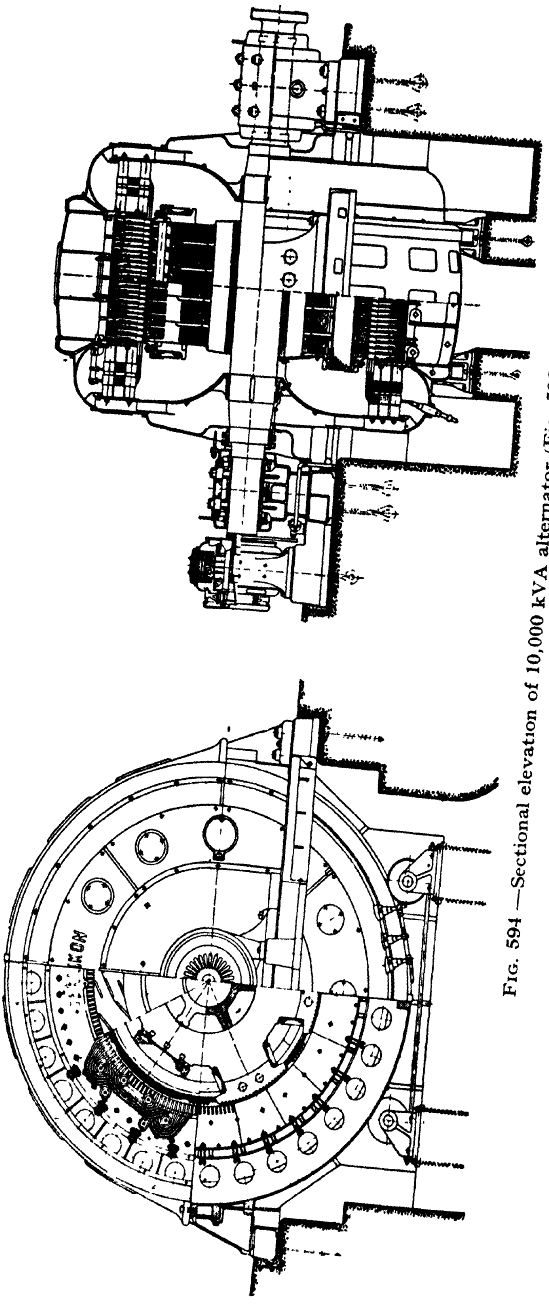

Fig. 594.—Sectional elevation of 10,000 kVA alternator (Fig. 593).

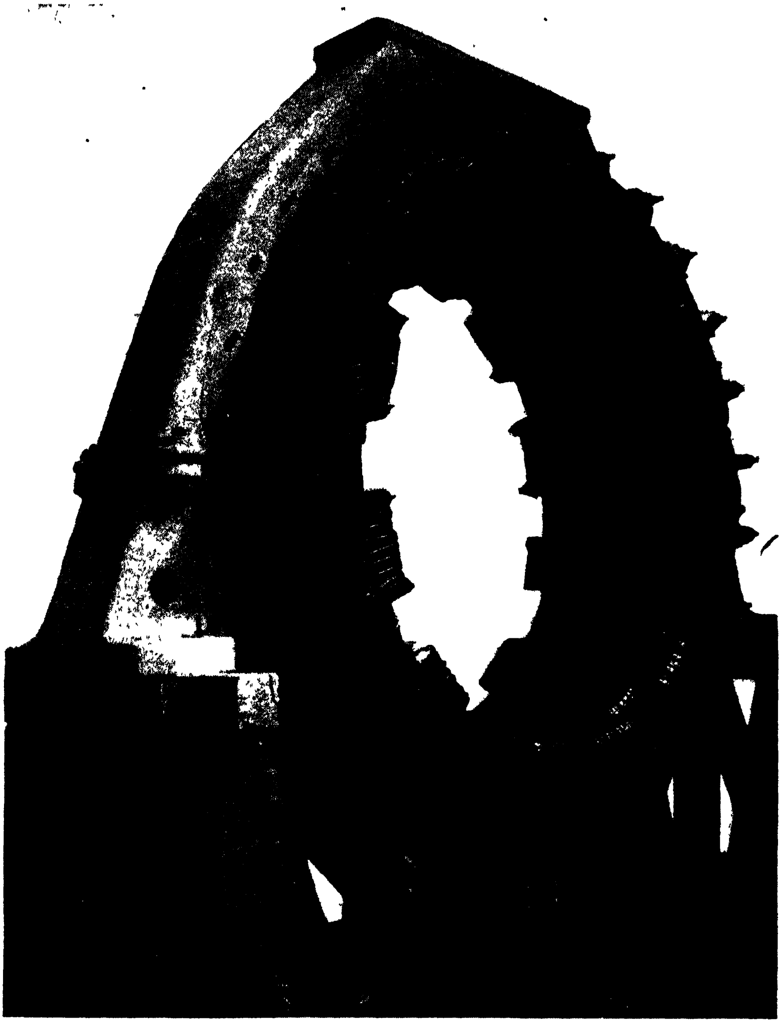

FIG. 595 —Stator of 3000 kVA water-wheel alternator from terminal end.

(Metropolitan-Vickers Electrical Co., Ltd.)

FIG. 596.—The same as Fig. 595 from the opposite end with lower half of end-guard.

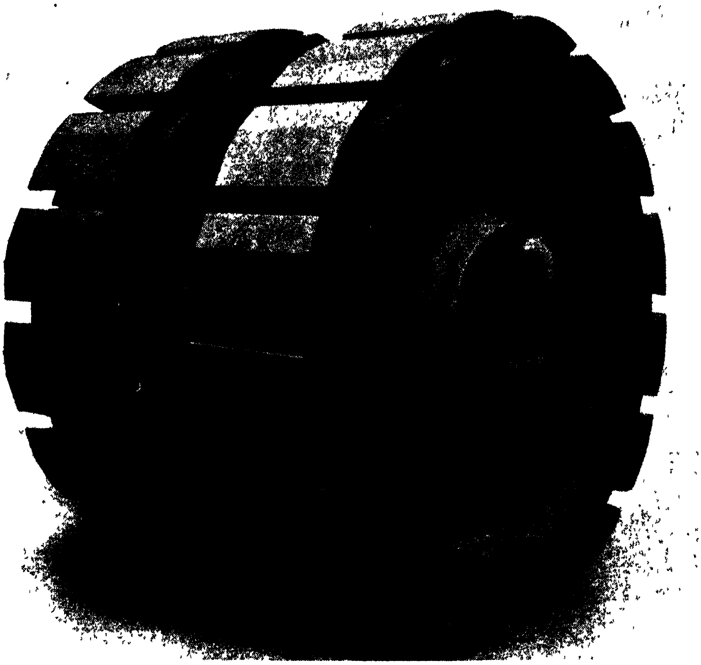

FIG. 597.—Magnet-wheels of 6000 kVA water-wheel alternators.
(Metropolitan-Vickers Electrical Co., Ltd.)

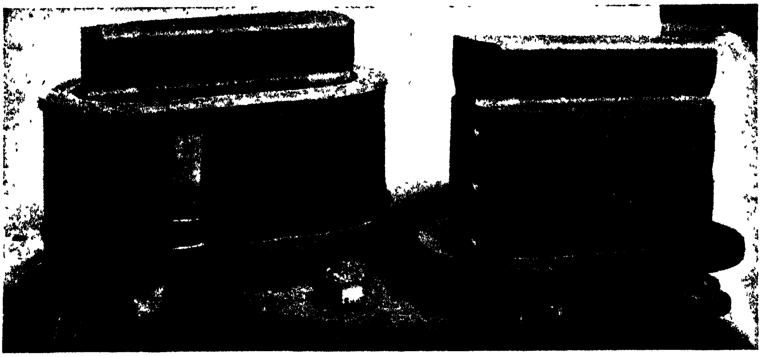

FIG. 598.—Dovetailed poles for Fig. 597.

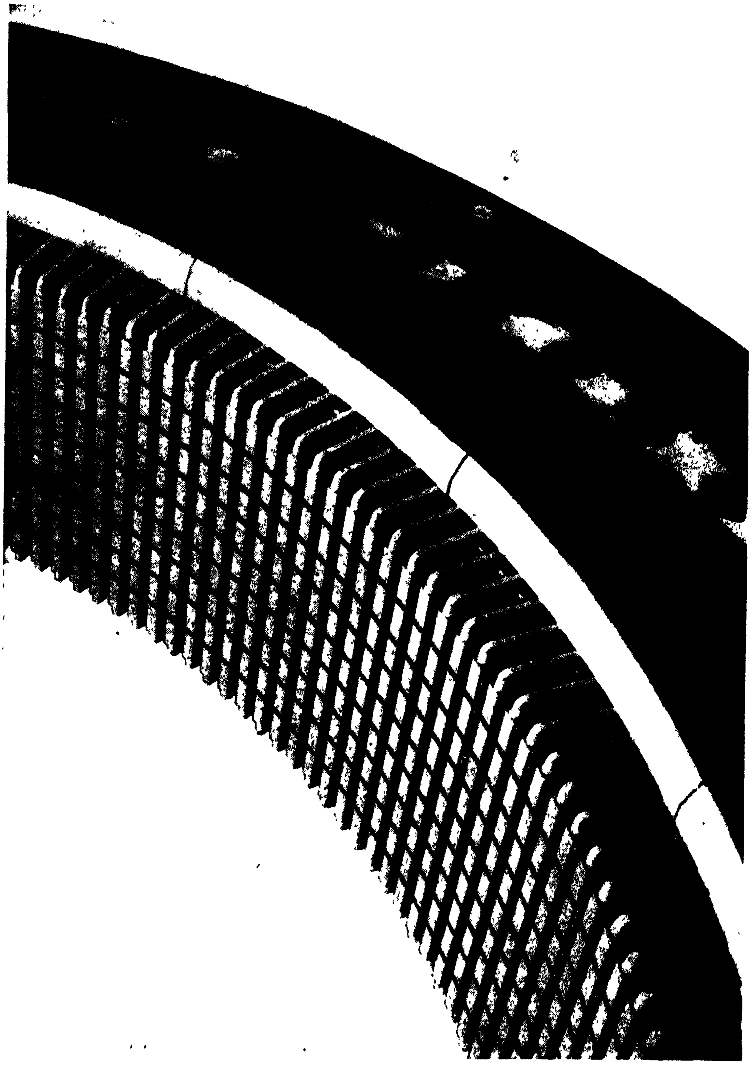

FIG. 599 —Unwound stator of 6000 kVA alternator.
(Metropolitan Vickers Electrical Co. Ltd)

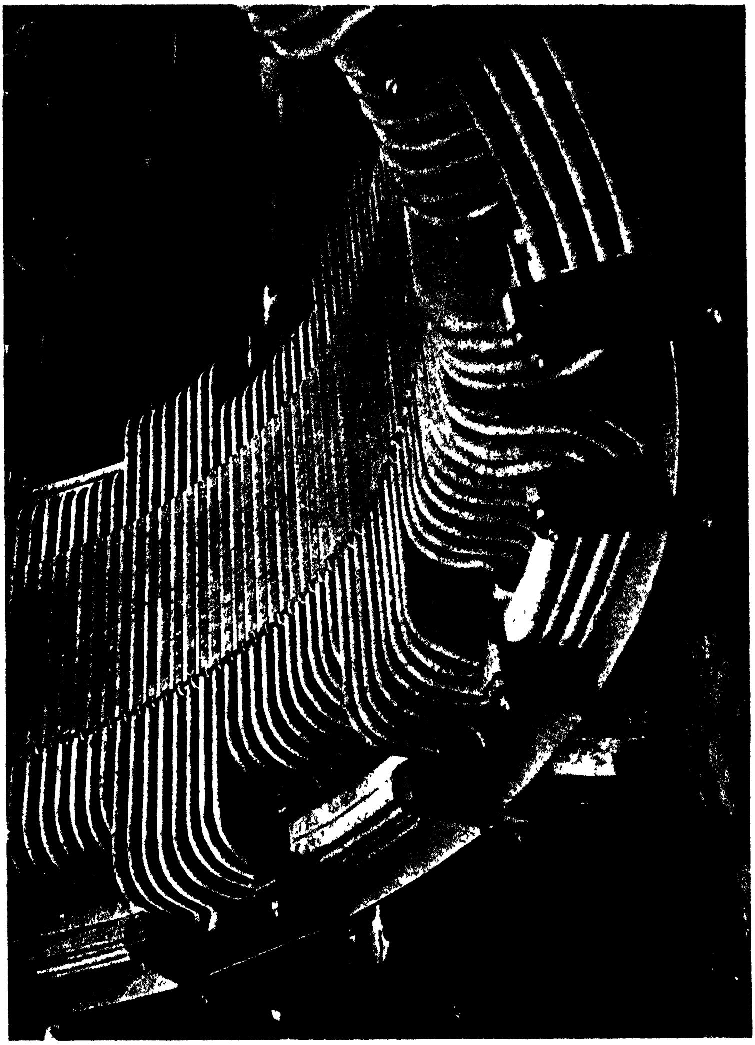

FIG. 600.—Winding of Fig. 599.

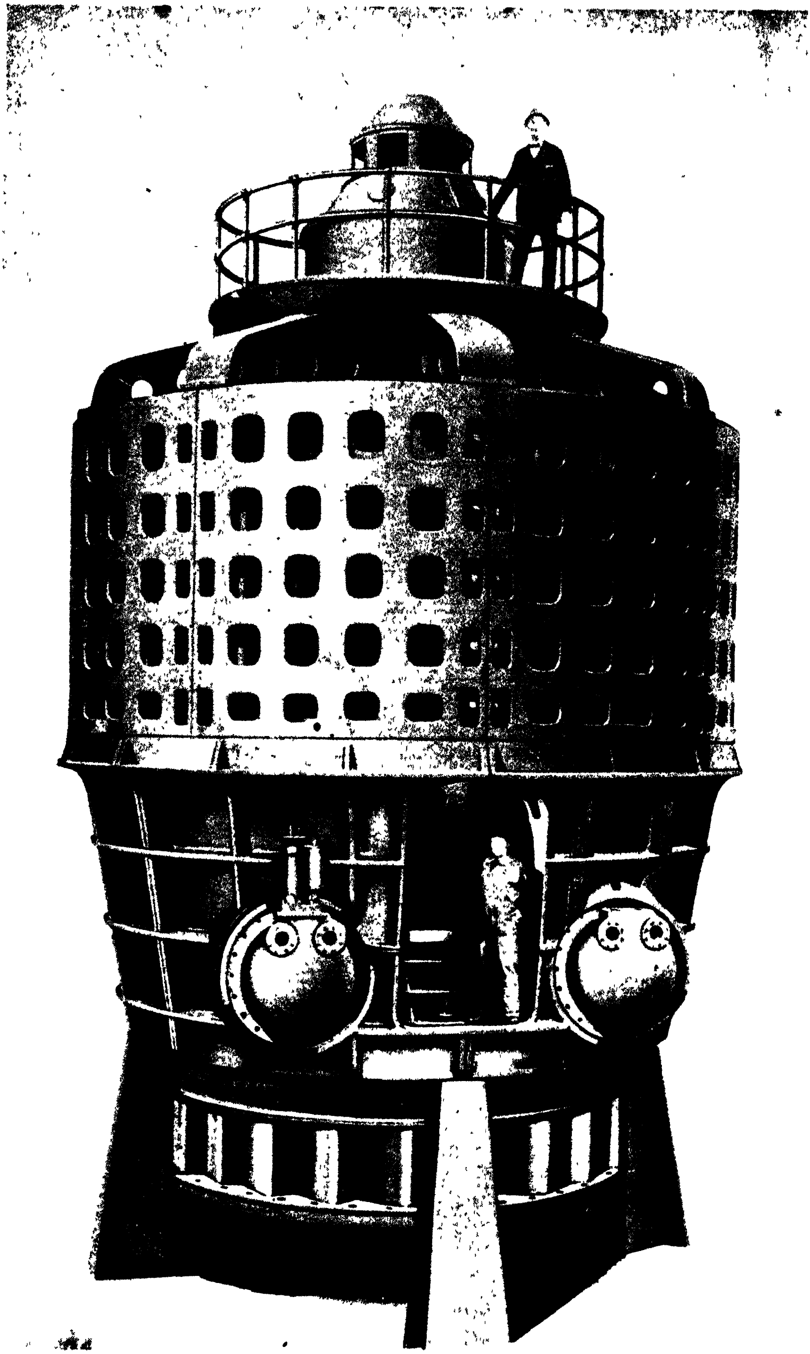

Fig. 601.—32,500 kVA hydro-electric unit, 150 r.p.m., built by the Allis-Chalmers Manufacturing Co. for the Niagara Falls Power Co.

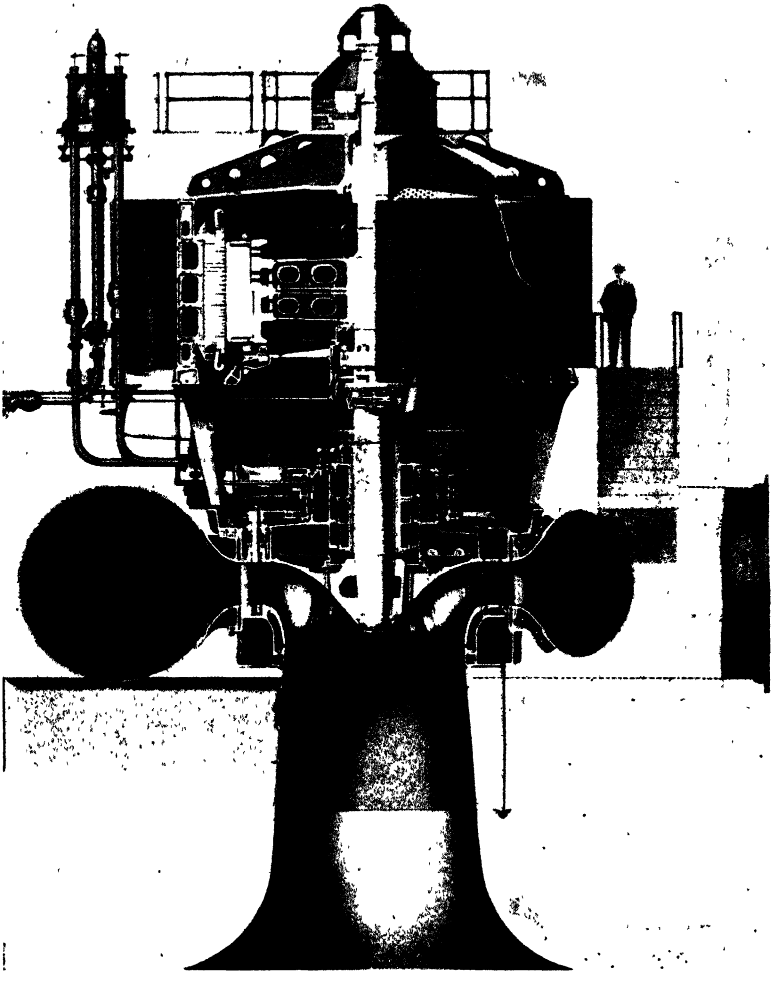

FIG. 602.—Section of 32,500 kVA hydro-electric unit (Fig. 601).

horizontally and carrying on its upper side massive radial arms uniting into a star frame, from which is hung the vertical shaft carrying the rotor of the generator and the waterwheel runner. The whole weight of the rotating parts and the unbalanced water thrust is then taken by a thrust bearing supported on the star frame.

Figs. 601 and 602 show in elevation and section one of the three 37,500 h.p., 32,500 kVA hydro-electric units in the " Station No. 3 Extension " of The Niagara Falls Power Co., completed in 1920 and situated in the gorge on the bank of the river on the American side about half a mile below the falls.

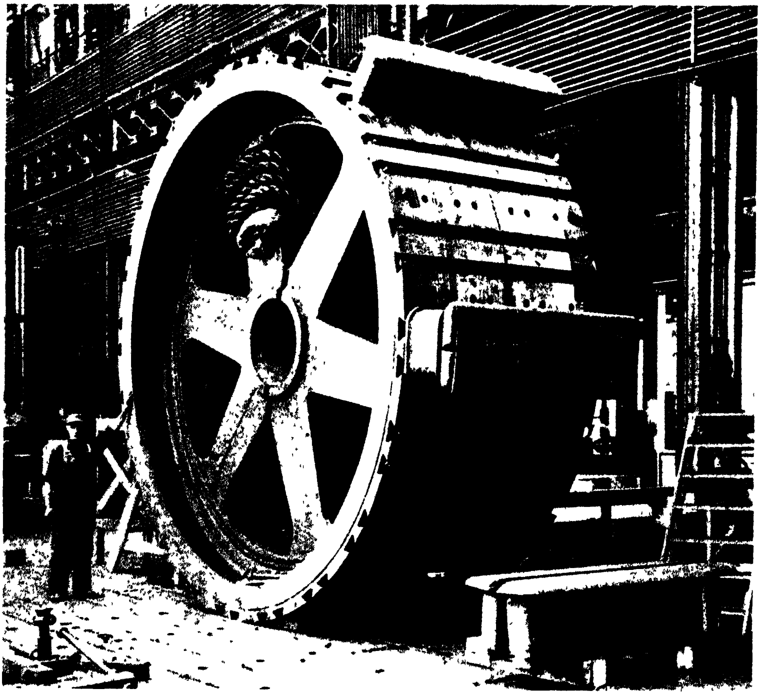

FIG. 603.—Rotor hub with field coil and pole of Fig 601.

The machine illustrated, which is three-phase 20-pole star-connected, was constructed by the Allis-Chalmers Manufacturing Co , and gives its output under the head of 214 ft at 12,000 volts and 25 frequency when running at 150 revs per min. The combined efficiency of turbine and generator, reckoned from the total energy in the water to the generator terminals, reaches 90 per cent. over a range from 80 to 90 per cent of the full-load of 37,500 h.p. and falls slightly below this value at full-load ; the turbine itself has an efficiency of about 93 per cent , and at 90 per cent power factor and full-load the efficiency of the generator is 98 per cent. The rotor hub is in two parts with yokes shrunk on and with dovetailed laminated poles in halves (Figs. 603 and 603*a*). The outside of the stator frame of box section, 21 ft. 6 ins. in diameter, is surrounded by a steel-plate ventilating housing connected to a duct system through a fan ; cooling air at the rate of some 70,000–90,000 cub. ft. per min. is taken in from the generator room and from the pit under the machine, and discharged outside the station in the summer, and partly within the station and control room in the winter. Out of 18 temperature indicators distributed in different parts of the stator windings, the six that show the highest temperatures are connected to an indicator in the control room. Both ends of each phase winding are brought out, so that a balanced relay system of protection operated from differentially-connected current transformers in the generator leads can be used ; by means of this, in case of trouble within the generator, its main oil-switch is opened practically instantaneously and its field circuit is killed. A shunt-wound 225 kW exciter driven by an induction motor supplies the excitation at 220 volts.[1] The neutral point is grounded through a resistance of about 5 ohms, given by a water rheostat.

§ 21. **45,000 kVA water-driven alternators.**—Five 25-frequency three-phase machines for 55,000 h p. 45,000 kVA at 187½ revs. per min., under a head of 305 ft., have been installed in the Queenston power-house of the Hydro-electric Power Commission of Ontario, situated on the Canadian side of the Niagara river. Fig. 604 shows one of two machines constructed for the above station by the Canadian General Electric Co., which gives 12,000 volts, 45,000 kVA at 80 per cent. power-factor or 49,500 kVA with either voltage or current 10 per cent. in excess of the rated values. The details of the construction are shown in Fig. 605. The casting for the stator frame, 25 ft. in outside diameter by 10 ft. high and weighing about 90 tons, is divided into three vertical sections, but the laminated core was built up within it in the power station as a continuous ring. The hollow rotor shaft is 2 ft. 6 ins. in diameter within the bearings, and the over-all height from the face of the coupling to the top of the exciter is 33 ft. 10 ins. or from the base ring is 28 ft. 3 ins. The diameter of the rotor over the pole-faces is approximately 18 ft., and the magnetic part of the spider carrying the poles is composed of 5 cast-steel wheels. Two additional sections, one above and one below, serve to increase the total fly-wheel effect of the complete rotor to 21,000,000 WR^2 in lb.-feet^2 units. The seven sections of the rotor spider weigh 190 tons. While the hubs of the rotor wheels fit close together, the rims of lesser width leave passages between them for air to reach the interpolar spaces.

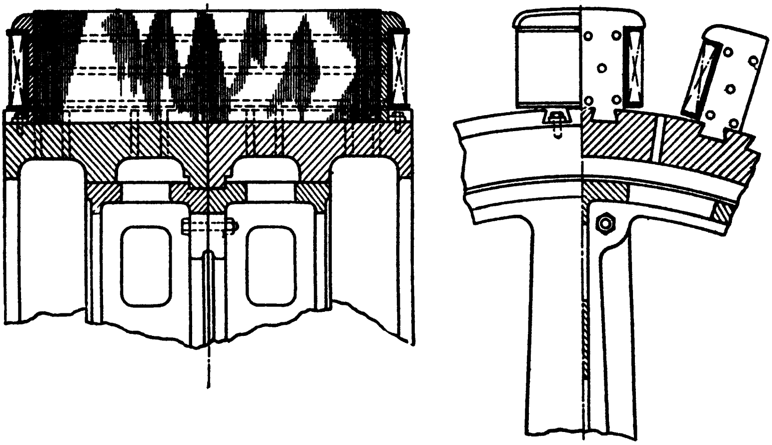

FIG. 603*a*.—Pole construction of Fig. 603.

The poles, of ¼-in. punchings riveted between heavy cast-steel end-plates, are attached by three T-shaped dove-tails designed to withstand the stresses due to 85 per cent. overspeed without exceeding half the elastic limit of the material. The field coils are of copper strip wound on edge with asbestos

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[1] "Hydro-electric Development at Niagara Falls," by J. L. Harper and J. A. Johnson, *Trans. Amer. I.E.E.*, Vol. 40, p. 881.

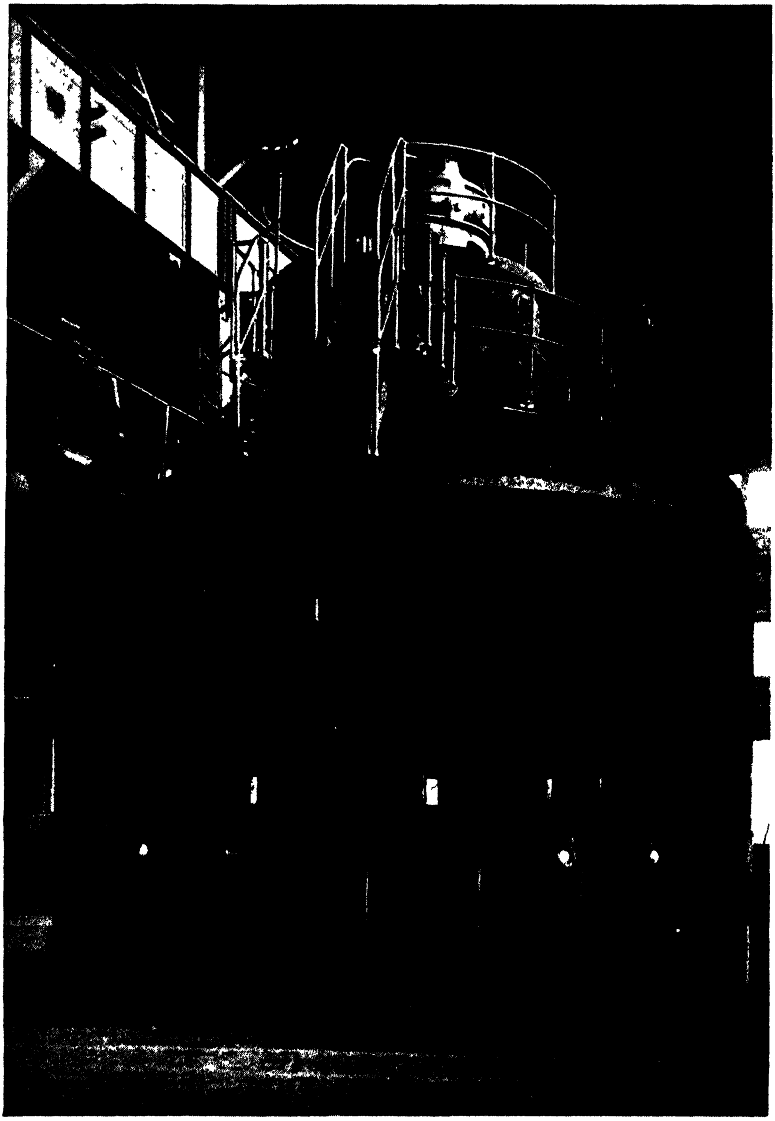

FIG. 604 —45,000 kVA three-phase generator in Queenston powerhouse, constructed by the Canadian General Electric Co.

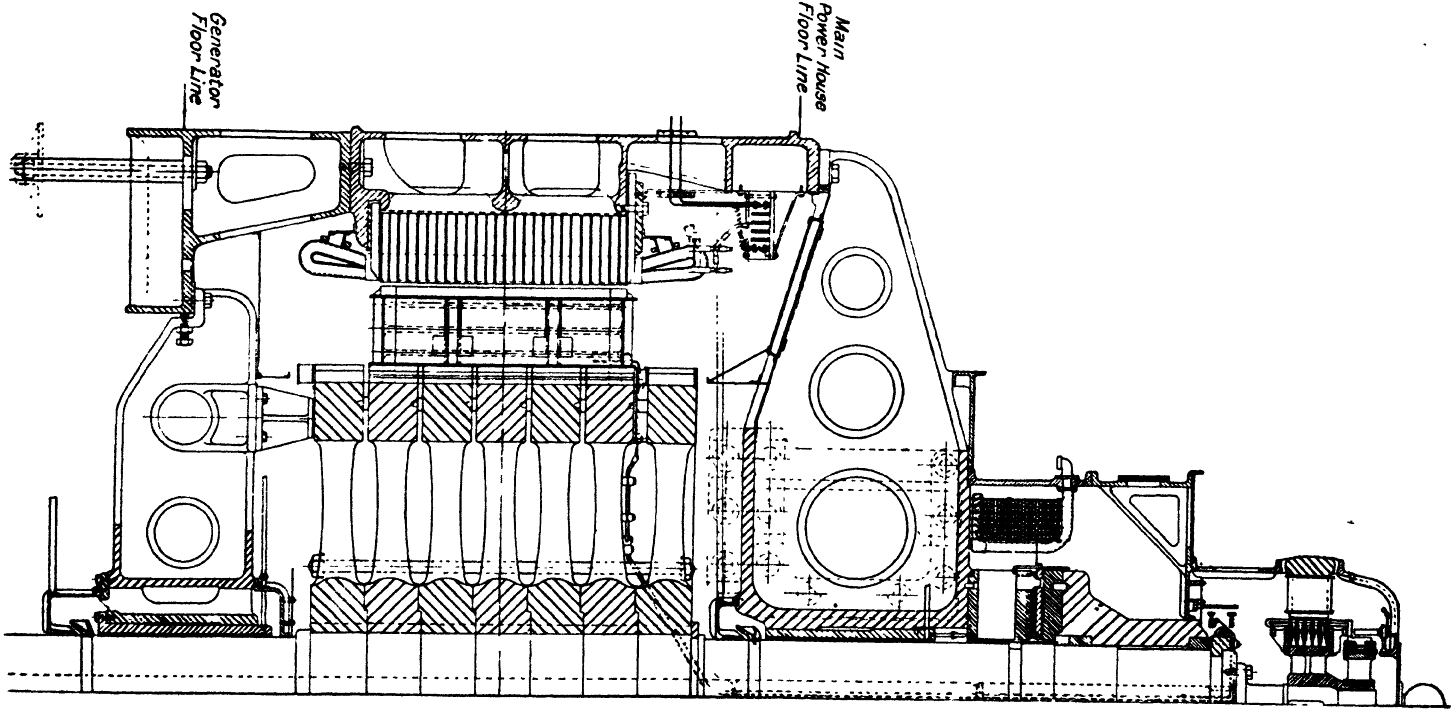

FIG. 605.—Half vertical section of 45,000 kVA alternator (Fig. 604).

and mica between adjacent turns, and between coil and pole-core, the fireproof character of the insulation being a precaution against possible fire arising from a fault in the stator winding. The weight of each coil is 2600 lb., and of pole and coil together 5 tons. At each end of the rotor are curved fan blades, and baffle plates and covers round the ends of the stator windings prevent re-circulation of the air. As much as 120,000 cub. ft. of air per min. is drawn in by the rotor through ducts from outside the station or from the generator room, is expelled through the openings in the stator frame into a chamber surrounding the generator, and is then exhausted by a fan through a ventilating shaft through the roof or into the building for heating purposes.

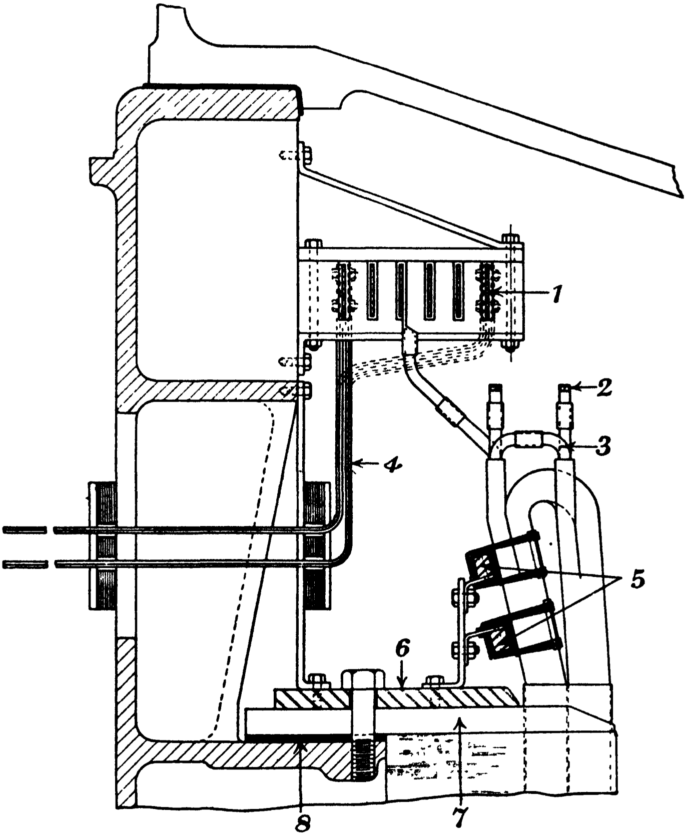

FIG. 606.—Stator coil bracing rings and phase bus-rings of 45,000 kVA alternator.

1. Phase bus rings.
2. Connexions from pole to pole.
3. Group connexions.
4. Terminal leads.
5. Bracing rings.
6. Clamping flange.
7. Clamping finger.
8. Removable plates to take up settlement of core.

The stator winding is of double-layer barrel type, with two coil-sides per slot. After the coil of subdivided copper conductor was formed, each turn was insulated throughout by hand with mica tape. To obtain a fractional pitch with a view to eliminate harmonics in the E.M.F. wave, the coils are chorded. The projecting ends are braced against one another by small wooden spacers placed between the sides of the coils, and are then bound with treated cord to two complete steel rings at each end, supported from the frame by iron brackets (Fig. 606). The phase connexions are made by flexible connecting leads to bus rings within the stator frame.

At the top of the generator is the 150 kW 250-volt direct-coupled exciter, shunt-wound with commutating poles. The full-load overall efficiency at 80 per cent. power factor is slightly over 97 per cent. With 40° C. in the surrounding air, the maximum observable temperature rise is guaranteed not to exceed 65° C.; it is ascertained by thermo-couples and indicated on a control pedestal near the generator and also in the control room.

The total weight of the rotor is 310 tons, and of the generator with exciter 625 tons.[1]

In the case of the three machines constructed for the same output by the Westinghouse Co., the rotor is of different construction. The inner cast-steel spider has no rim, but to its arms are dovetailed overlapping $\frac{1}{16}$-in. rolled steel plates, so as to form a rim of material of proved and uniform quality. The rim is further clamped between heavy steel end-plates by bolts passing through the whole. Two large air-ducts pass radially through the central part of the spider.

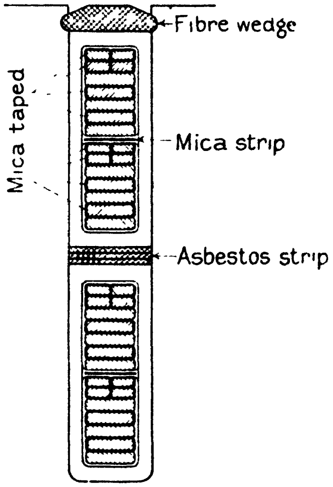

FIG 607.—Slot of 45,000 kVA alternator.

(Westinghouse Co., U.S.A.)

The stator, which is divided vertically into four 90° sections, has 19½ slots per pole. Each slot contains two coil-sides in an upper and lower layer respectively, and in each coil-side there are two conductors. Each of these conductors is subdivided into 9 divisions in parallel, i.e. into 7 layers, the upper two of which are again subdivided across the width of the slot. The divisions are maintained continuously throughout the length of the coil, and insulation between them is ensured by mica tape on those marked in Fig. 607. The group of wires forming a single conductor was insulated with mica tape, and mica strips were placed between the two conductors forming a coil. The straight part of the coil-side was then brushed with bakelite and hot-pressed to solidify it. A wrapper of mica pasted to very thin paper to hold it was then loosely applied to the straight part by hand and subsequently ironed on by electrically-heated irons revolved round it. Lastly, the projecting end-connexions were insulated with mica tape and varnished cloth tape on the outside to seal the whole.[2]

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[1] B. L. Barns and F. Bowness, *Trans. Amer. I.E.E.*, Vol. 41, p. 491; and F. A. Gaby, *Trans. Amer. I.E.E.*, Vol. 41, p. 481.

[2] "Design of 45,000-kVA Generators, Queenston Plant," by R. A. McCarty and H. U. Hart, *Trans. Amer. I.E.E.*, Vol. 41, p. 496.

§ 22. 65,000 kVA water-driven alternator.—In 1924 there was put into service for the Niagara Falls Power Company the largest hydro-electric unit that has been built up to the present (1925). The three-phase alternator,[1] (Fig. 608), constructed by the General Electric Co., Schenectady, N.Y., has a rated output of 65,000 kVA at 0·8 power-factor, 12,000 volts, 25 cycles, 107 revs. per min., so that its volt-amperes per rev. per min. reach the high figure of 607,000; but it is capable of giving continuously 13,200 volts, 68,250 kVA at 0·8 power-factor, and was designed with special view to the highest efficiency, the losses at the full rated load being approximately 2 per cent. of the output.

The cast-iron stator frame is made in 4 sections, measuring 10 ft. in height and 22 ft. across the arc. The core laminations, of silicon steel enamelled on both sides, are held by keys screwed to the ribs of the stator frame, and were assembled on site with staggered breaks to form a complete circle without stator joints, and with I spacing blocks to give ventilating ducts. The heavy end-flanges at top and bottom are clamped together by bolts running the entire length of the core between core and stator frame. The double-layer winding consists of 360 coils, so that the number of coils per pole, $12\frac{6}{7}$, is fractional to reduce harmonics; each coil of 3 turns, spanning 70 per cent. of the pole-pitch, is wound of 36 rectangular wires in parallel, arranged four abreast and nine deep in the slot. Each phase has four branches in parallel. The insulation of the turns and of a coil as a whole was designed to eliminate air-pockets and avoid internal corona; it is of mica tape, giving some flexibility during assemblage or in case of removal in the event of any repairs becoming necessary. The coils were moulded to size for insertion into the deep radial slots with open mouths closed by wooden wedges. Each complete insulated coil weighed 186 lb.

The rotor spider consists of 5 cast-steel wheels, each in two halves fastened at each joint of the rim by two nickel-steel shrink keys and at the nave by nickel-steel shrink bolts. Between adjacent wheels there are circumferential ventilating spaces at the rim. The arms of the top and bottom wheels are partly covered with steel plates to reduce windage and increase the flow of air through the rim. Each of the 28 poles consists of sheet-steel laminations, $\frac{1}{8}$ in. thick, clamped between cast-steel end-plates by through-bolts, and has two nickel-steel keys running through its base and through the end-plates; into each of these keys enter 10 nickel-steel tap bolts passing radially through the rim of the wheel, the pole being thus fastened to each of the 5 wheels by 4 bolts. The field coils are of copper strap 0·43 in. thick by $2\frac{5}{8}$ ins. deep, wound on edge,

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[1] See "The 65,000–Kv-a Generator of the Niagara Falls Power Co," by W. J. Foster and A. E. Glass, *Journ. Amer. I.E.E.*, Vol. 43, p. 365, from which the following particulars are abbreviated.

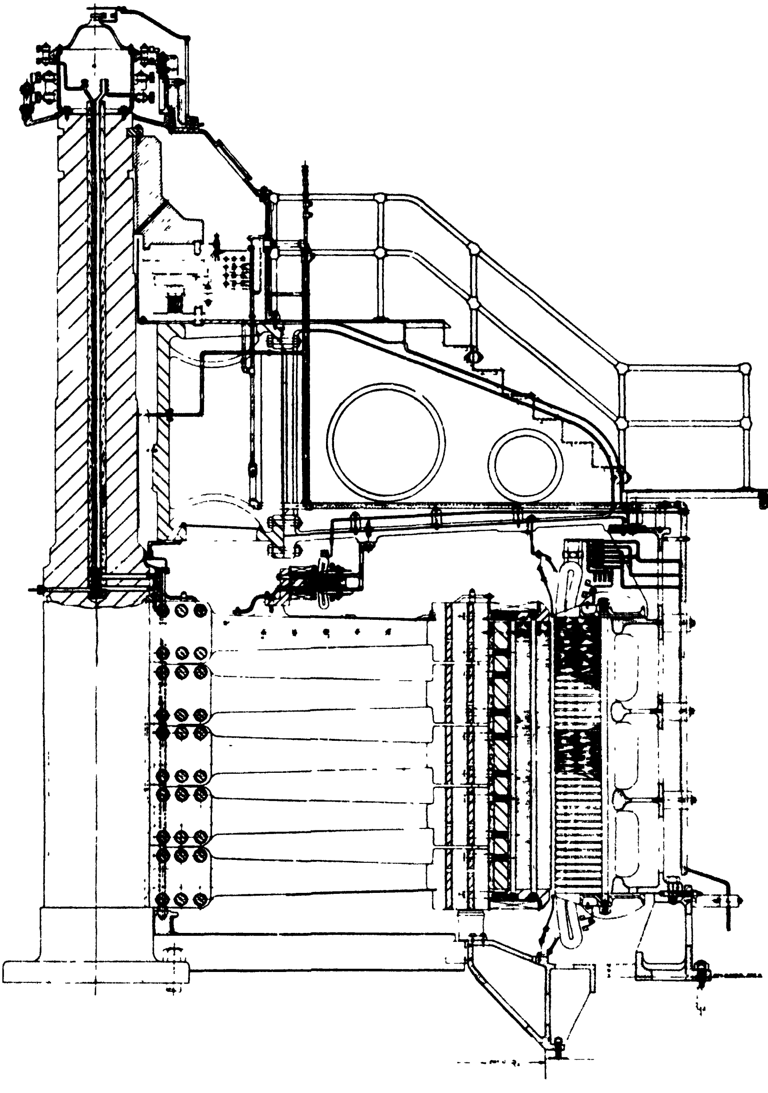

FIG. 608.—Half cross-section of 65,000 kVA alternator.

(General Electric Co., U.S.A.)

the weight of copper per pole being 2800 lb. and of pole and coil 10,900 lb. Supporting brackets between the poles prevent outward bulging of the coils. Special tests were made at 214 revs. per min. to meet the requirement that the rotor should withstand an overspeed of twice the normal speed. The rotor, complete with shaft, weighed 350 tons, and this, together with the weight of the turbine runner—a total of approximately 535 tons—is supported by a single Kingsbury thrust bearing, resting on the upper bearing bracket which also carries the generator guide bearing. The central hub of this bracket is a steel casting; to it are rabbeted, doweled and bolted 10 radial arms of cast-steel, which at their ends are fastened into the stator frame by steel bolts. Brackets and bolts are insulated from the stator frame to prevent pitting of the guide bearing by stray currents. The stator frame rests on a base-ring, cast in 4 sections, and of smaller diameter within this is a second ring, which supports the brakes for shutting down the rotor, and also serves to take the weight of rotor and runner when jacks are applied under the rotor rim to relieve the weight on the thrust bearing.

A novel feature is the introduction of a 650 kVA 2200-volt alternator, with stator suspended from the upper bearing bracket, and with rotor attached to the arms of the uppermost wheel of the main generator. The auxiliary generator drives the motor-generator set that excites the field of the main generator and supplies power for other motor-driven auxiliaries. Above the thrust bearing are mounted two pairs of slip rings, the one pair to supply exciting current to the 650 kVA alternator, and the other to supply current to the main rotor field by means of rectangular copper strips running down a hole in the rotor shaft and of cables running outwards along the arms of the rotor spider. At the extreme top is a speed-limiting device. The over-all diameter of the generator is 30 ft. 4 ins., and its over-all height from coupling face 26 ft. 4½ ins. The weight of the stator frame, core, and winding is 210 tons, and of the whole generator, complete with excitation generator, approximately 686 tons. The WR^2 is 65,000,000 lb.-ft.^{2}—numerically the same as the volt-amperes of the generator.

The armature reactance is calculated to be 26 per cent., and for the full-load rated current on short-circuit the field amperes are slightly in excess of the amount (approximately 600 amperes) required to give 12,000 volts on no-load.

§ 23. Calculation of leakage permeance with salient poles.—Following in many respects the analysis of Messrs. R. E. Doherty and O. E. Shirley[1] and the facts represented in Fig. 237 (Vol. II, p. 452), the leakage permeance in salient-pole machines may be considered under the following heads—

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[1] "Reactance of Synchronous Machines and Its Applications," *Trans. Amer. I.E.E.*, Vol. 37, Part II, pp. 1275–1278.

(1) Between the pole-tips along the polar edges, much of the flux enters the stator as an interpolar fringe. For the average depth of the *leakage* path Messrs. Doherty and Shirley give the approximate expression $d_p = d_t - c/2$, where c is the half interpolar gap (Fig. 609, *cp.* Fig. 272)—a quantity which with thin pole-tips and low ratio of polar arc to pole-pitch may become zero or negative, and is then to be neglected. In relation to $1{\cdot}257\ AT_p \times 2$ and reckoning for both edges of a pole, the leakage permeance with L_f in inches is

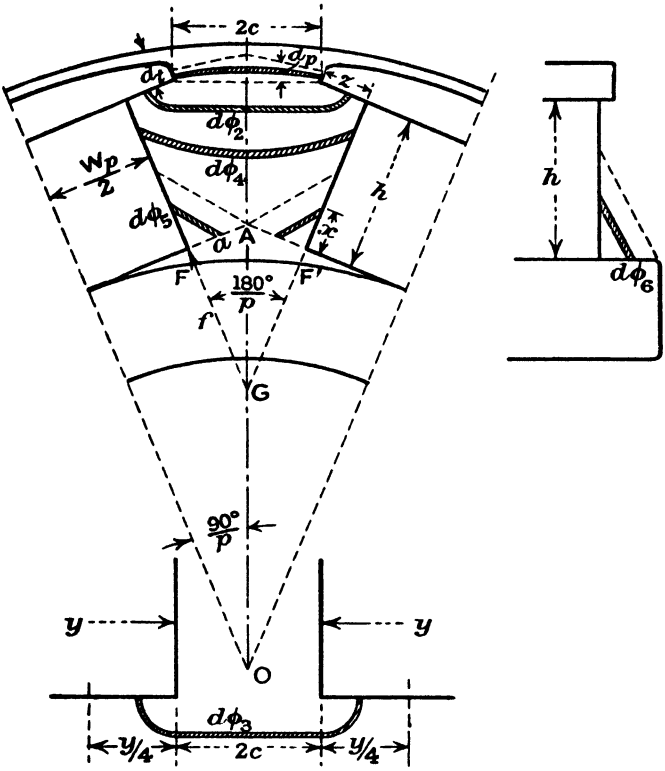

FIG. 609.—Calculation of leakage permeance in salient-pole machine.

$$\mathcal{S}_1 = \frac{d_p}{2c} \times L_f'' \times 2{\cdot}54 \times 2$$

(2) From the underside of the overhanging pole-tip of width z in curves and straight lines, by equation $(122a)$[1]

$$\mathcal{S}_2 = 1{\cdot}86 \times \frac{p}{p-1} \log_{10} \frac{\pi z\,(p-1)/p + 2c}{2c} \times L_f'' \times 2$$

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[1] For more accurate expressions, see Prof. W. Cramp and N. L. Calderwood *Journ. I.E.E.*, Vol. 61, p. 1061.

For if the two undersides of adjacent poles are produced until they meet, the angle enclosed by them is $180^\circ\left(1-\frac{1}{p}\right)=\pi\frac{p-1}{p}$

(3) From the middle of the flanks of the pole-tips, the flux bends upwards into the stator core and cuts the armature winding, so that it is not to be reckoned as leakage. The leakage flux across from pole-tip to pole-tip may be assumed to end at a distance from the pole-tip of about one-quarter of the polar arc, v. The permeance of the 4 strips, two on each side of a pole, is then by equation (122*a*)

$$\wp_3 = 1\cdot86\log_{10}\frac{\pi v/4 + 2c}{2c}\times d_p''\times 4$$

(4) Along the pole sides the leakage flux divides (Fig. 237), part crossing the gap into the adjacent pole of opposite sign in curved lines, and part bending down into the yoke. The line of demarcation may approximately be set at a distance from the pole-base of $h/\sqrt{p}$. The depth of the pole-core from which the former portion of the flux springs is thus $h\left(1-\frac{1}{\sqrt{p}}\right)$. The path of the present flux is curved inwards to the yoke, but its length with 6 or more poles is not far from equality with the length of the outward-curving arc of a circle intercepted between the pole-sides, and struck from the centre G at which the pole-sides meet when produced (Fig. 609). The angle $FGF' = 180^\circ/p$, and the length of such an arc $= (f+x)\times\pi/p$, where x is measured upwards from $f = FG$. The latter without reference to a drawing is approximately

$$f = 0\cdot47\,D - h - \frac{W_p/2}{\tan\,(90^\circ/p)}$$

where W_p is the width of the pole-flank and itself is approximately one-half of the pole-pitch, or $\pi D/4p$. An element of the leakage flux across from pole to pole is then

$$d\phi_4 = 1\cdot257\,AT_p\times 2\times\frac{x}{h}\times\frac{B\,dx}{(f+x)\,\pi/p}$$

where B is the length of the pole-core along the axis of the machine in cm. When integrated between the limits $x = h/\sqrt{p}$ and $x = h$, the total flux leaking from *both* sides of the pole is

$$1\cdot257\,AT_p\times 2\times\frac{p}{\pi}\left\{\left(1-\frac{1}{\sqrt{p}}\right)-2\cdot3\frac{f}{h}\log_{10}\frac{f+h}{f+h/\sqrt{p}}\right\}$$
$$\times B''\times 2\cdot54\times 2$$

The whole of this flux does not, however, pass through the entire length of the pole-core. If the linkages of the flux are calculated and divided by T_f, an equivalent flux passing through all the turns on the pole is obtained ; the corresponding leakage permeance in parallel with that across the pole-shoes is then

$$\mathcal{P}_4' = \frac{p}{\pi}\left[\frac{1}{2}\left(1-\frac{1}{p}\right)-\frac{f}{h}\left\{\left(1-\frac{1}{\sqrt{p}}\right)-2.3\frac{f}{h}\log_{10}\frac{f+h}{f+h/\sqrt{p}}\right\}\right] \times B'' \times 2.54 \times 2$$

This will, however, under-estimate the ampere-turns required, since with the rising density the ampere-turns per cm. length increase fairly rapidly, and the length of air-path not increasing greatly towards the top of the pole, the higher proportion of AT_P yields the greater part of the flux near the top. Further, the depth of air through which the present flux passes at the centre of its path is somewhat more than the depth of iron from which it springs, although the difference is not great owing to the fact that when the number of poles is few, and the flux (2) from the overhanging pole-tip is large, it occupies a considerable portion of the space between the pole-cores. The larger value of the equivalent permeance as first calculated[1] may thus approximately be retained, and regarded as in parallel with the preceding items is

$$\mathcal{P}_4 = \frac{p}{\pi}\left\{\left(1-\frac{1}{\sqrt{p}}\right)-2.3\frac{f}{h}\log_{10}\frac{f+h}{f+h/\sqrt{p}}\right\} \times B'' \times 2.54 \times 2$$

(5) For the remaining flux from the pole-side, the base $FA = a = f \tan (90°/p)$, and as the number of poles increases, it quickly approaches the half of the length FF'. For any height x up the pole-core, the length of an assumed straight path is then equal to x increased in the ratio $\frac{\sqrt{a^2+h^2/p}}{h/\sqrt{p}} = \frac{\sqrt{a^2p+h^2}}{h}$. An element of the flux is then

$$d\,\phi_5 = 1.257\,AT_P \times \frac{x}{h} \times \frac{h\,Bdx}{x\sqrt{a^2p+h^2}}$$

Integrating from $x=0$ to $x=h/\sqrt{p}$, the equivalent permeance from *both* sides in relation to $AT_P \times 2$ is

$$\mathcal{P}_5 = \frac{h/\sqrt{p}}{\sqrt{a^2p+h^2}} \times B'' \times 2.54$$

(6) For the similar leakage from the pole-flanks into the field yoke, the height up the pole-core to be considered is on the average[2] about $\frac{3}{4}h$. Beyond this, the flux bends up into the stator core and mostly cuts the armature winding. Reckoning the length of

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[1] The expression being the difference of two quantities, when p is large, the ratio of which the logarithm is taken must be accurately calculated, or preferably the logarithm of the denominator subtracted from that of the numerator. For parallel pole-cores of circular cross-section, see B. Hague, *Journ. I.E.E.*, Vol. 61, p. 1072.

[2] As given by Messrs. Doherty and Shirley.

the downward path roughly as $1\cdot3\,x$, the equivalent permeance from *both* flanks is in relation to $AT_p \times 2$

$$\mathcal{P}_6 = W_p \int_0^{\frac{3}{4}h} \frac{x}{h} \times \frac{dx}{1\cdot3\,x} = 0\cdot576\, W_p'' \times 2\cdot54$$

When the fluxes resulting from the last items are added as passing through the whole length of the magnet core, their effect is over-estimated. They are not, however, of great amount, and the use of the equivalent permeances that would yield linkages with T_f equal to the actual number, viz.

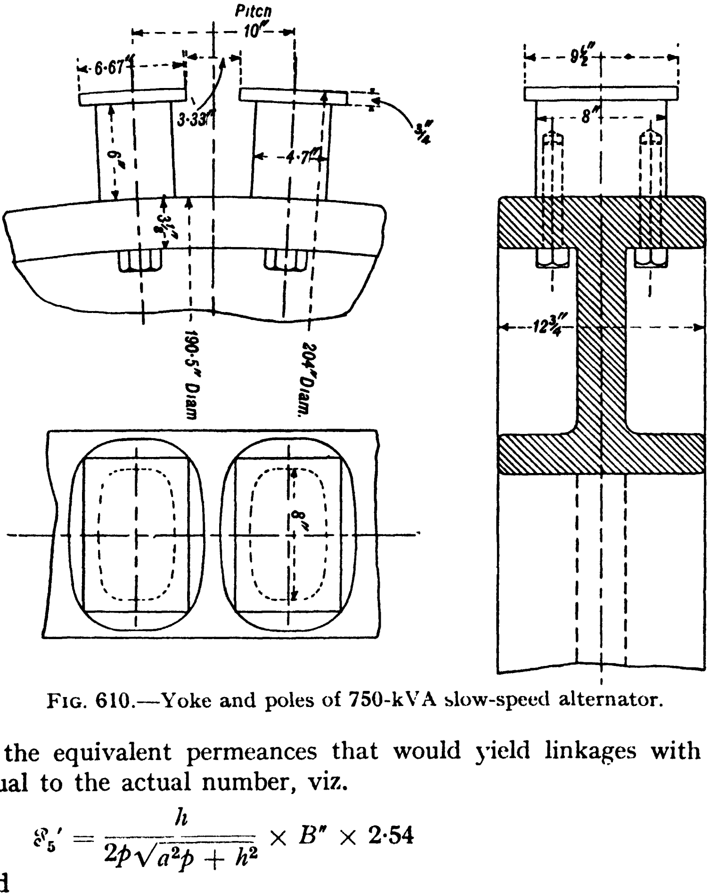

FIG. 610.—Yoke and poles of 750-kVA slow-speed alternator.

$$\mathcal{P}_5' = \frac{h}{2p\sqrt{a^2p + h^2}} \times B'' \times 2\cdot54$$

and

$$\mathcal{P}_6' = \int_0^{\frac{3}{4}h} \frac{x^2}{h^2} \cdot \frac{dx}{1\cdot3\,x} W_p'' \times 2\cdot54 = \frac{1}{1\cdot3} \times \frac{9}{32} W_p'' \times 2\cdot54$$
$$= 0\cdot216\, W_p'' \times 2\cdot54$$

gives such reduced values as to lead to an under-estimate of the effect of the saturation at the root of the pole.

As an illustrative example, take the magnetic circuit of a slow-speed 64-pole 750 kVA alternator in Fig. 610.

(1) The air-gap being 0·375 in., $d_p = d_t - c/2 = 1{\cdot}125 - 0{\cdot}833 = 0{\cdot}292$ in., and

$$\mathfrak{P}_1 = \frac{0{\cdot}292}{3{\cdot}33} \times 9\tfrac{1}{2} \times 2{\cdot}54 \times 2 = 4{\cdot}24$$

(2) The width z of the over-hanging pole-tip $= \tfrac{1}{2}(6{\cdot}67 - 4{\cdot}7) = 0{\cdot}985$ in. Thence

$$\mathfrak{P}_2 = 1{\cdot}86 \times \frac{32}{31} \times \log \frac{\pi \times 0{\cdot}985 \times 0{\cdot}97 + 3{\cdot}33}{3{\cdot}33} \times 9\tfrac{1}{2} \times 2 = 10{\cdot}2$$

(3) The polar arc y being 6·67 ins.,

$$\mathfrak{P}_3 = 1{\cdot}86 \log \frac{\pi \times 1{\cdot}67 + 3{\cdot}33}{3{\cdot}33} \times 0{\cdot}292 \times 4 = 0{\cdot}9$$

(4) The angle 90°/32 being 2·81°, its tangent = 0·049, and

$$f = 95{\cdot}25 - \frac{2{\cdot}35}{0{\cdot}049} = 47{\cdot}25 \text{ ins.}$$

The division of the flux occurring at $h/\sqrt{p} = h \times 0{\cdot}177 = 1{\cdot}06$ in.

$$\mathfrak{P}_4 = \frac{32}{3{\cdot}14}\left\{0{\cdot}823 - 2{\cdot}3\,\frac{47{\cdot}25}{6}\log\frac{53{\cdot}25}{48{\cdot}31}\right\} \times 8 \times 2{\cdot}54 \times 2$$
$$= 10{\cdot}2\,(0{\cdot}823 - 0{\cdot}766) \times 40{\cdot}6 = 23{\cdot}6$$

(5) The length $a = f \tan 2{\cdot}81° = 2{\cdot}32$ ins.

$$\mathfrak{P}_5 = \frac{1{\cdot}06}{\sqrt{32 \times 2{\cdot}32^2 + 6^2}} \times 8 \times 2{\cdot}54 = 1{\cdot}49$$

(6) The width of the pole being $W'_p = 4{\cdot}7$ ins.,

$$\mathfrak{P}_6 = 0{\cdot}576 \times 4{\cdot}7 \times 2{\cdot}54 = 6{\cdot}88$$

The total leakage permeance is thus

$$\mathfrak{P}_l = \mathfrak{P}_1 + \mathfrak{P}_2 + \mathfrak{P}_3 + \mathfrak{P}_4 + \mathfrak{P}_5 + \mathfrak{P}_6$$
$$= 4{\cdot}24 + 10{\cdot}2 + 0{\cdot}9 + 23{\cdot}6 + 1{\cdot}49 + 6{\cdot}88 = 47{\cdot}31, \text{ say } 50.$$

§ 24. The exciting coils.—In the case of revolving field-magnets, the exciting coils, owing to their high peripheral speed, are best formed of thin copper strip wound on edge with paper insulation, about 0·01 in. thick, coiled round between the turns (Fig. 611); the effect of the centrifugal force is then only to compress the whole spiral towards the periphery and the insulation does not suffer by chafing. After varnishing between the layers, the whole can be baked at a high temperature and under considerable pressure, so that the winding becomes a solid mass. Copper band wound on the flat is not so economical, since a number of turns of thin sheet nearly equal in width to the length of the poles implies a considerable amount of space lost in insulation. Round wires should never be employed on rotating magnets, owing to the danger of the wire slipping on itself and damaging the insulation. Coils of rectangular or square wire necessitate insulating bobbin cases, and are not so self-supporting as spirals of copper strip. The coils or their cases are held in place by the overhanging edges of the pole-shoes, or by special gun-metal bridges, which also serve in some slight measure to steady the machine for parallel working by their damping action. The current-density in the field wire usually ranges from 1300 to 1900 amperes per sq. in., or higher than in continuous-current dynamos, owing to the greater cooling action of the rotating magnet. With rotating field-coils the influence of the peripheral speed is more than in Fig. 398, though less than in Fig. 399, owing to the greater depth of winding. The power of the peripheral speed which should be employed is not far different from that of equation (212), Fig. 399, but with a lesser coefficient of, say, 0·5 instead of 0·8. For the ***mean*** rise of temperature of an ordinary bobbin wound with d.c.c. wire in a number of layers, and, therefore, with considerable depth of winding, the heating coefficient in terms of degrees Centigrade and sq. ins. of cooling surface may then approximately be given as

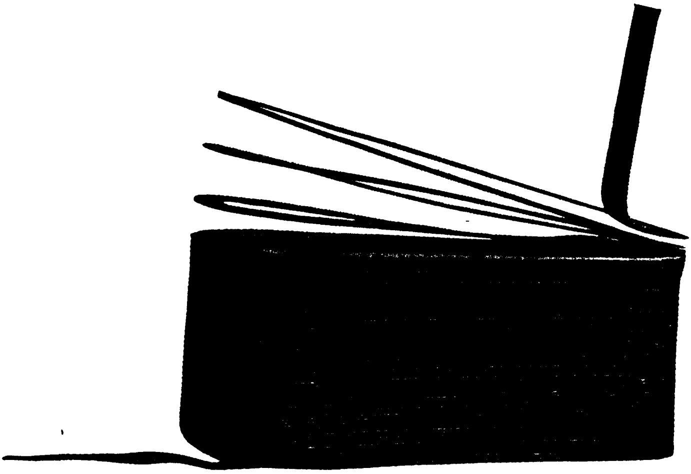

FIG 611.—Field-coil of flat strip wound edgewise.

$$k = t^{\circ} C \times \frac{S_c}{W} = \frac{144}{1 + 0{\cdot}5\left(\frac{v}{1000}\right)^{1{\cdot}2}} \quad . \quad . \quad (269)$$

as plotted in the upper dotted curve of Fig. 612. Here v is the *mean* peripheral speed at the centre of the height of the bobbin, and S_c is the cooling surface of the sides and ends of the coil winding, *plus* the surface of the curved ends only of one flange at one end of the pole. The end flange abutting on the seating of the pole is of less value, and the influence of the flanges as a whole is less than in continuous-current machines (Chapter XXI, § 12). At 5000 ft. per min. mean speed with cooling surface thus reckoned, a ratio of about 0·8 to 1 sq. inch per watt will give a mean rise of 35 to 40° C., when the axial dimension of the pole parallel to the axis of the rotor is about equal to the pole-pitch, as is usual from considerations of economy in the field copper.

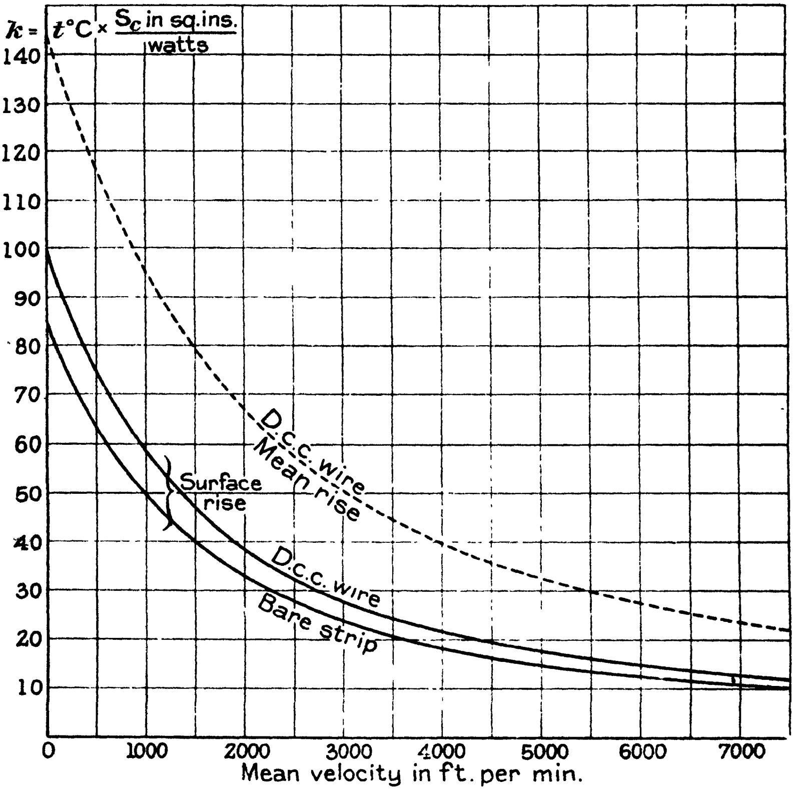

FIG. 612—Heating coefficient of rotating field-magnet coils.

But though indicating the order of the figures, the above does not take separate account of a greater or lesser axial dimension of the pole, or of variation in the number of poles.

The sides of the coils are for a given area by no means so effective in dissipating the heat as the ends which are fully exposed to the powerful fanning action which arises from their rotation. Consequently if the axial length on a machine of given cross-section and poles is trebled and the periphery of a coil is thereby doubled, the same heating coefficient will not hold; its value, if, say, 20 for the short machine, must be raised to, say, 25 for the long machine.

As the number of poles is increased, the sides of the coils approach closer, and become more nearly parallel; the air then tends to be imprisoned between the coils, its temperature is raised, and the cooling of the sides is not so good as in the machine with few widely diverging poles.

The combined effect of both influences may be taken approximately into account[1] by assuming only the fraction $\sqrt[3]{s/l}$ of the side cooling surfaces to be on a par with the end surfaces and equally effective, s being the *mean* distance between the sides of neighbouring coils of width l. The calculated cooling surface being thus greatly reduced, the constant must also be reduced, and the two lower curves of Fig. 612 give approximately the *surface* temperature-rise when only the above fraction of the sides is used in conjunction with the full value of the end-surfaces and end-flange, for the two cases of d.c.c. wire coils and flat copper strip presenting bare edges to the air, plotted respectively from the expressions

$$k = \frac{100}{1 + 0{\cdot}7\left(\frac{v}{1000}\right)^{1{\cdot}2}} \text{ and } \frac{85}{1 + 0{\cdot}7\left(\frac{v}{1000}\right)^{1{\cdot}2}} \quad . \quad (270)$$

Bare copper strip has an advantage of about 20 to 30 per cent. in the matter of heating, the difference of temperature between the hottest section and the outside being greatly reduced.

In high-speed machines, especially for a frequency of 25, the number of poles is small, and for small outputs of, say, 500 kW, their neighbouring sides slant inwards so much that "stepped" coils, wound with an increasing number of layers in steps towards the pole-shoes, become necessary. In such cases for the reason above given the ventilation is poor owing to the sides of the coils becoming closely crowded together, and it becomes difficult to put enough copper in the coils to keep the temperature rise within reasonable limits, especially at low power-factors. For high-speed rotors driven by water-wheels, Messrs. Ganz & Co.[2] have employed field coils thoroughly ventilated by numerous holes passing through the coil from top to bottom (Fig. 588), and thereby have effected a saving in weight.

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[1] See Prof. Miles Walker (*Specification and Design of Dynamo-electric Machinery*, pp. 233, 234), who introduced as a factor the square root $\sqrt{s/l}$ for use with a different and more detailed method of calculating the relative efficiency of the surfaces, external and internal, in dissipating the heat.

[2] *Electr.*, Vol. 73, p. 894.

In large slow-speed machines for 50 frequency and 1000 kW or more, the excitation is fixed rather by the efficiency than by the heating. The voltage of the exciting current varies according to circumstances from 50 to 200 volts, yet good insulation of the entire circuit of coils and exciter is necessary, since owing to the great inductance of the circle of bobbins there is a considerable strain on the insulation when the circuit is broken ; indeed, on this account the opening of the circuit should be effected through a non-inductive shunt and carbon-break switch.

§ 25. Methods of field excitation.—The field-magnet system of the alternator must necessarily be excited by a continuous current. Separate excitation by an auxiliary continuous-current dynamo or *exciter* is, therefore, almost universally employed, its great recommendation being that the exciting current of any alternator can readily be altered by means of a rheostat in series with its field-winding, or in the field-circuit of the exciter.

The alternating current supplied by the alternator itself can also be used to excite the field if *rectified* or converted into a direct form ; the machine can then be made self-exciting, or, if given an initial excitation from a separate source, a compounding effect is obtained which increases with the load just as in the compound-wound continuous current dynamo. Two sets of magnet bobbins are then required, the one separately excited and the other self-excited by the armature current. But none of the methods which have been proposed for compounding alternators have found much favour in practice, and are not, therefore, here described.[1] While the series turns may be adjusted to suit the normal load and normal load-factor so as to maintain the voltage fairly accurately, there remains the difficulty that the compounding effect is really required to vary not merely with the value of the current, but also with the angle of lag of the current according as the external load is inductive or non-inductive. Hence an alternator which is adjusted for a load of lamps is not so well adjusted for a load of motors, and will require an alteration of its separate excitation.

§ 26. Excitation and exciters.—Standard exciter voltages range from 125 volts for small and medium-sized plants to 250 volts for large waterwheel-driven generators ; the choice is largely determined by consideration of the necessary area of the copper strap, which may become too thin or too thick to be wound easily on edge, or of the loss of space by the insulation of many turns of small wire. For waterwheel-driven generators the kW of excitation required, expressed as a percentage of the kVA rating of the alternator, range from 1½ per cent. to ½ per cent. in the cases of a 1000 kVA and 20,000 kVA slow-speed generator respectively. In every case the full-load rating of the exciter must allow of a good margin over the full-load excitation to meet abnormal emergencies, say, 10 to 15 volts for 125-volt excitation or 20 to 30 volts for 250-volt excitation.

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[1] See Prof. Miles Walker, *Journ. I.E.E.*, Vol. 34, p. 417.

The exciters may be either individual to the generator and in this case may be either directly connected to the generator shaft as usually in hydro-electric units or driven by motor, separate steam engine or turbine ; or common to several alternators, a number of exciters feeding in parallel into common bus bars, and each alternator having its own field-regulating resistance. Direct coupling to the shaft of the alternator has the disadvantage that any change of speed of the prime mover also causes a change of speed and of voltage in the exciter.[1] Reliability of the exciting system is the first consideration, and has led to the settlement of the question in many different ways,[2] sometimes in combination with the driving of auxiliary plant off the exciter bus bars.

Usually the exciter is shunt-wound, but here again there is room for difference of opinion and practice according to the circumstances of each case,[3] and compound-wound exciters also find advocates.

When the armature reaction of the generator approaches the ampere-turns of excitation for full voltage at no-load, the range of exciting current will be about 1½ to 1 at unity power-factor, or 2 to 1 at 80 per cent. power-factor. It is evident, therefore, that when the alternator excitation is controlled not by a rheostat in the main alternator field but by the exciter field rheostat, the individual exciter must work over a voltage range of 2 to 1, or, to allow of some margin, of 2½ to 1. And this condition is usually met by the automatic-voltage regulator. But in this connexion it is very important that the exciter at its lowest voltage should not approach too closely to the straight-line part of its magnetization curve. The low degree of saturation then implies a danger of instability,[4] by reason of which there may be " creeping " of the voltage, up or down, without apparent cause; working on the straight part of the curve (*cp.* Fig. 291, and Vol. I, Chapter XVII, § 11) is in fact only possible through the presence of residual magnetism.

If the exciter voltage is lowered still further, as by the increase of the exciting current which follows upon a short-circuit of the alternator, or by a sudden decrease in the resistance of the alternator field rheostat, the energy stored in the alternator field in virtue of its large inductance will maintain the field current at a higher value than that at which the voltage and the stored energy of the exciter field would by themselves maintain it. Hence when the flux of the exciter has died away to zero, current is maintained through its armature by the voltage of the alternator field; the voltage across the armature and, therefore, also across the shunt field is then reversed, and consequently as voltage is recovered, it is built up with reversed polarity. The not infrequent occurrence of this trouble or in a lesser degree of oscillation of the voltage from the same cause, shows the importance of securing complete stability by working the exciter above the bend of the saturation curve.

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[1] But for arguments in favour of direct-coupled exciters, *cp. Electr.*, Vol. 73, p. 895.

[2] For the *pros* and *cons*, see a number of papers in *Trans Amer. I.E.E.*, Vol. 39, Part II, pp. 1521-1644.

[3] For the arguments against the use of compound-wound exciters when they are operated in parallel, see J. T. Barron and A E. Banhan, *Trans. Amer. I.E.E.*, *loc cit.*, p. 1545 ; and for arguments in favour of series winding as a protection against instability under sudden short-circuits, etc., R. E. Doherty, *Trans. Amer. I E.E* , Vol. 41, p. 770.

[4] See " Exciter Instability," by R. E. Doherty, *loc. cit.*, p. 768, with bibliography.

On the other hand, the responsiveness of the exciter or the rapidity with which it answers to the automatic-voltage regulator of the vibrating contact type depends on the vertical distance between the inclined straight line (*OH* in Fig. 290, Vol. I, Chapter XVII, § 9) which is the characteristic of the shunt field circuit, and the volt-curve of the armature. So far as these diverge, the difference between the volts supplied by the armature and the volts acting at the moment on the shunt field is absorbed by a counter E.M.F. due to the inductive effect from the change in the flux linked with the field turns, *i.e.*, by $L\,di/dt$. The above vertical distance therefore, so far as L is constant, is a measure of the rate at which the field-current will build up in amperes per second.[1] If the field line approaches the straight part of the magnetization curve, the smallness of the divergence means that the exciter is sluggish in its response to the automatic-voltage regulator. The writer cited in the preceding note gives as desirable figures for a 250-volt machine a flux linkage not exceeding 15×10^9, so that when the actual machine is set for a maximum of 290 volts, it will in combination with a Tirrill regulator,[2] build up over a range of 60 to 250 volts in 4 seconds or under.

§ 27. The induction generator.—The induction generator, or, as it is sometimes called, "asynchronous generator," is an induction machine mechanically driven above the speed of synchronism. It has found some applications in connexion with steam or water turbines, as the simple and strong construction of its squirrel-cage rotor renders it specially suitable for high speeds, but since it also requires a large synchronous machine such as a converter to serve as an alternating exciter, and the latter must itself be excited by a direct current, the expense of the whole plant has up to the present prevented its commercial adoption.[1]

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[1] See C. A. Boddie, *Trans. Amer I.E E*, Vol. 39, Part II, p. 1620.

[2] For automatic voltage regulators, see *Hydro-electric Engineering* (Blackie & Son, Ltd.), Vol 2, Chapter VIII, by W. A. Coates, p. 151; *Brown, Boveri Rev.*, Vol. 10, p. 199.

If the stator coils of an induction motor are connected across mains between which an alternating voltage is maintained, and if further a prime mover is coupled to the rotor of the motor, by which its speed may be raised, the first result of so raising its speed is to reduce the current taken by the stator and to cause the driving torque from the motor to decrease until, when it is running at the exact speed of synchronism, no torque is exerted by the rotor and the prime mover simply supplies the losses from friction and windage, while the stator coils take only a magnetizing current from the mains. But when the speed is still further raised by the prime mover above that of synchronism, so that there is a negative slip and the rotor runs faster than the rotating magnetic field, currents are generated in the rotor at this slip frequency, and these in turn will induce watt currents in the stator, in phase with the voltage of the mains, and supplying part or the whole of the load on the mains. It is evident that for the operation of the system there must be some synchronous machine (either generator, motor, or converter) already running on the mains to supply the wattless magnetizing current that the machine is itself unable to supply, and to set the voltage and frequency of the system.[2]

§ 28. The standards of frequency.—The standard frequency adopted in Europe is 50, and in the United States of America 60, a frequency of 25 being also but less commonly used both in Europe and America.[3] While in the early days of electric lighting frequencies of 125 or 133 which suited small transformers well were not uncommon, these are no longer now found. A disadvantage of high frequencies in the case of engine-driven sets is that they do not work so well in parallel; the maximum angle by which it is permissible for the rotor to be mechanically displaced from the position corresponding to perfectly uniform speed of rotation varies inversely as the number of poles, and if the machines for high frequencies are to be run in parallel the engines must generally either be better balanced as regards their moving parts or have larger and more expensive fly-wheels. If the alternators are to be used for the supply of energy to polyphase induction motors, still another reason is found for the adoption of a low frequency, since with high frequencies the power-factor and efficiency of the motors are less, at least if their speed is to be moderate. Lastly, if rotary converters form a large part of the load, uniformity of the angular velocity becomes of special importance in order to prevent hunting.

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[1] Except occasionally in small water-power stations, F. Drexler, *E.u M* Vol. 40, p. 139; *cp.* W. L. Waters, *Trans. Amer. I.E.E*, Vol. 27, Part I, p. 157.

[2] The action of the induction generator, together with A. Heyland's method of rendering it self-exciting and of compounding it, is more fully described by A. F. T. Atchison, "Some Properties of Alternators under various Conditions of Load," *Journ. I.E.E*, Vol. 33, pp. 1093 1104.

[3] *Cp.* D. B. Rushmore, "Frequency," *Trans. Amer. I.E.E.*, Vol. 31, Part I, p. 955.

In the other direction there are, however, again certain limits. The eye can detect and is fatigued by a variation in the light from an incandescent lamp if the frequency is less than 30. Thus for systems combining the supply of both light and of energy to motors, and in which the former is an important part of the whole, a frequency of 50 is found to be the best compromise. Except on very long transmitting lines the reactance is then moderate, and the regulation of voltage is good, while polyphase induction motors can be built to run well and at reasonable speeds with such a standard. Rotary converters of large size can now be operated successfully at 50 or 60 cycles. At 60 cycles per second the higher speed necessitated for a given number of poles renders the mechanical design of large turbo-alternators a somewhat more difficult problem than at 50 cycles, and the design of the rotary converter at 60 cycles is also more difficult. But these difficulties have been practically overcome, the uniform rotation of the steam turbine as prime mover having greatly conduced to the success of the 60-cycle rotary converter.[1]

The frequency of 25 cycles per second, adopted after much discussion by the engineers for the original Niagara power plant, has proved of service in cases where a lower frequency may be advantageous, as *e.g.* for low-speed induction motors direct-connected to rolling mills in steel works. Frequency-changers for the purpose of exchanging power between 60- and 25- cycle systems have been built for as much as 35,000 kW.[2]

For heavy railroad work, single-phase commutator motors have a better power-factor, and are more easily designed for sparkless running, at still lower frequencies, and for such traction work $16\frac{2}{3}$ and 15 cycles per second have both been employed. Although not suited to the alternator driven by steam turbine, they are often obtained conveniently from water-driven plants. The question of the best frequency for railway work cannot, however, as yet be regarded as entirely settled, and when the supply is to be taken from neighbouring works, 25 or 50 may require to be considered in conjunction with non-commutator machines, such as induction motors, used with phase converters.

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[1] See B. G. Lamme, "The Technical Story of the Frequencies," *Trans. Amer. I.E.E.*, Vol. 37, Part I, p. 65.

[2] *Electr Journ.*, Vol. 21, p. 343, *Journ. Amer. I E.E*, Vol. 43, p. 709, and Vol. 42, p. 1324. If a 25-cycle synchronous machine is directly connected to an induction motor with 60-cycle stator and rotor held by the synchronous machine to run at 35 cycles, so that its slip frequency is 25 cycles per sec., power is transferred from the 60-cycle to the 25-cycle system partly through the shaft and partly as electrical output from the rotor of the induction motor. See also J. W. Dodge, *Gen. El. Rev.*, Vol. 26, p. 435.

§ **29. Bearing currents.**—A M.M.F. encircling the shaft causes flux to pass along the shaft, completing its circuit through the journals, pedestals, and base-plate. Thence, if the M.M.F. is constant, there results a *homopolar* E.M.F. in the rotating journals of the shaft, and currents through shaft and brasses, so far as breaks in the oil film or contact through end-play or oil-rings permit. But in addition a difference in the reluctance presented to the field-magnet at different positions in a revolution may virtually cause a changing flux to be linked with the electric circuit of the shaft, bearings and base-plate, and thence by transformer action there results an *alternating* E.M.F. along the shaft.[1] The latter is the more usual cause of shaft or bearing currents of such magnitude as to cause pitting of the brasses and scoring of the journals, and the change of linked flux giving rise to it is due to certain combinations of poles and joints in the stator core. A stator divided into sections by butt joints naturally is most prone to the trouble, but it may also occur in a core built up of discs in symmetrical segments; if each segment has two dovetails or keys, and the segments are laid round a circle, overlapping in alternate layers so as to break joint, there are twice as many joint-places as there are segments, and at the radial section through each joint-place the area of the iron is only one-half of the full amount for the core elsewhere. At high densities, therefore, the greater reluctance at the joint-sections has an effect exactly analogous to that of a butt joint. If $j =$ either the number of uniformly spaced butt joints in a sectionalized stator, or *twice* the number of segments in a stator built up of symmetrical segmental laminations, and $p =$ the number of pole-pairs, let F be the H.C.F. of j and p, and let $j = Fx$; then if x is odd, there will be shaft currents, of frequency x times the frequency of the magnetic flux alternations in the core, *i.e.*, in synchronous machines x times the machine frequency. If x is even, and the reluctances of all the joints are precisely alike, there will be no shaft currents. The latter condition for a satisfactory number of joints may also be stated as follows:—if $f =$ the highest odd factor in p (including p when its value is odd), shaft currents will be absent if $j = pn_1/f$, where n_1 is any integer.[2] By the use of dovetails or keys at $\frac{1}{8}$ and $\frac{5}{8}$ of the arc of a segment instead of at $\frac{1}{4}$ and $\frac{3}{4}$, four joints per segment are obtained instead of two, and a cross-section at the joint-place $\frac{3}{4}$ of the full amount,[3] and for many combinations of poles and joints a cure can thus be effected.

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[1] For a further account of both actions, see Prof. Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 133-145; and "Shaft Currents in Electric Machines," by P. L. Alger and H. W. Samson, *Journ. Amer. I.E.E*, Vol. 42, p. 1325, with bibliography.

[2] A. Fraenckel, *El. Rev.*, Vol 93, p. 488.

[3] Alger and Samson, *loc. cit.*, *qu.v.* also for other remedies.

CHAPTER XXIX

THE DESIGN OF THE SALIENT-POLE ALTERNATOR

§ 1. The torque-coefficient of the alternator.—The preliminary determination of the dimensions, D and L, of an alternator will follow a similar procedure to that already employed (Chapter XXII) for the continuous-current machine, but with this difference, that the number of pole-pairs is already fixed by the frequency, $f = pN/60$. With average values for x_a and r_a', and an external power-factor of 0·8, E_t will be about 1·05 to 1·07 times V_e, the value of the terminal voltage per phase, or less with unity power factor; in the latter case, the loss of volts over the resistance of the armature Ir_a' is additive to V_e, but the self-induced E.M.F. Ix_a, being then at right angles to V_e (*cp.* Fig. 508), adds but little to it. The induced volt-amperes per phase may, therefore, roughly be reckoned as 1·05 V_eI where I is the virtual current per phase, and the rate of development of electrical energy in all phases in watts is

$$1{\cdot}05\ V_eI \cos\phi_t \times N_{ph} = E_t \,.\, I \cos\phi_t \times N_{ph},$$

so that the actual torque in watts per rev. per min. is $E_t \,.\, I \cos\phi_t \times N_{ph}/N$. For this the alternative expression has been found (Vol. I, pp. 49, 50)

$$k_f \,.\, k_d \,.\, k\, B_{g\ max}\, JZ \frac{\pi}{60} DL \cos\phi_t \times 10^{-8} \quad . \quad . \quad . \quad (5e)$$

Assuming a sinusoidally distributed field or taking B_{g1} the fundamental only of the flux curve, $k = \frac{2}{\pi}$ and $k_f = \frac{\pi}{2\sqrt{2}}$, while k_d becomes k_{s1}; therefore

$$\frac{E_tIN_{ph}}{N} = \frac{VA}{N} = \frac{k_{s1}}{\sqrt{2}} \times \frac{\pi}{60} \times B_{g1}\, JZ\, DL \times 10^{-8} \quad . \quad (270)$$

Substituting $ac \times \pi D$ for JZ,

$$\frac{E_t\, IN_{ph}}{N} = \frac{k_{s1}}{\sqrt{2}} \times \frac{\pi^2}{60} \times B_{g1} \,.\, ac \,.\, D^2L \times 10^{-8} \quad . \quad . \quad (271)$$

analogously to equation 216 (Vol. II, p. 229). For a distributed 3-phase winding, k_{s1} may be taken as 0·955, so that $\frac{0{\cdot}955}{\sqrt{2}} \times \frac{\pi^2}{60} = 0{\cdot}111$ has the same value as the corresponding quantity for the continuous-current machine (eq. 218), *i.e.*

$$G = \text{volt-amperes}/D^2LN = 0{\cdot}111\, B_{g1}\, ac \times 10^{-8} \text{ in C.G.S. units}$$
$$= \text{volt-amperes}/D_u{}^2L_uN = 0{\cdot}72\, B_{g1}\, ac_u \times 10^{-8} \quad . \quad . \quad (272)$$

if $ac_{''}$ = ampere-conductors per inch of periphery, and $D_{''}$ and $L_{''}$ are measured in inches, while B_{g1} is retained in C.G.S. lines per sq. cm. The only differences from the continuous-current case are that in place of the true torque or true watts a nominal torque corresponding to the induced volt-amperes per rev. per min. is at present alone in view, and that the fundamental amplitude B_{g1} is here used instead of $B_{g\ max}$.

§ 2. The value of the fundamental amplitude, B_{g1} of the flux-density.—The advantage of retaining the flux-density rather than the total flux in the above equations has already been explained generally (Vol. I, pp. 168, 501). In the alternator case, although B_{g1} is used as a starting-point, there will later be required for magnetic calculations a knowledge also of $B_{g\ max}$, and for determining the true density in armature core and yoke a knowledge of Φ_a, in place of Φ_1 corresponding to B_{g1}.

If the spacial flux-density peripherally over a pole-pitch is plotted, starting from $B_{g\ max}$ for a tooth-pitch centrally under a pole, the fundamental amplitude extracted therefrom (Fig. 613) exceeds $B_{g\ max}$ when the polar arc bears a normal proportion to the pole-pitch not less than 0·5, and we have such figures as are given in the second column below for the ratio of the fundamental to $B_{g\ max}$.

Ratio, β, of polar arc to pole-pitch	$\dfrac{\textit{Fundamental amplitude}}{B_{g\ max}}$	$\times\ 0{\cdot}96 = \dfrac{B_{g1}}{B_{g\ max}}$
0·7	1·172	1·125
0·6	1·105	1·06
0·5	1·006	0·965

(*cp*. Fig. 79 and Fig. 618*a*, for the latter of which with a ratio β of 0·69, the height of the fundamental exceeds $B_{g\ max}$ by 16 per cent.). For any given value of β, the ratio is also dependent in a minor degree upon the length of the air-gap and upon the ratio of the maximum to the minimum when the air-gap is increased towards the pole-edge.

But the amplitude so extracted is related only to the reduced length of core, $L_f + K_2 l_g - K_3\, w_d\, n_d$, just as $B_{g\ max}$ is so related, whereas the factor k has throughout been arranged to reduce $B_{g\ max}$ to an average value over the whole of the pole-pitch area YL, including the full over-all length L of the armature core. Consequently the fundamental amplitude shown in the second column above must be reduced in the proportion

$$\frac{L_f + K_2 l_g - K_3\, w_d\, n_d}{L}$$

to give the fundamental B_{g1} for insertion in (270) as present over the full length L. Assuming the reducing factor to be 0·96, the third column above gives the final value thus obtained for use with L.

The values of the second column can be checked as follows. From Chapter XXV, § 4 (*cp.* also Vol. I, p. 190), the average value of the real flux-density is

$$B_{g\,av} = B_{g1} \times \frac{2}{\pi}\left(1 + \frac{1}{3}x_3 + \frac{1}{5}x_5 + \ldots.\right)$$

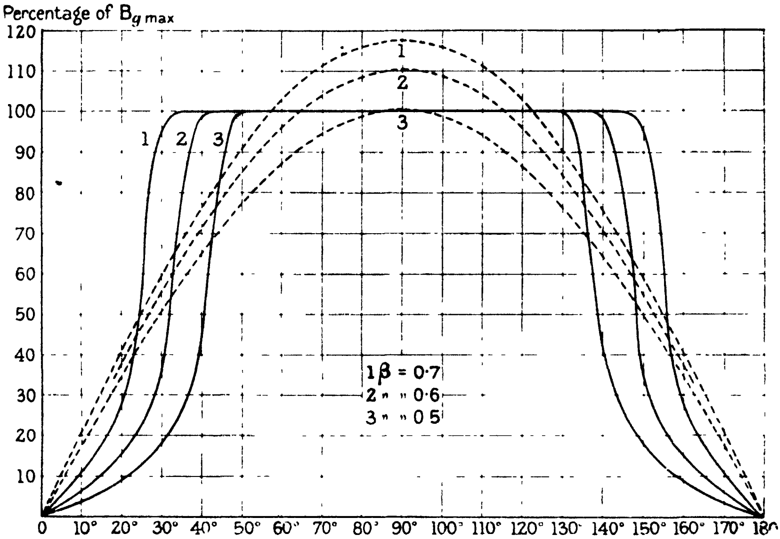

FIG 613.—Fundamentals of B_g curve for different ratios of pole arc to pole-pitch and uniform air-gap.

where x_n is the ratio of the amplitude of a harmonic in the flux-density curve to that of the fundamental. Also from equation (115), and assuming an air-gap of uniform length

$$B_{g\,av} = k\,B_{g\,max} = B_{g\,max}\frac{(A' + K_1 l_g)\,(L_f + K_2 l_g - K_3 w_d n_d)}{\pi\,(D + l_g)\,L/2p}$$

$$\text{and}\quad A' = \pi(D + l_g)\,\frac{\phi^\circ}{360^\circ} = \frac{\pi\,(D + l_g)}{2p}\,\beta$$

Thence by equating the two expressions for $B_{g\,av}$

$$B_{g1} = B_{g\,max} \times \frac{\frac{\pi}{2}\left(\beta + \frac{\text{fringes}}{\text{pole-pitch}}\right)}{(1 + \frac{1}{3}x_3 + \frac{1}{5}x_5 + \ldots)} \times \frac{L_f + K_2 l_g - K_3 w_d n_d}{L} \qquad (273)$$

As the ratio of the polar arc to the pole-pitch is increased, the denominator $(1 + \frac{1}{3}x_3 + \frac{1}{5}x_5 + \ldots.)$ falls from about 1·01 for

$\beta = 0{\cdot}7$ to 0·97 for $\beta = 0{\cdot}6$ and 0·93 for $\beta = 0{\cdot}5$, its exact value depending upon the same varying factors as have been mentioned above.[1] The same expression also gives the ratio[2] of the actual flux of a pole-pitch to the flux of the fundamental B_{g1}, since if both of the expressions for $B_{g\ av}$ are multiplied by the whole pole-pitch area $\pi(D + l_g)\,L/2p$,

$$B_{g1} \times \frac{2}{\pi} \times \frac{\pi\,(D + l_g)\,L}{2p} \times \left(1 + \frac{1}{3}x_3 + \frac{1}{5}x_5 + \ldots\right)$$

$$= B_{g\ max}\,(A' + K_1 l_g)\,(L_f + K_2 l_g - K_3 w_d n_d)$$

$$\Phi_1\left(1 + \frac{1}{3}x_3 + \frac{1}{5}x_5 + \ \ldots\right) = \Phi_a$$

But since it has been shown that the E.M.F. is practically determined solely by B_{g1}, and the required real value of $B_{g\ max}$ can thence be estimated with reasonable accuracy, it suffices to multiply the latter by the reduced area of a pole-pitch to find Φ_a without calculation of, or use of the calculated value of, Φ_a/Φ_1.

§ 3. **The influence of the degree of regulation specified.**—The temperature limit is fundamental to the life of the machine, and with this alone in view, some such values as those of Fig. 616 for the ampere-conductors of the stator per unit length of its periphery may be reached under favourable conditions of conductor dimensions as dependent on the voltage and on the number of poles as fixed by the frequency. But of two machines for the same apparent output in kilovolt-amperes at the same speed and power-factor, one may require to be considerably larger than the other, if the specification for the former calls for a closer degree of inherent regulation. This difference between the two machines must, therefore, be taken into account, and should appear in the preceding

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[1] For the curves of Fig 613, the lower harmonics in terms of $B_{g\ max}$ as unity have such values as the following—

	$\beta =$ 0·5	0·6	0·7
3rd harmonic . . .	0·142	+ 0·0226	+ 0·16
5th ,, . . .	− 0·184	− 0·1658	− 0·0746
7th ,, . . .	+ 0·018	− 0·1127	− 0·1106
9th ,, . . .	+ 0·088	+ 0·0184	− 0·0472
11th ,, . . .	+ 0·007	+ 0·0614	+ 0·0047

A symmetrical flux-density wave can be very closely represented by a small number of trapezia mounted on one another; for these the fundamental and harmonics can be quickly calculated mathematically by the formulæ given by Dr. A. E. Clayton in his paper on "Magnetomotive Force Calculations for Cylindrical-Field Alternators, etc.," *Journ. I.E.E.*, Supplement to Vol 57, p. 82; and up to the 7th or 9th harmonic good accuracy is obtained, although the curved sides are replaced by the sloping sides of the small number of trapezia.

[2] Curves for the ratio $\Phi_a/\Phi_1 = 1 + \frac{1}{3}x_3 + \frac{1}{5}x_5 + \ldots$ under different conditions are given in Fig 20 of the paper by R. E. Doherty and O. E. Shirley on "Reactance of Synchronous Machines and its Applications," *Trans. Amer. I.E.E.*, Vol. 37, Part II, pp. 1242, 1282.

formulæ, even if only approximate dimensions are to be obtained from them.

Now a close degree of regulation will depend on the armature ampere-turns bearing a small ratio to the field ampere-turns at full-load, and on the magnetic saturation being fairly high. For either or both reasons, therefore, the field ampere-turns placed on the internal revolving magnet-system will be carried to the maximum that is found to be reasonably economical in copper and reasonably cool in working, in order that the D^2L of the machine may not be larger than is necessary. Hence, in regard to regulation it is the maximum of the *field ampere-turns per unit length of the rotor periphery* rather than the maximum armature ampere-conductors per unit length of stator periphery that decisively fixes the D^2L of the alternator. From this point of view it is still possible to retain the same forms of expression if JZ or ac are expressed in terms of the field ampere-turns in order to introduce some comparison between the relative magnetic strengths of field and armature. Values for JZ or ac in dependence upon the degree of regulation called for by the specification will thence result.

It has already been mentioned (Chapter XXVII, § 40) that with the introduction of automatic voltage regulators the need for a close degree of inherent regulation has become less stringent. Nevertheless it is here proposed to proceed with the approximate determination of dimensions by way of the regulation, the assumption being made that with the larger values of ac permissible by the coarser regulations the temperature limit in the stator conductors is reached simultaneously with the limit imposed by the regulation.

§ 4. **Preliminary calculation of dimensions of alternator.**—For a logical comparison of the relative strengths of field and armature, either ampere-turns should be compared with ampere-turns, or ampere-conductors with ampere-conductors. The total field ampere-turns on full-load are $2pAT_{fl}$, or per pole AT_{fl}. The total armature ampere-turns, all reckoned as of equal value without reference to phases or magnetic circuits, are $JZ/2$, or per pole $JZ/4p$.

The greater the value of the ratio

$$\frac{\text{Field ampere-turns per pole on full-load}}{\text{Armature ampere-turns per pole}} = \frac{AT_{fl}}{JZ/4p} = K_r$$

the better the regulation, but the greater the cost of the field copper, and the greater the difficulty of keeping the temperature of the field coils within permissible limits. As a general rule, K_r is greater than 2, and usually approaches about 3, although under favourable conditions with good design it is often not more than 2·5. The estimate of the relative strengths is, however, also often made by comparing field ampere-turns against armature ampere-conductors,

$$i.e.\ \frac{\text{field ampere-turns per pole on full-load}}{\text{armature ampere-conductors per pole}} = \frac{AT_{fl}}{JZ/2p} = \tfrac{1}{2} K_r$$

when $\frac{1}{2} K_r$ ranges from 1 to 1·5.

Thus $JZ = 4p\,AT_{fl}/K_r$ and equation (270) becomes

$$\frac{1{\cdot}05\ \text{kVA}}{N} = \frac{k_{s1}}{\sqrt{2}} \frac{\pi}{60} B_{g1} \frac{2p\,AT_{fl} \times 2}{K_r} DL \times 10^{-11} \quad . \quad . \quad (274)$$

Lastly, the form of expression (271) may be retained by converting the field ampere-turns per pole into ampere-conductors per unit length of periphery. One field ampere-turn being equivalent to two ampere-conductors on the periphery, we return to $K_r = \frac{2p\,AT_{fl} \times 2}{JZ}$. Thus to be comparable with ac of the stator, ac' on the rotor $= \frac{4p\,AT_{fl}}{\pi D}$, whence $JZ = ac'\,\pi D/K_r$, or $ac = ac'/K_r$. Equation (271) then becomes

$$\frac{1{\cdot}05\ \text{kVA}}{N} = \frac{k_{s1}}{\sqrt{2}} \frac{\pi^2}{60} B_{g1} \frac{ac'}{K_r} D^2L \times 10^{-11}$$

$$= 2{\cdot}22\, k_{s1} \frac{\pi}{60} B_{g1} \frac{ac'}{K_r} D^2L \times 10^{-11}$$

which for $k_{s1} = 0{\cdot}955$

$$= 0{\cdot}111\, B_{g1} \frac{ac'}{K_r} D^2L \times 10^{-11} \quad . \quad . \quad (275)$$

The only difference from equation (271) is in the substitution of ac'/K_r for ac.

It is then in the value assumed for K_r that adjustment will be made for differences in the degree of regulation specified.

§ 5. **The division of** D^2L.—In the alternator the axial length of the core usually bears a somewhat higher ratio to the pole-pitch than in the continuous-current dynamo, especially when the number of poles is few. Thus the ratio $\frac{L}{\pi D/2p}$ may be as much as 2 with 4 poles, falling to 1·5 with 6 poles. But when the number of poles is 10 or more, L is usually about equal to the pole-pitch, although in some cases a somewhat large diameter must be chosen in order to obtain such a pole-pitch that the field coils do not become crowded. A very general value for the pole-pitch with medium speeds is about 10 to 12 ins., although, as will be seen later, in high-speed turbo-alternators this may become 50 ins. or more. As a rule, a long machine of small diameter means more copper and less iron and less total weight than a machine of greater diameter and shorter length for the same output; the labour and freight are also less, but on the other hand the ventilation is not so good, and the increase of copper and increased loss in excitation quickly limit the possible length of core that can be economically employed.

Assuming then $L = \pi D/2p$ as a preliminary starting-point when $p = 5$ or more, from the known value of D^2L calculated by equation (275)

$$D^3 = \frac{2p}{\pi}(D^2L) = 0{\cdot}636p\,(D^2L) \qquad . \quad . \quad . \quad . \quad . \quad (276)$$

The value of D thence found must then be checked by considerations of the peripheral velocity of the rotating field-magnet. Except in the case of steam-turbo-alternators or alternators driven by waterwheels, this seldom exceeds 8000 ft. per min. (40 metres per sec.), and as a normal value for the maximum permissible without any special construction may be taken 6000 ft. pei min. (30 metres per sec.). If the value of D is found to be too high, then in expression (275) L must be lengthened, and the assumed ac' must again be checked with Fig. 615.

From the known value of DL, calculated by equation (274), we also have $D^2 = 0{\cdot}636p(DL)$. But DL can only be found from equation (274) after insertion of AT_{fl}, and it is obvious that for a given number of poles AT_{fl} will not have nearly the same measure of constancy as ac' (*cp.* Figs. 614, 614*a* with Fig. 615); it must depend upon D, as fixing the pole-pitch, and D has still to be found. Consequently a design *ab initio* can only be begun from equation (275).

But in most cases for each number of poles a frame will be to hand, having a normal pole-pitch of, say, 10 ins., giving a favourable relation between diameter and poles, for which AT_{fl} can be laid down at the outset. Equation (274) can then be used to determine the separate value of L.

§ 6. The field ampere-turns per pole.—The number of ampere-turns that can be disposed on a given rotating field-magnet is affected by the question of whether the winding is of square d.c.c. wire or of bare copper strip wound on edge and insulated by layers of paper, and also by the peripheral velocity of the rotor in ft. per min. The figures adopted in Figs. 614–615 are based on the valuable discussion of the whole subject by Prof. Miles Walker in *The Specification and Design of Dynamo-electric Machinery,*[1] and refer solely to coils wound with square d.c.c. wire suitable for excitation at 110 volts. When required to give free circulation of air between the poles, it is assumed that the coils are tapered in one or more steps. But as soon as the dimensions of the conductor permit of a bare copper strip being satisfactorily wound on edge, the ampere-turns may be increased by about 20 per cent.

[1] Pp. 299–304.

In Figs. 614 and 615 it is assumed that in every case the speed of the rotor is such as to give 50 cycles with the number of poles stated, so that other similar curves would be required for 25 cycles (Fig. 614*a*). Lastly, it is assumed throughout that the axial length of core is 12½ ins.; if wider, the ampere-turns or ampere-conductors per inch must be reduced; if narrowed, say, to 7½ ins., they may be increased about 7 per cent. for 16 poles, or 11 per cent. for 32 poles.[1]

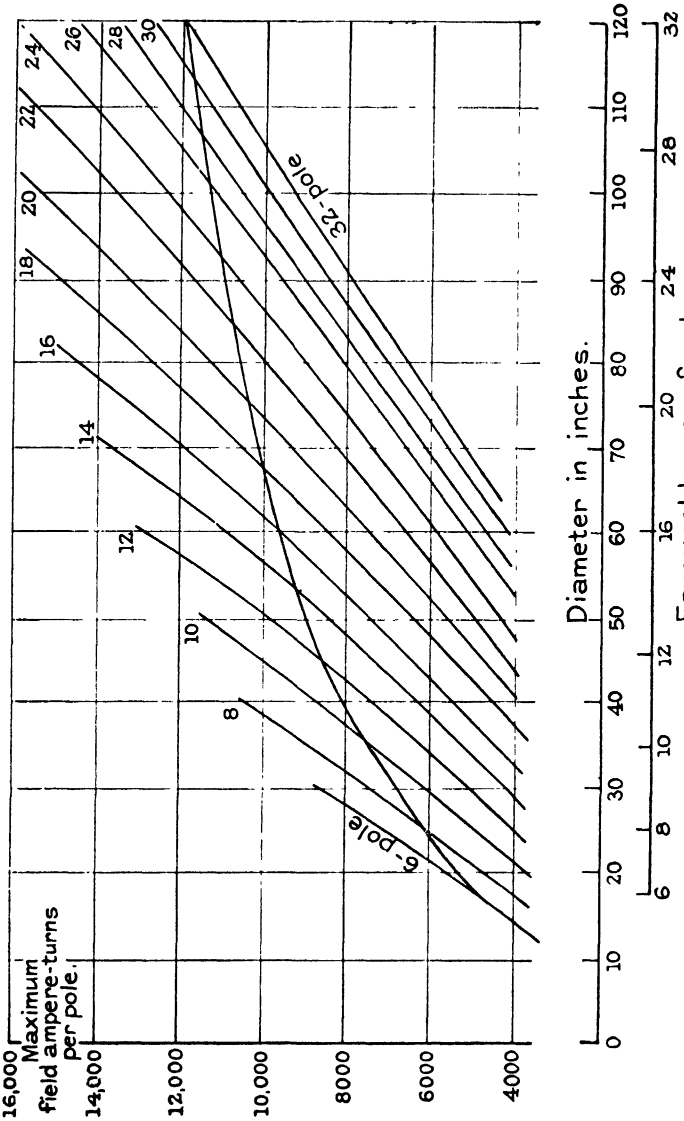

FIG 614 —Field ampere-turns per pole with square d c c wire for 50-cycle 3-phase generators and 12½ ins. axial length of core.

(For strip copper wound on edge increase 20 per cent)

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[1] Prof. Miles Walker, *loc. cit.*, p. 303.

Obviously the maximum possible for the field ampere-turns per pole will increase with the pole-pitch, and in Fig. 614 the limits of pole-pitch considered are from 6·3 ins. to 16 ins. But since a pole-pitch of about 10 ins. is practically found to be advantageous, the favourable number of poles for each diameter can be marked off as a second horizontal scale for pole-pitches rising from 9 ins. for 6 poles to 12 ins. for 32 poles. Corresponding thereto is the single intersecting curve of Fig. 614, which gives the maximum field AT per pole for the favourable number of poles given in the lower of the two horizontal scales.

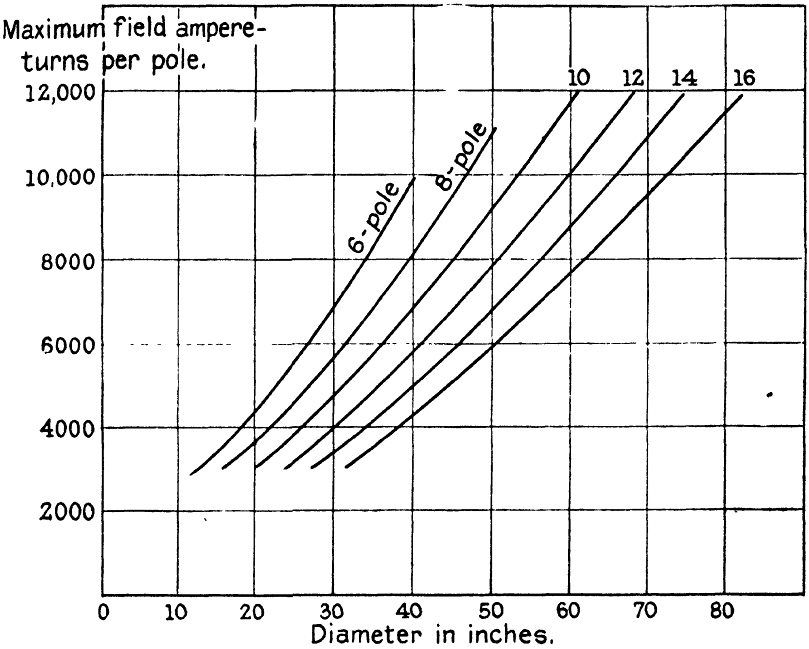

Fig. 614a.—Field ampere-turns per pole with square d.c c wire for 25-cycle 3-phase generators and 12½ ins. axial length of core.

Conversion into ampere-conductors per inch of periphery yields Fig. 615. Numerals at the end of a curved line, say, for 6 poles, mark the same limits as in Fig. 614, and the numerals central to each component curve which are linked up into a single curve mark again the maximum value of ac' for the favourable number of poles given in the lower horizontal scale.

It only then remains to be added that after selection of the maximum ac' or $AT_{f\ max}$ for the given number of poles, some allowance must be made to enable the normal voltage to be maintained at low power-factors and ordinary overloads; if the specified

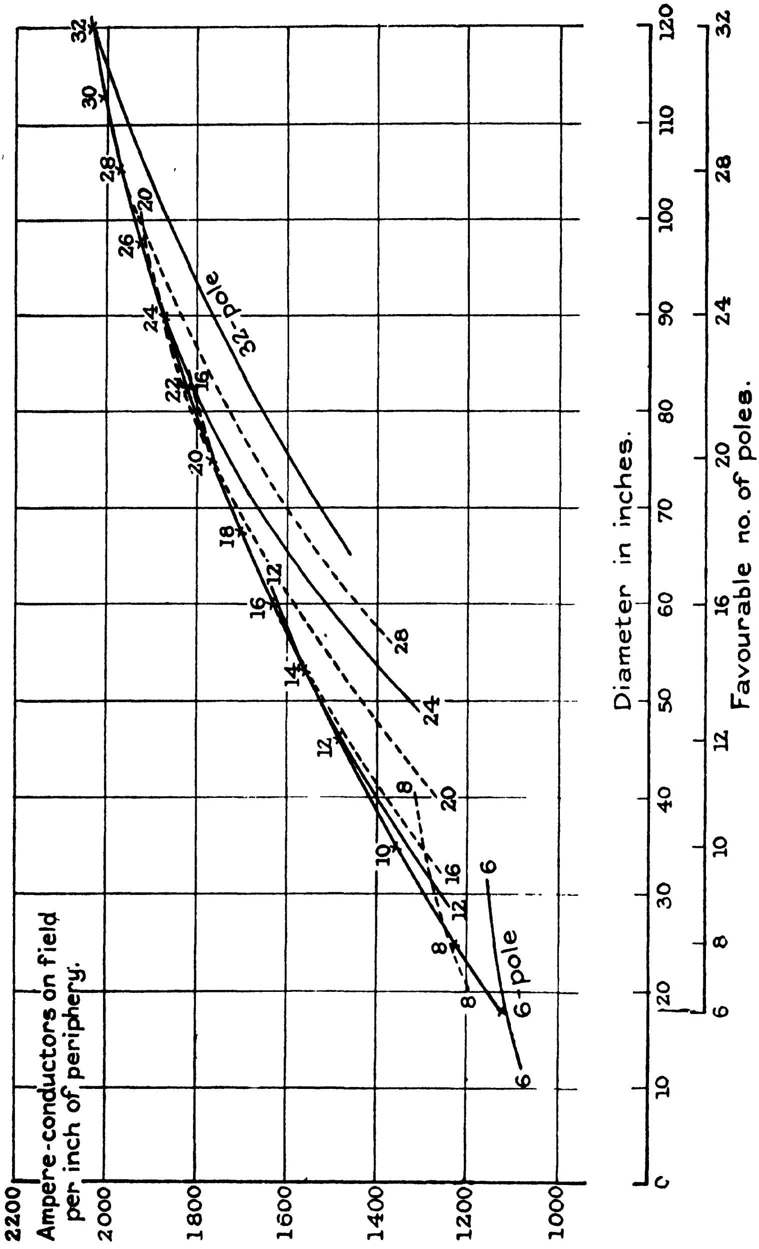

FIG. 615.—Ampere-conductors on field per inch of periphery, *ac'*, for 50-cycle 3-phase alternators, and 12½ ins. axial length of core.

power-factor is 0·8, an increase of the exciting current to give 1·25 AT_{fl} will usually cover all contingencies, so that for insertion in equations (275) and (274) as ac' or AT_{fl} about 80 per cent. of the values obtained from Figs. 615 and 614 will be used, or 90 per cent. when actually the field coils are of bare copper wound on edge.

Owing to the need for obtaining fairly large outputs with the smaller number of poles in 25-cycle machines, the pole-pitch must be taken to larger values than in 50-cycle machines, and it is not usually possible to secure so favourable a ratio of length of core to pole-pitch. The circumstances of each case have, therefore, to be considered more in detail than is necessary with 50-cycle machines, and no intersecting line has been drawn in Fig. 614*a*.

§ 7. The completion of the preliminary estimate of dimensions.—Assuming the regulation to be specified, to proceed further without some assurance that it will be complied with, may only lead to loss of time, so that a provisional estimate of it should now be undertaken.

When values for ac' and D and, therefore, for AT_{fl} have been obtained which are in agreement, the value of $Z = 4p\,AT_{fl}/JK_r$ is fixed, since K_r has been assumed as a datum. Let the increased volts when full-load is thrown off be E_o exceeding V_e by the specified amount or, say, 18 per cent. when $\cos \phi_e$ has been 0·8. There can now be calculated for the two open-circuit cases of (*a*) V_e and (*b*) E_o, the corresponding values of B_{g1}, $B_{g\,max}$, Φ_a, and $AT_g = 0{\cdot}8\,B_{g\,max}\,Kl_g$ upon a known or assumed value of l_g. We thus reach AT_{go} in the former case, and after addition of some ampere-turns for $AT_c + AT_t$, reach AT_{Po} in the latter case.

The *full-load* case (*a'*), still with V_e, can now be completed. The ratio $JZ/(4p\,.\,AT_{go})$ is fixed, and from Fig. 540 is obtained the ratio $(AT_{gl} + AT_{DM})/AT_{go}$; this will be slightly exceeded by the ratio AT_{Pl}/AT_{go}, so that AT_{Pl} is approximately found.

The differences $AT_{fl} - AT_{Pl}$ in case (*a'*) and $AT_{fl} - AT_{Po}$ in case (*b*), and their reasonableness as $AT_m + AT_y$ in the two cases can now be simultaneously checked through B_m and B_y from a knowledge of the pole-core area and leakage permeance $\mathfrak{S}_l$ estimated from analogous machines. The width of the pole-core rises from about 0·45 of the pole-pitch in a 6-pole machine, 0·48 in a 8-pole machine to 0·5 in a 12-pole machine, and thence rises but little more. On an average, whether with solid poles having rounded ends or with laminated poles of rectangular shape when allowance is made for insulation between the laminæ, the area of the magnet core is about 0·45 of the polar area. The yoke, being given ample proportions, only adds on a few hundreds of ampere-turns.

A comparison of the total AT under the two conditions (*a'*) and (*b*) then shows at once whether alteration of the data is necessary, and also enables some conclusion to be drawn as to where it had best be made, whether in K_r, l_g, or the saturation *of the magnet-core. An example is given in the preliminary* design of § 10.

The reactance voltage not being directly related to the number of armature ampere-turns per pole, in different cases something may be done to vary the one or the other so as to suit the exact nature of the load. Thus if the load-factor varies through a wide range, it is especially important that the armature ampere-turns per pole should be low, although the true armature reactance voltage may be comparatively high; such a case is met by the employment of a very small number of turns per coil, which may then be entirely embedded in the iron of the armature in closed slots or tunnels. On the other hand, if the power-factor is high and does not vary greatly, and especially on non-inductive loads, it would be better to employ a larger number of armature ampere-turns per pole, arranged in shallow open slots, so that the actual reactance voltage is not so high; in other words, it becomes then of more importance to minimize the electric part of the reaction due to the leakage reactance rather than the magnetic part due to the demagnetizing ampere-turns.

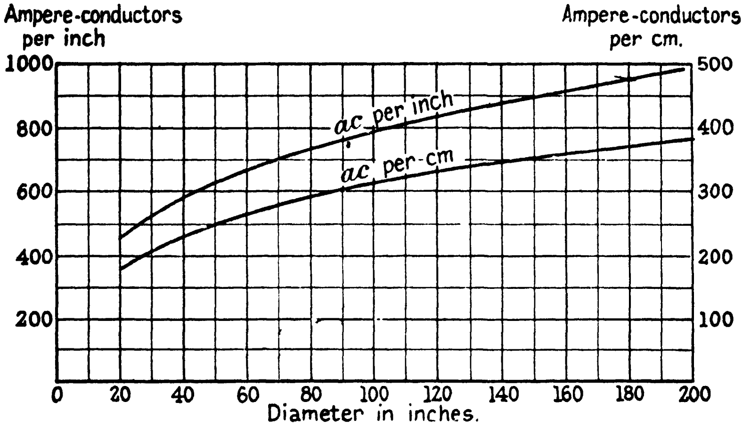

FIG 616—Approximate average values of *ac* in alternator armatures.

§ 8. The armature ampere-conductors per unit length of periphery.—Owing to the great diversity of conditions that are found in alternators, it is only to be expected that curves of *ac* such as those of Fig. 616 will not show nearly so great a measure of uniformity as the similar curves of Figs. 331 or 405 in the continuous-current case. In the alternator not only may there be one, two, or three phases, but the voltages in use cover a very wide range. Again the degree of regulation specified may vary greatly, and if this is close, the heating of the armature is not so decisive as in the continuous-current dynamo. But even if the same degree of regulation and the same temperature limits are maintained throughout, there remains the great difference between the alternator and the continuous-current dynamo, in that for the former we are not at liberty to choose the number of poles that would best suit the output. Thus while a 540-kVA 500 r.p.m. alternator at a frequency of 50 has 12 poles and an output of 45 kVA per pole, a machine for the same output and speed at a frequency of 25 has 6 poles and an output of 90 kVA per pole; the latter would then be the more difficult machine to build.

Fig. 616 is, therefore, only to be regarded as indicating possibilities for frequencies of 50 or 25 under favourable conditions in 3-phase machines. In large machines with coarse regulation even as much as 1200 ampere-conductors per inch of periphery have been employed. But for moderate outputs on the other hand, $ac = 650$ per inch or 250 per cm. may be taken as a normal value, which is often found extending over a wide range from 20 ins. to 120 ins. diameter without necessarily implying any departure from the conditions for economy in material and cost.

If the regulation on inductive loads is to be the same in a single-phase as in a 3-phase generator, the value of ac under conditions comparable with those last-mentioned must be reduced to about 400 per inch or 155 per cm. But if the single-phase alternator is to be used mostly on lighting circuits with high power-factors and is compared with a 3-phase generator on circuits of only moderate power-factor, ac in the former may be high as 70 per cent. of its value in the latter.

§ 9. The curve of the torque coefficient or size coefficient.— The variations in the power-factor of the output have been eliminated by considering only the apparent torque but, this done, there still remain the diversities of conditions that have been mentioned in the preceding section. For the same reasons, therefore, that are given in that section, it is not surprising that the values of the dimensional torque coefficient,[1] or the ratio of the induced volt-amperes per rev. per min. to D^2L, although deduced broadly on the same principles as in the continuous-current dynamo, will not have the same uniformity in the case of the alternator. A single curve connecting either the dimensional torque coefficient, or its reciprocal the size coefficient, as shown in Fig. 617 is only, therefore, added as showing approximately the order of the figures, and wide deviations from it may arise according to the circumstances of the design.

Certain general conclusions may, however, be drawn therefrom. The apparent useful torque per cubic inch of D^2L naturally tends to be less in the alternator than in the continuous-current machine, since the diameter of the armature in the former must allow room for the internal poles, and owing to the number of these to give the usual frequencies of practice an efficient design necessitates a short armature core of large diameter. The square of the diameter has thus a preponderating influence upon the size coefficient. In many cases, too, a large diameter and narrow width of core is of no disadvantage, since it gives a large fly-wheel effect for a given mass of copper and iron. With short magnet-cores the cost of the magnetically and electrically effective portions of the alternator is also more independent of the diameter than is the case with the continuous-current dynamo.

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[1] See F. Emde, *E.T.Z.*, Vol. 43, p. 1430.

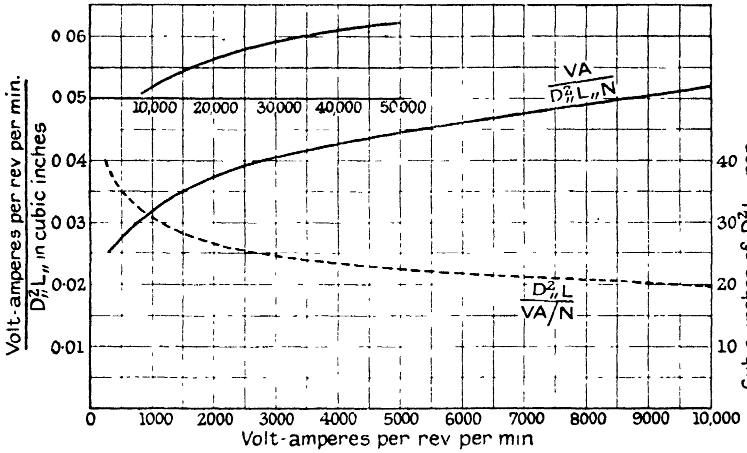

FIG 617 —Torque coefficient and size coefficient for 3-phase alternators

The dimensional torque coefficient does not so quickly reach a maximum value in alternators as in continuous-current dynamos, but continues to ascend slowly, owing to the gradually increasing value that may be assigned as mentioned above to *ac* with increasing size. Apart from turbo-alternators with artificial ventilation, the maximum value of *G* for very large machines approaches 0·06, yet so low a figure as 0·035 is not incompatible with economical design for machines even up to 10,000 volt-amperes per rev. per min. On the other hand, 0·03 may be obtained even with so small an output as 1000 volt-amperes per rev. per min.

Speaking generally, with low speeds and high frequency, *i.e.* with poles 30 or more in number, the preponderating effect of the diameter of the armature bore which has already been mentioned produces figures which appear low; yet the machines may not be in reality expensive to build for their output. Speeds and frequencies which call for poles from 10 to 20 in number, when it is possible to combine with them a high value of *ac*, give a high torque coefficient. But if the voltage is low and heavy bar copper winding is necessary, this again is not universally true; for then the eddy loss may limit the output by rise of temperature, or it may not be possible to balance the armature ampere-turns by sufficient excitation on the rotor without overheating the latter, as *e.g.* in 25-cycle alternators at 300 revs. per min. for small outputs up to 500 kW.

Approximately speaking, the output of a single-phase alternator may be reckoned as 70 per cent. of a three-phase generator of the same size.

§ 10. Design of a three-phase alternator for 800 kVA at 300 revs. per min.—As in Chapter XXII on the design of continuous-current dynamos, a design of a 3-phase alternator is here worked out in detail to show the order of procedure from first principles and to illustrate many of the points to which attention has been directed. For many excellent designs the reader is again referred to other works supplementing all that space here permits.[1]

Three-phase, star-connected—

Kilowatts	640
Power-factor	0·8
kVA	800
Volts between terminals	2300
Amperes per phase	201
Revs. per min.	300
Regulation when full-load at 0·8 power-factor is removed not to exceed 20 per cent.	
Frequency, f	50
No. of pole-pairs, $p = 60f/N$	10
Induced kVA per rev. per min., $1{\cdot}05 \times 800/300$	2·8

Estimate of approximate dimensions—

(1) Assume $B_{g1} = 9700$, $ac'_u/K_r = 1760/2{\cdot}9 = 607$

$$G = 0{\cdot}72 \times 9700 \times 607 \times 10^{-8} = \quad 42{\cdot}4 \times 10^{-3}$$

$$D_u^2 L_u \text{ in cubic inches} = 2800/42{\cdot}4 \times 10^{-3} = \quad 66{,}100$$

$$L = \pi D/2p = 0{\cdot}157\, D$$

$$D_u^3 = (66{\cdot}1 \times 10^3)/0{\cdot}157 = 420 \times 10^3$$

$$D_u = 75 \text{ ins.} \quad L_u = 0{\cdot}157 \times 75 \quad = 11{\cdot}75 \text{ ins.}$$

$$AT_{fl} = \tfrac{1}{2}(ac'_u \times \pi D)/2p = \tfrac{1}{2}(1760 \times 236)/20 = 10{,}390$$

ac'_u, D and AT_{fl} are, therefore, in sufficient agreement to warrant proceeding on the above lines, with $D = 75$ ins. and $L = 11\tfrac{3}{4}$ ins.

(2) But *alternatively*, it will be assumed that a 20-pole magnet frame with stator having a bore of 75 ins. diameter is available, on which $AT_{fl} = 10{,}400$ is permissible by Fig. 614.

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[1] Especially Prof. Miles Walker's *The Specification and Design of Dynamo-electric Machinery*, and Prof. S. P. Smith's "Notes on the Theory and Design of Alternating-current Generators" in *Papers on the Design of Alternating-current Machinery* by Hawkins, Smith, and Neville (Pitman & Sons).

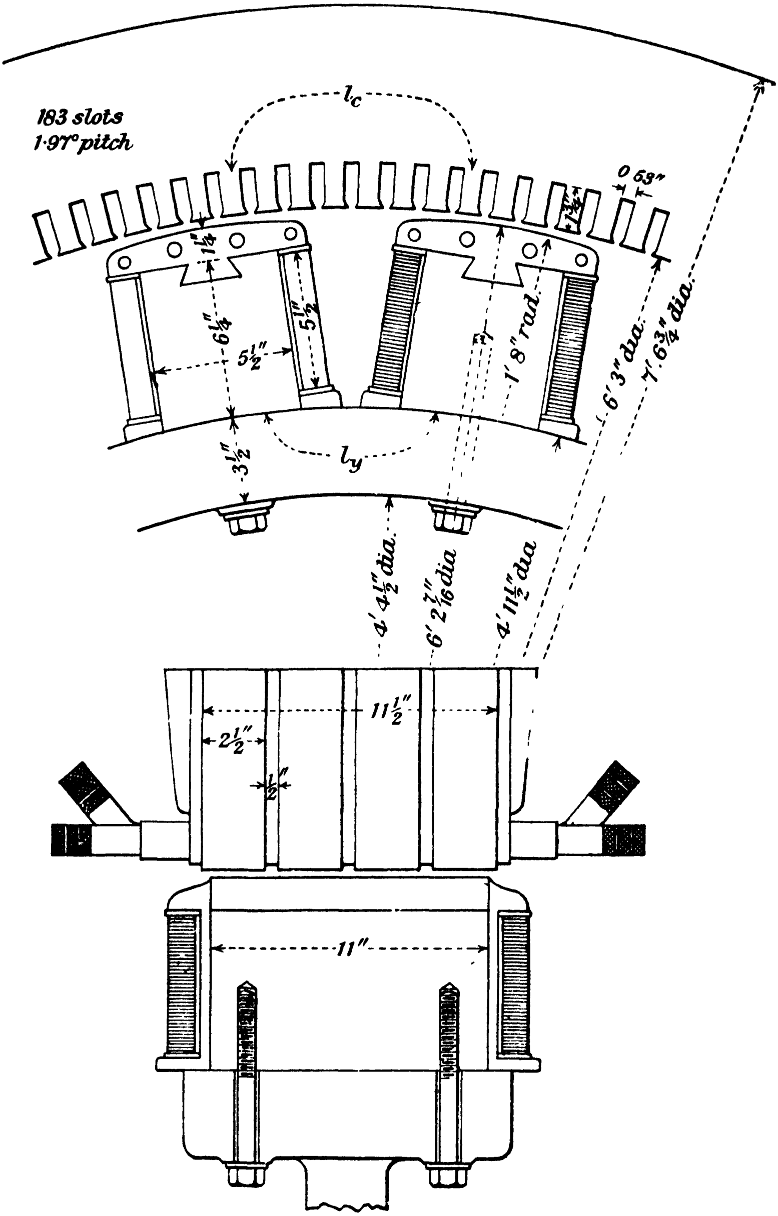

FIG. 618.—Outline design for 800 kVA alternator.

Hence by equation (274)

$$2{\cdot}8 = \frac{0{\cdot}955}{\sqrt{2}}\,\frac{\pi}{60}\,9700\,\frac{20 \times 10{,}400 \times 2}{2{\cdot}9} \times 75 \times L_u \times 6{\cdot}45 \times 10^{-11}$$

$L_u = 11{\cdot}72$ ins.

The over-all length of the given stator core being 11·5 ins., it will be used in with an increase of B_{g1} to 9900.

Therefore finally, diameter of bore of stator	75 ins.
Over-all length of stator core	$11\frac{1}{2}$ ins.
Peripheral velocity, slightly less than	5890 ft. per min.
Pole-pitch, $75 \times 3{\cdot}14/20$	$11\frac{3}{4}$ ins.

With a single armature path per phase, $Z = \dfrac{4p\,AT_{fl}}{JK_r}$

$$= \frac{40 \times 10{,}400}{201 \times 2{\cdot}9} = 715, \text{ or } 238 \text{ per phase.}$$

Provisional estimate of regulation—

Multiplier, *m*, to convert total sinusoidal flux of a pole-pitch into volts per phase, *i.e.* $E_a = m\,\Phi_1$

$$m = \frac{\pi}{\sqrt{2}}\,k_{s1}\,f\,\frac{Z}{N_{ph}\,q} \times 10^{-8}$$

approx. $= 2{\cdot}22 \times 0{\cdot}955 \times 50 \times 238 \times 10^{-8} = 252 \times 10^{-6}$

$= 252$ if Φ_1 is in megalines.

Factor to convert volts per phase into total sinusoidal flux of a pole-pitch

$$\frac{1}{m} = \frac{1}{252 \times 10^{-6}} = 3965$$

$$B_{g1} = \frac{p}{6{\cdot}45\,D_u L_u}\,\Phi_1 = \frac{10}{6{\cdot}45 \times 75 \times 11{\cdot}5}\,\Phi_1 = 1797 \times 10^{-6}\,\Phi_1$$

$$\therefore\ B_{g1} = \left(\frac{1}{m} \times \frac{p}{D_u\,L_u \times 6{\cdot}45}\right)E_a = \frac{1797 \times 10^{-6}}{252 \times 10^{-6}}\,E_a = 7{\cdot}15\,E_a$$

Let Kl_g be assumed as $1{\cdot}02 \times 0{\cdot}3 \times 2{\cdot}54 = 0{\cdot}777$ cm., and $B_{g\,max}$ as $B_{g1}/1{\cdot}12$.

		(*a*)	(*b*)
Open-circuit voltages per phase	$V_e =$	1328	$E_o = 1328 \times 1{\cdot}18$ $= 1567$
Φ_1 in megalines $= E_a/252$		5·27	6·22
$B_{g1} = 1797\,\Phi_1 =$		9490	11,200
$B_{g\,max} = B_{g1}/1{\cdot}12$		8460	10,000
$0{\cdot}8\,B_{g\,max} \times 0{\cdot}777 =$	**AT$_{go}$ =**	**5265**	**6220**
Allowance for stator core and teeth		**AT$_c$ + AT$_t$ =**	**600**
		AT$_{po}$ =	**6820**

$JZ/(4p\,.\,AT_{go}) = (201 \times 715)/(40 \times 5265) = 0{\cdot}682$

$\therefore$ by Fig. 540 $(AT_{gl} + AT_{\text{DM}})/AT_{go}$ for 0·8 power factor $= 1{\cdot}42$

Under load, $1{\cdot}42AT_{go} + 400$ for $AT_{c+t} = \mathbf{AT}_{pl} = \mathbf{7880}$

Area of magnet-core at 45 per cent. of $\pi DL/2p$

$= 0{\cdot}45 \times 11{\cdot}75 \times 11{\cdot}5 \times 6{\cdot}45 = 393$ sq. cm.

	(a′)	(b)
$\mathcal{F}_l = 63$		
$\phi_l = 1{\cdot}257\,AT_p \times 2 \times 63 =$ in megalines	1·25	1·08
$\Phi_m =$	6·52	7·3
$B_m = \Phi_m/393$	16,600	18,600
$at_m =$	50	110
$l_m = 7{\cdot}5$ ins. $\times 2{\cdot}54 = 19$ cm. $\quad \mathbf{AT}_m =$	**950**	**2090**
$\mathbf{AT}_y =$ say,	**90**	**150**
Total $\mathbf{AT}_f =$	**8920**	**9060**

Since the total AT of column (b) slightly exceeds that of (a′) for full-load, the specified regulation is just reached, but since the flux-density in the field-magnet is fairly low at full-load it will, if the poles are solid steel, be better to decrease their area slightly which will, if anything, better the regulation; at the same time let Z be raised to $240 \times 3 = 720$ to give slots holding 4 or 6 conductors.

The stator winding—

The ampere-conductors of the stator in a pole-pitch are then $(720 \times 201)/20 = 7236$.

(i) If 2 slots per pole per phase, *ac* per slot $= 7236/6 = 1206$, which would be permissible but is rather high.

(ii) If 3 slots per pole per phase, *ac* per slot $= 7236/9 = 804$, which will be easier to cool and will give a better wave-shape of E.M.F.

(i) By the first alternative there would be 6 conductors in a slot and these could be arranged either (a) 2 abreast in 3 layers with single-layer composite coils, or (b) 3 abreast in a double-layer opened wave-winding. In case (a) the number of wound slots to take the assumed total number of conductors would be $2 \times 3 \times 20 = 120$, to which should be added 3 empty slots to reduce harmonics in the E.M.F., *i.e.* $S = 123$ which has no C.F. with $p = 10$. The same number of slots, all filled, would equally well suit (b), since by Table VII (p. 259, Vol. I) $123 = 30 \times 4 + 3$. In case (b) we could also have $S = 30n_1 + 9 = 30 \times 4 + 9 = 129$ and 258 conductors per phase, with a reduced $B_{g\ max}$.

(ii) By the second alternative, there would be 4 conductors in a slot, and these could be arranged one above the other. The nearest total number of wound slots to take the assumed number of conductors would be $3 \times 3 \times 20 = 180$, to which should be added

3 empty slots, for the same reason as above, making a total of $S = 183$, which has no C.F. with $p = 10$.

Trial then shows that with the double-layer arrangement if a suitable depth of copper conductor is not to be exceeded, and assuming that apart from the slot insulation each separate conductor had better receive a thin wrapping of micanite owing to the comparatively high voltage between adjoining conductors in the same slot, the slot-width must be nearly 0·9 in. out of a tooth-pitch of 1·929 ins. at the bottom of the wedge. This makes the uncorrected density in the teeth at that level above 19,000, which is too high.

Choice will, therefore, be made of (ii) with concentric coils in one layer which will also give good cooling surface to the embedded length of the conductor and will not call for much thickness of insulation on the separate conductors. The stator core will be divided into 6 segments, each containing 183/6 = 30½ tooth-pitches. 183 slots, overhung at the top, leaving a slot-opening $w_3 = 0{\cdot}14$ in. wide and 0·04 in. deep. Coils pushed through and fastened by a wooden wedge 0·23 in. thick in deepest part. From the calculated tooth-pitch below wedge, make the slots 1·75 ins. deep × 0·53 in. wide.

		Tooth-pitch.	*Width of tooth.*
At top of slot,	$\pi \times 75/183$	= 1·285 in.	1·145 in.
Below wedge,	$\pi(75 + 0{\cdot}54)/183$	= 1·295 ,,	0·765 ,,
At centre of tooth,	$\pi(75 + 1{\cdot}75)/183$	= 1·315 ,,	0·785 ,,
,, root ,, ,,	$\pi(75 + 3{\cdot}5)/183$	= 1·345 ,,	0·815 ,,

Insulation round the group of 4 conductors as a whole—

	In width.	*In depth.*
One half-lapped spiral of cotton tape, 0·006 in. thick	4 × 0·006 = 0·024 in.	0·024 in.
Wrapping of micanite, 0·02 in. thick, folded over at top	2 × 0·02 = 0·04 ,,	3 × 0·02 = 0·06 ,,
Two half-lapped spirals of oiled linen, 0·006 in. thick	8 × 0·006 = 0·048 ,,	0·048 ,,
Wrapping of paper, 0·015 in. thick, folded over at top	2 × 0·015 = 0·03 ,,	3×0·015 = 0·045 ,,
	0·142	0·177
Depth of overhanging edge of slot opening		0·04 ,,
Thickness of wedge		0·23 ,,
Insulating strip of micanite at bottom of slot		0·04 ,,
		0·487 ,,

$0{\cdot}53 - 0{\cdot}142$	$= 0{\cdot}388$	$\dfrac{1{\cdot}75 - 0{\cdot}487}{4}$	$= 0{\cdot}316$
insulation of conductor	$= 0{\cdot}024$	insulation of cond.	$= 0{\cdot}024$
	$0{\cdot}364$		$0{\cdot}292$
clearance	$= 0{\cdot}024$	clearance per layer	$= 0{\cdot}032$
width of conductor	$= 0{\cdot}340$	depth of conductor	$= 0{\cdot}260$

By equation (264), $m = 2{\cdot}42\sqrt{\dfrac{0{\cdot}34 \times 50}{0{\cdot}53 \times 314{\cdot}5}} = 0{\cdot}775$

$h = 0{\cdot}26 \times 2{\cdot}54 = 0{\cdot}66$. $mh = 0{\cdot}66 \times 0{\cdot}775 = 0{\cdot}51$.

By Fig. 560 for $n_l = 4$, 14 per cent. increase in loss.

Area of conductor, $0{\cdot}34 \times 0{\cdot}26 = 0{\cdot}0885$ sq. in.

Resistance per 1000 yds. $= 0{\cdot}02445/0{\cdot}0885 = 0{\cdot}276$ ohm at 20° C. (68° F.).

Projection of coil, 3 ins. slot-insulated + 3 ins. = 6 ins. at each end.

Mean pole-pitch, $\pi(75 + 1{\cdot}75)/20 = 12$ ins.

Length of a conductor, $11{\cdot}5 + 12 + 12 = 35{\cdot}5$ ins. $= 0{\cdot}985$ yd.

Yds. per phase $= 240 \times 0{\cdot}985 = 236$.

Resistance of one phase $= 0{\cdot}276 \times 0{\cdot}236 = 0{\cdot}065$ ohm.

$14\% \times$ slot length/total length $= 14 \times 10/35{\cdot}5 = 3{\cdot}94\%$, with some addition for end-connexions, say 4·5 per cent.

r_a' cold $= 0{\cdot}065 \times 1{\cdot}045 = 0{\cdot}068$ ohm.

r_a' hot $= 0{\cdot}068 \times 1{\cdot}175 = 0{\cdot}08$ ohm.

$Ir_a' = 201 \times 0{\cdot}08 = 16{\cdot}1$ volts.

Heating of the stator—

$3I^2r_a' = 3 \times 201 \times 16{\cdot}1 =$ 9720 watts.

Mean diam. of core behind slots $= 75 + 3{\cdot}5 + 6 = 84{\cdot}5$ ins.

$V_c = 348 \times \pi \times 84{\cdot}5 \times 2{\cdot}54 = 234{,}500$ c. cms.

Mean apparent tooth density (p. 348) $= 18{,}100 \times \dfrac{3 \times 17{,}000}{2 \times 17{,}000 + 18{,}100}$
$= 17{,}700$

$V_t = 0{\cdot}785 \times 1{\cdot}75 \times 0{\cdot}9\,(11{\cdot}5 - 1{\cdot}5) \times 6{\cdot}45 \times 183$
$= 14{,}600$ c. cms.

By equation (277)

$$0{\cdot}71\left\{\left(\frac{7960}{1000}\right)^2 \times 234{\cdot}5 + \left(\frac{17{,}700}{1000}\right)^2 \times 14{\cdot}6\right\}$$

$= 0{\cdot}71\,(14{,}800 + 4575) =$ 13,750 watts.

23,470 „

For an estimate of the temperature rise of the stator, the division of the cooling effects and the method of calculation recommended by Prof. Miles Walker[1] will be here employed. By this it is assumed that the end-projections of the stator coils will dissipate their own heat, leaving only the embedded portion to add heat to the stator core. This amount is $9720 \times \frac{10 \times 1{\cdot}16 + 1{\cdot}5}{35{\cdot}5 \times 1{\cdot}045} = 3430$. The total rate at which heat must be dissipated by the stator core when a constant temperature is reached is thus $13{,}750 + 3430 = 17{,}180$ watts.

For the internal cylindrical surface of the stator face, when v is the peripheral speed of the rotor in ft. per min., and S_c is in sq. ins., approximately the watts dissipated are

$$W = \frac{S_c\left\{1 + 0{\cdot}5\left(\frac{v}{1000}\right)\right\}}{50} \times t^\circ \text{ C.}$$

where t° C. is the difference in temperature between the iron and the air. $S_c = 75 \times \pi \times 11{\cdot}5 = 2710$ sq ins., and $v = 5850$ ft. per min., whence for a temperature rise of 30° C. (the air reaching the surface being assumed to have already been heated 10°),

$$W_1 = \frac{30 \times 2710 \times 3{\cdot}92}{50} = 6370 \text{ watts out of the total of } 17{,}180.$$

For the 3 ventilating ducts (Fig. 618), assuming the speed of the air through them to be one-tenth of the peripheral speed of the rotor, and reckoning both sides of the ducts as the cooling surface, the experimental formula of Prof. Miles Walker[2] gives (see eq. 294)

$$W = 0{\cdot}0458 \times 0{\cdot}1\left(\frac{v}{1000}\right)S_c \times t^\circ \text{ C.}$$

where t° is the mean difference in degrees Centigrade between the temperature of the iron and that of the air. The latter has already been assumed to reach the duct with a temperature rise of 10° C.; if the iron has risen 40° C., and the air leaves the duct with 30° rise, the mean difference t° is $\frac{1}{2}(40 - 10 + 40 - 30) = 20^\circ$.

The mean diameter of a duct $= 75 + 7{\cdot}75 = 82{\cdot}75$ ins.

S_c of both sides of the 3 ducts $= 7{\cdot}75 \times 82{\cdot}75 \times 3{\cdot}14 \times 2 \times 3 = 12{,}100$ sq. ins., whence

$$W_2 = 0{\cdot}00458 \times 5{\cdot}85 \times 12{,}100 \times 20 = 6470.$$

For the rest of the stator core surface, *i.e.* the two end-plates and back, $S_c = \pi \times 90{\cdot}75 \times 11{\cdot}5 + 7{\cdot}75 \times \pi \times 82{\cdot}75 \times 2 = 7300$ sq.

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[1] *Specification and Design of Dynamo-electric Machinery*, pp. 254, 325.

[2] *Journ. I.E.E.*, Vol. 48, p. 703.

ins., and allowing about ¾ watt per sq. in. for radiation and convection $W_3 = 5470$.

The total, $W_1 + W_2 + W_3 = 18{,}310$ is then greater than 17,180, and a temperature rise of 40° C. is not likely to be exceeded.

Calculation of armature reactance, x_a—

By equation (254)

$$\lambda_s = \frac{12{\cdot}57}{3}\left(\frac{1{\cdot}263}{3 \times 0{\cdot}53} + \frac{0{\cdot}089}{0{\cdot}53} + \frac{2 \times 0{\cdot}23}{0{\cdot}53 + 0{\cdot}14} + \frac{0{\cdot}04}{0{\cdot}14} + 0{\cdot}2\right)$$

$$= 4{\cdot}19\,(0{\cdot}795 + 0{\cdot}168 + 0{\cdot}686 + 0{\cdot}286 + 0{\cdot}2)$$

$$= 8{\cdot}95$$

$$l\lambda_s = 10 \times 2{\cdot}54 \times 8{\cdot}95 = \qquad 227$$

By equation (255)

$$\lambda_c = 10 + \frac{9{\cdot}2}{9}\log\frac{1{\cdot}285}{2 \times 0{\cdot}14} = 10{\cdot}68$$

$$L\lambda_c = 11{\cdot}5 \times 2{\cdot}54 \times 10{\cdot}68 = \qquad 315$$

Width of two tooth-pitches 2·63 ins.
,, one bare conductor 0·34 ,,
2·97 ,,
Height of 4 conductors 1·263 ,,
$a + b$ of projections 4·233 ,,

By equation (256)

$$\lambda_p = 4{\cdot}6\log\frac{4{\cdot}04 \times 11{\cdot}75}{4{\cdot}233} = 4{\cdot}82$$

$$(l_1 - l)\lambda_p = 10{\cdot}6 \times 2{\cdot}54 \times 4{\cdot}82 = \qquad 130$$

Width of three bare conductors = 1·02 ins.
,, two insulations = 0·333 ,,
1·353 ,,
Height of 4 conductors = 1·263 ,,
$a' + b'$ of ends, say = 2·65 ,,

By equation (257)

$$\lambda_e = 4{\cdot}6\log\frac{1{\cdot}57 \times 8{\cdot}7}{2{\cdot}65} = 3{\cdot}27$$

$$l_3\,\lambda_e = 11{\cdot}75 \times 2{\cdot}54 \times 3{\cdot}27 - \qquad 98$$

By equation (253) 770

$$x_a = 2\pi \times 50 \times 2880 \times 770 \times 10^{-9} = \qquad 0{\cdot}696$$

Calculation of x_b.

$$\frac{YL_f}{Kl_g}\left\{1-\cos\left(\frac{\pi}{2}\times\frac{y}{Y}\right)\right\} = \frac{11{\cdot}75\times 11\times 6{\cdot}45}{1{\cdot}02\times 0{\cdot}762}\left\{1-\cos\left(90^\circ\times\frac{8{\cdot}84}{11{\cdot}75}\right)\right\}$$

$$= \frac{835}{0{\cdot}777}\,(1-\cos 67{\cdot}6^\circ) = 665$$

$$k_{s1}' = \frac{\sin 30^\circ}{3\sin 10^\circ} = \frac{0{\cdot}5}{3\times 0{\cdot}1736} = 0{\cdot}96$$

$$k_{s1}'' = \sin\left(\frac{\pi}{2}\times\frac{y'}{Y}\right) = \sin\left(90^\circ\times\frac{9}{9{\cdot}15}\right) = \sin 88{\cdot}5^\circ = 0{\cdot}999$$

$$\frac{8f}{2p\,N_{ph}}\left(\frac{Zk_{s1}'\,k_{s1}''}{q}\right)^2 = \frac{8\times 50}{20\times 3}\left(\frac{720\times 0{\cdot}96}{1}\right)^2$$

$$= 6{\cdot}66\times 47{\cdot}8\times 10^4 = 318\times 10^4$$

By equation (248*a*) therefore

$x_b = 665\times 318\times 10^{-5} = 2{\cdot}11$

$Ix_b = 201\times 2{\cdot}11 = 424$

V_e per phase $= 2300/1{\cdot}732 = 1328$

$Ix_b/V_e = 424/1328 = 0{\cdot}32$

$\therefore$ by Fig. 539, E_a volts per phase at full-load = about $1{\cdot}055\,V_e = 1400$.

Accurate calculation of volts and flux-density at full-load.

$x_a + x_b = 0{\cdot}696 + 2{\cdot}11 =$ $2{\cdot}806$

By equation (249), $\tan\phi_o = \dfrac{1328\times 0{\cdot}6 + 201\times 2{\cdot}806}{1328\times 0{\cdot}8 + 201\times 0{\cdot}08}$

$= \dfrac{1362}{1076} =$ $1{\cdot}265$

$\phi_o = 51{\cdot}66^\circ \qquad \sin\phi_o = 0{\cdot}7844$

$OD = \sqrt{1362^2 + 1076^2} = 1735$

$E_a = OD - Ix_b\sin\phi_o = 1735 - 424\times 0{\cdot}7844 =$ 1403

Accurately with 3 slots per pole per phase,

$m = 2{\cdot}22\times 0{\cdot}96\times 50\times 240\times 10^{-8} = 255{\cdot}7\times 10^{-6}$

$$B_{g1} = \frac{1797\times 10^{-6}}{255{\cdot}7\times 10^{-6}}\,E_a = 7{\cdot}03\,E_a$$

Magnetic circuit of stator core and air-gap.

Radial depth of iron at back of teeth 6 ins.

Three ventilating ducts, each $\frac{1}{2}$ in. wide.

Double section of stator core,

$2 \times 6 \times 0{\cdot}9(11{\cdot}5 - 1{\cdot}5) \times 6{\cdot}45 =$ 696 sq. cm.

Single air-gap slightly chamfered, 0·281 in. at centre of pole opening out to 0·33 in. at extreme edges, mean length $l_g = 0{\cdot}3 \times 2{\cdot}54 =$ 0·762 cm.

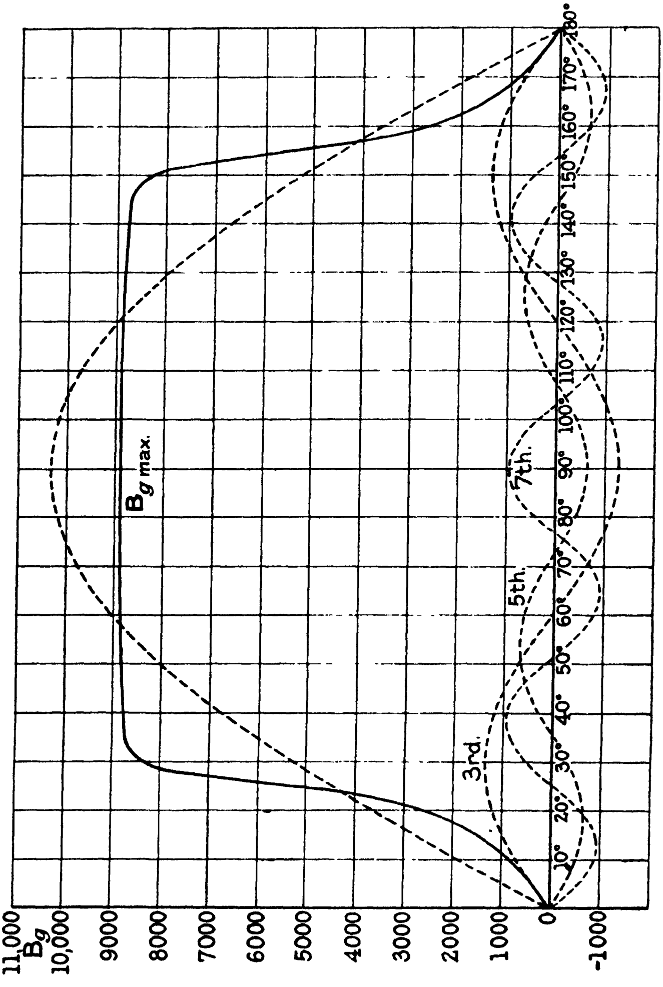

FIG. 618*a* —Flux-density curve.

$B_{gx} = 10{,}314 \sin \alpha + 1308 \sin 3\alpha - 689 \sin 5\alpha - 989 \sin 7\alpha + \ldots$
$= 10{,}314 (\sin \alpha + 0{\cdot}1265 \sin 3\alpha - 0{\cdot}0666 \sin 5\alpha - 0{\cdot}0956 \sin 7\alpha + \ldots)$

Ratio of polar arc to pole-pitch, $\beta = 0{\cdot}69$

Polar arc 8·12 ins.

Interpolar gap, $11{\cdot}75 - 8{\cdot}12 =$ 3·63 „

Half of one interpolar gap, $c = 1{\cdot}815$

$c/l_g = 1{\cdot}815/0{\cdot}3 = 6{\cdot}05$ $K_1 = 2{\cdot}3$ by Fig. 253

$A' + K_1 l_g = 8{\cdot}15 + 2{\cdot}3 \times 0{\cdot}3 =$ 8·84 „

$a/l_g = 0{\cdot}25/0{\cdot}3 = 0{\cdot}83$. By Fig. 254, $K_2 = 1{\cdot}1$
$w_d/l_g = 0{\cdot}5/0{\cdot}3 = 1{\cdot}66$. By Fig. 256, $K_3 = 0{\cdot}21$
$L_f + K_2 l_g - K_3 w_d n_d = 11 + 1{\cdot}1 \times 0{\cdot}3 - 0{\cdot}21 \times 3 \times \frac{1}{2} = 11{\cdot}02$ ins.
Effective area of air-gap = $8{\cdot}84 \times 11{\cdot}02 \times 6{\cdot}45 =$ 630 sq. cm.
$w_3/l_g = 0{\cdot}14/0{\cdot}3 = 0{\cdot}46$. $w_{t1}/w_3 = 1{\cdot}085/0{\cdot}14 = 7{\cdot}75$
By Fig. 262, K = say, 1·02
$AT_g = 0{\cdot}8\, B_g \times 1{\cdot}02 \times 0{\cdot}3 \times 2{\cdot}54 = 0{\cdot}622\, B_g$.

Calculation of pole-face potential, AT_p, at no-load (normal volt and 20 per cent. increase) and at full-load.

	No-load.		*Full-load.*
Volts per phase	1328	1600	1403
$B_{g1} = 7{\cdot}03\, E_a =$	9340	11,250	9865
B_g (corresponding to mean air-gap length) $= B_{g1}/1{\cdot}12 =$	8320	10,040	8800
Φ_a in megalines $= 630\, B_{g\ max}$	5·24	6·34	5·55
$0{\cdot}622\, B_{g\ max} =$ **AT$_g$** =	**5170**	**6250**	**5470**
$B_c = \Phi_a/696$	7520	9100	7960
$at =$	1·8	2·6	2
$l_c/2 = 6{\cdot}25$ ins. $\times 2{\cdot}54 = 15{\cdot}9$ cm. **AT$_c$** =	**29**	**41**	**32**
Q (Vol. I, p. 501) =			
$1{\cdot}285 \times 11{\cdot}02/\{0{\cdot}9\,(11{\cdot}5 - 1{\cdot}5)\} = 1{\cdot}575$			
Q' (Vol. I, p. 502) $= 11{\cdot}5/9 = 1{\cdot}28$			
$Q \times B_g =$	13,100	15,810	13,850
Uncorrected density at crown of tooth			
$B_{t1}' = QB_g/w_{t1} = QB_g \div 1{\cdot}145$	11,450	13,820	12,100
Corrected, say, $0{\cdot}9\, B_{t1}$	10,300	12,450	10,900
at_1	3·6	6·6	5
Uncorrected density at level of bottom of wedge, $QB_g/0{\cdot}765$	17,150	20,700	18,100
Slot ratio, K_s (equation 117) = $1{\cdot}295 \times 1{\cdot}28/0{\cdot}765 = 2{\cdot}17$			
Corrected density =	—	20,300	17,950
at	66	280	100
Uncorrected density at centre of tooth			
$QB_g/0{\cdot}785$	16,700	20,200	17,650
Slot ratio, $K_s = 2{\cdot}11$			
Corrected density =	—	19,900	17,630
at_{tc}	52	236	83

	No-load.		*Full-load.*
Uncorrected density at root of tooth			
$QB_g/w_{t2} = QB_g/0{\cdot}815$	16,100	19,400	17,000
Slot ratio $K_s = 2{\cdot}04$	—	19,200	—
at_2	38	170	60
Average at_t	50	225	79
$l_t = 1{\cdot}75$ ins. × 2·54 = 4·45 cm. $\mathbf{AT}_t =$	**222**	**1000**	**351**

By equation (243), $k_D =$

$$0{\cdot}574 \times \frac{11{\cdot}75}{8{\cdot}84} \times \sin 67{\cdot}6^\circ \times 0{\cdot}96 \times 0{\cdot}999$$

$$k_D \frac{JZ}{4p} \sin \phi_o = 0{\cdot}675 \times 3618 \times 0{\cdot}7844 = \mathbf{AT}_{DM} = \mathbf{0} \qquad \mathbf{0} \qquad \mathbf{1912}$$

$$AT_g + AT_c + AT_t + AT_{DM} = \mathbf{AT}_P \quad = \quad \mathbf{5421} \qquad \mathbf{7291} \qquad \mathbf{7765}$$

The field-magnet.

Pole-cores, solid steel, $5\frac{1}{2}$ ins. wide × 11 ins. parallel to shaft with corners rounded to 1 in. radius.

Area $= 3\frac{1}{2} \times 11 + 2 \times 9 + \pi \times 1^2 = 59{\cdot}64$ sq. ins. = 384 sq. cm.

Magnet wheel of cast-steel, rim 13 ins. × $3\frac{1}{2}$ ins.

Double section of yoke = 13 × 7 = 91 sq. ins. = 586 sq. cm.

$\mathfrak{S}_l = 63$ in pole-core, and say 90 in yoke.

Check on estimate of $\mathfrak{S}_l$

By the expressions of Chapter XXVIII, § 23.

$$d_p = d_t - c/2 = 1{\cdot}25 - 0{\cdot}907 = 0{\cdot}343 \text{ in.}$$

$$\mathfrak{S}_1 = \frac{0{\cdot}343}{3{\cdot}63} \times 11'' \times 2{\cdot}54 \times 2 = \qquad 5{\cdot}3$$

$$\mathfrak{S}_2 = 1{\cdot}86 \times \frac{10}{9} \log \frac{\pi \times 1{\cdot}1875 \times 0{\cdot}9 + 3{\cdot}63}{3{\cdot}63} \times 11'' \times 2 = \qquad 12{\cdot}85$$

$$\mathfrak{S}_3 = 1{\cdot}86 \log \frac{\pi \times 2{\cdot}03 + 3{\cdot}63}{3{\cdot}63} \times 0{\cdot}343 \times 4 = \qquad 1{\cdot}12$$

$$\tan 90^\circ/10 = 0{\cdot}1584$$

From the drawing, $f = 30 - \frac{2{\cdot}75}{0{\cdot}1584} = 30 - 17{\cdot}4 = 12{\cdot}6$ ins.

$$h/\sqrt{p} = 6 \times 0{\cdot}316 = 1{\cdot}9 \text{ ins.}$$

$$\mathfrak{S}_4 = \frac{10}{3{\cdot}14} \left\{ 0{\cdot}684 - 2{\cdot}3 \frac{12{\cdot}6}{6} \log \frac{18{\cdot}6}{14{\cdot}5} \right\} \times 11'' \times 2{\cdot}54 \times 2 = \qquad 29{\cdot}4$$

$$a = 12{\cdot}6 \times 0{\cdot}1584 = 2 \text{ ins.}$$

$$\mathfrak{S}_5 = \frac{1{\cdot}9}{\sqrt{4 \times 10 + 36}} \times 11'' \times 2{\cdot}54 = \qquad 6{\cdot}1$$

$$\mathfrak{S}_6 = 0{\cdot}576 \times 5{\cdot}5 \times 2{\cdot}54 = \qquad 8{\cdot}05$$

$$\mathfrak{S}_l = 62{\cdot}82$$

say, 63

Ampere-turns for pole and yoke.

	No-load.		*Full-load.*
Φ_a = in megalines	5·24	6·34	5·55
ϕ_l in pole-core = $1{\cdot}257\,AT_p \times 2 \times 63$ =	0·86	1·155	1·23
Φ_m =	6·1	7·495	6·78
$B_m = \Phi_m/384$	15,900	19,500	17,650
at_m =	24	160	76
l_m = 7·5 ins. × 2·54 = 19 cm. $\mathbf{AT}_m$ =	**456**	**3040**	**1445**
ϕ_l in yoke = $1{\cdot}257\,AT_p \times 2 \times 90$	1·23	1·65	1·76
Φ_y =	6·47	7·99	7·31
$B_y = \Phi_y/586$	11,050	13,600	12,450
at_y =	5	12	8
$l_y/2$ = 4 ins. × 2·54 = 10·16 cm. $\mathbf{AT}_y$ =	**51**	**122**	**81**
$\mathbf{AT}_p$ =	**5421**	**7291**	**7765**
Per pole $\mathbf{AT}_f$ =	5928	10,453	9291
say,	**5930**	**10,500**	**9300**

From Fig. 541 the regulation is found to be 19 per cent., and the specified condition is fulfilled. The next step will show that, so far as heating of the field bobbins is concerned, there is a good margin of permissible increase in the excitation above 9300 AT, but the possible overload or lowering of the power-factor is not large, as shown by Fig 535. The design illustrates the case of a machine with field pole well saturated.

Field-magnet winding.

Net winding length, 5½ ins.

Magnet winding, flat strip, $1\frac{3}{16}$ in. × $\frac{1}{16}$ in. = 0·0743 sq. in.

Resistance per 1000 yds. = 0·02445/0·0743 = 0·33 ohm.

Length of mean turn = $2(3\frac{1}{2} + 9) + 2\pi \times 1{\cdot}97 = 37{\cdot}35$ ins. = 1·036 yd.

Length of outer turn = $2(3\frac{1}{2} + 9) + 2\pi \times 2{\cdot}565 = 41{\cdot}1$ ins.

Insulation, 0·01 paper between turns.

Turns per coil, 5·5/0·0725 = 76, less one, = 75.

Yds. of 20 coils, 75 × 20 × 1·036 = 1555.

Resistance of 20 coils, hot = 0·33 × 1·555 × 1·17 = 0·6 ohm.

Amperes = 124.

Volts = 74·5.

Watts per coil = 462·5.

Total watts in excitation = 9250 = 1·45 per cent. of output.

If at 100 volts with rheostat in circuit, 12,400 watts = 1·94 per cent.

Surface of rounded ends of coil = $(41{\cdot}1 - 22) \times 5\frac{1}{2} =$ 104·5 sq. ins.
Surface of one end-flange = $37{\cdot}37 \times 1{\cdot}1875 =$ 44·4 ,,
$\sqrt[3]{s/l} = \sqrt[3]{2{\cdot}6/11} = 0{\cdot}619$
Reduced surface of sides = $22 \times 5\frac{1}{2} \times 0{\cdot}619 =$ 74·9 ,,
$S_c = 223{\cdot}8$,,

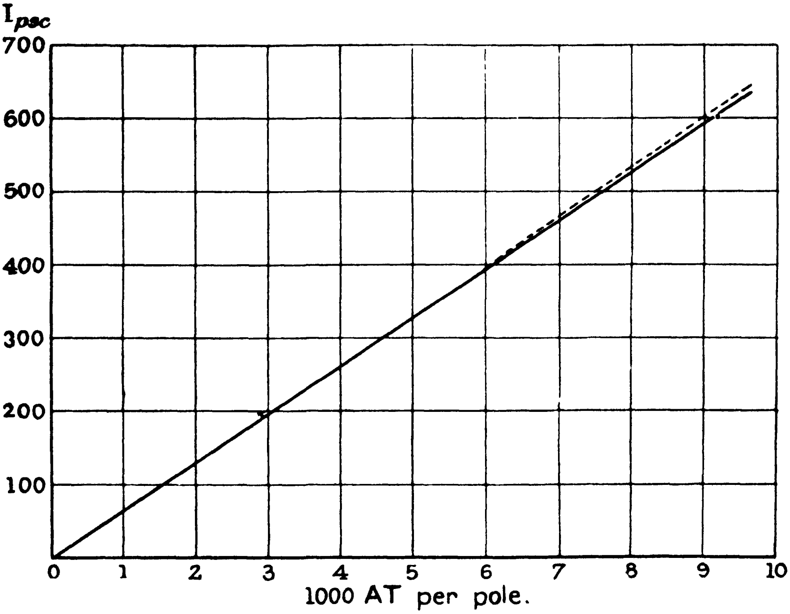

Fig. 618*b*.—Permanent short-circuit current under varying field-excitation.

Mean velocity of coil = 5200 ft. per min.
By equation (270), $0{\cdot}7\ (v/1000)^{1{\cdot}2} = 5{\cdot}06$
$k = 85/6{\cdot}06 = 14.$
$t°\text{ C.} = (14 \times 462{\cdot}5)/223{\cdot}8 = 29°$ C. rise of outside.
At 30° C. temperature of room and, say, 31·5° C. mean rise,

$\frac{R_f \text{ hot}}{R_f \text{ at } 20°\text{ C.}} = \frac{234{\cdot}5 + 61{\cdot}5}{234{\cdot}5 + 20} = 1{\cdot}16$, so that the original allowance of 1·17 is more than sufficient.

Permanent short-circuit current.

In equation (251*a*), $k_D \frac{Z}{4pq} = 0{\cdot}675 \times \frac{720}{40} = 12{\cdot}15.$

In relation to Φ_a, $m = 253 \times 10^{-6}$, and

$$\frac{0{\cdot}8 \times 0{\cdot}696}{0{\cdot}000253} = 2200.$$

$\mathfrak{R}_g + \mathfrak{R}_c + \mathfrak{R}_t = 0{\cdot}00125$, and $\mathfrak{R}_m + \mathfrak{R}_y$ rises from 0·000024 with $I_{sc} = 100$ to 0·000048 with $I_{sc} = 600$, while ν remains constant at 1·87. Consequently the true value of $\mathfrak{R}_{do}$ rises from 0·001295 to 0·00134, and the permanent short-circuit current in relation to increasing excitation is as shown by the full line of Fig. 618*b*. It is thus practically represented by

$$I_{psc} = \frac{AT_{fsc}}{12{\cdot}15 + 2{\cdot}2 \times 1{\cdot}3} = \frac{AT_{fsc}}{15}$$

as shown by the dotted straight line. Under the full-load excitation of 9300 AT, its true value is 618 amperes = 3·08 times the normal full-load current of 201 amperes.

§ 11. Experimental measurement of armature inductance.—By means of the oscillograph, the true angle of lag ϕ_o between E_a and the current can be measured[1] when the alternator is at work on a non-inductive resistance for which $\phi_e = 0$. It then follows from equation (249*a*) that under these conditions

$$2\pi f(\mathcal{L}_a + \mathcal{L}_r) = q\left(r_a' + \frac{V_e}{I}\right)\tan\phi_o$$

The inductance of one phase of the armature varying according to its position relatively to the poles has, when plotted, the characteristic shape of Fig 619. The electrical degrees are here reckoned from a zero position when the axes of poles and coils coincide; although not always the case, the smallest figure is then usually obtained, the largest value occurring when the axes of coils and poles are crossed.[2] The temporal value may be expressed as $\mathcal{L} = A + B\cos 2\pi f\,.\,2t + \mathcal{L}_a$, where $A + B = \mathcal{L}_c$ and $A - B = \mathcal{L}_d$. If, therefore, an alternating current of the normal frequency is passed through one phase, and the voltage across the ends with the corresponding current be noted, the quotient V/I first with the axes of the coils of the phase exactly midway between the poles and secondly with the axes of the coils coincident with the centres of the poles, gives the impedance for the two positions, from which can be determined the two values of the reactance or of the inductance, namely, $\mathcal{L}_c + \mathcal{L}_a$, and $\mathcal{L}_d + \mathcal{L}_a$. As the field-system becomes saturated, the inductances diminish, especially when the axes of coils and poles are crossed, so that it is very necessary to excite the field-magnet at the same time with its normal current. The value of the alternating current through the phase is of lesser importance, as the armature current has but little effect in saturating the magnetic circuit. From the values of $A + B + \mathcal{L}_a$, and $A - B + \mathcal{L}_a$, B can be determined, and also $A + \mathcal{L}_a$, but it is still necessary to calculate A in order to separate out $\mathcal{L}_a$. This is best done by means of an approximate estimate of $\mathcal{L}_c$, *i.e.* of $A + B$. In calculating $\mathcal{L}_c$ from the data of Chapter XXVII, § 35, it must be borne in mind that only a single-phase current is used for the test, and that k_C must correspond thereto. The calculation of l_d is still more liable to error through the indefiniteness of the amount of damping The pulsating character of either the cross or direct flux with a single-phase current (*cp*. Chap XXVII, § 34) introduces, in fact, considerable doubt as to the validity of the results when applied to 3-phase alternators On the other hand, if a 3-phase current were used for the same experiment with the rotor stationary in its two crucial positions, the current passing through its alternations in each position does not truly represent the facts when the field-magnet rotates at the speed of synchronism. To supplement the above test, therefore, even better information is obtained, *e g.* in the case of a star-connected 3-phase armature by sending the alternating current through phase *A*, and collecting it again from the two other phases *B* and *C* in parallel, and again by passing the alternating current through two of the phases, say, *A* and *B*, in series, the rotor in each experiment being set relatively to the coils of phase *A* in its correct position corresponding to these momentary cases The phase *A* is thus at least placed under the conditions which would actually hold at the moments when the working current passes through its maximum or through 0·866 of its maximum value, and the inductance includes, as it should, any mutual inductance in these positions from the other phases

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[1] A. Blondel, "Tests of Alternators," *Trans. Intern. Electr. Congress, St. Louis*, 1904, Vol. 1, p. 631.

[2] *Cp.* especially Rushmore, *Trans. Intern. Electr. Congress, St. Louis*, 1904, Vol. 1, p. 724 ff.; and Hobart and Punga, *Elektrische Kraftbetriebe u. Bahnen*, Vol. 5, p. 611 ff.; *cp.* also Chapter XXVII, §§ 48 and 49, for other references.

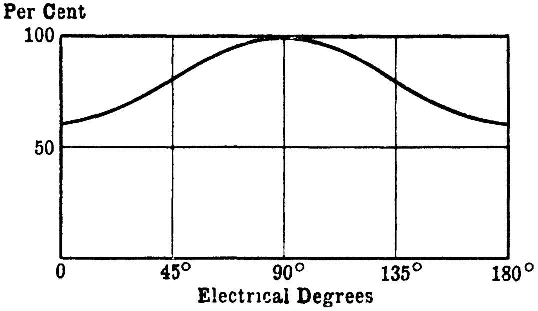

FIG 619.—Armature inductance as dependent upon position relatively to field-poles

If a current is passed through the armature of an alternator and the field-magnet is rotated without excitation, the pulsation of the ammeter and voltmeter readings which have been mentioned in Chapter XXVII, § 34, can be used to obtain an approximate estimate of the impedances corresponding respectively to $2\pi f(l_c + l_a)$ and $2\pi f(l_d + l_a)$ even in a polyphase machine The rotor must be coupled to a motor and driven very slightly faster or very slightly slower than the speed of synchronism ; a small polyphase current (not enough to allow the machine to develop synchronous action) is then supplied to the armature, and when the beats are slow, the limiting values of the voltmeter and ammeter beats can be read , their instantaneous quotients will then give the maximum and minimum impedance, but a clue as to which corresponds to coincidence of the axes of poles and coils is still wanting, and with the normal field-excitation the conditions are again altered. There must also be no residual magnetism in the field-system

It has been shown in Chapter XXVII, § 12 (equation 234), that with several slots per pole per phase, and with each phase-belt covering 60° (electrical), the amplitude of the fundamental of the joint M M F wave is $\frac{4\pi t}{10} \times 0{\cdot}911\,\mathbf{I}$. If, therefore, a 3-phase current of normal frequency *f* is supplied to a 3-phase alternator with rotor removed, the self-induced flux across the bore is practically constant and rotates with frequency *f*. As shown by M. Schenkel,[1] the value of this flux issuing from one pole-pitch is then $2 \times \frac{4\pi}{10} t \times 0{\cdot}911\,\mathbf{I} = 3{\cdot}24tI$ per cm. length of core, where I is the virtual value of the current. If, therefore, L is the length of the armature core, the flux cutting the stator winding generates a phase E.M.F. (assuming a differential factor of 0·955)

$$2{\cdot}12 \times 2ptf \times 3{\cdot}24tIL \times 10^{-8} = 13{\cdot}75pft^2\,LI \times 10^{-8}$$

and an interlinked E.M.F. with star connection

$$\sqrt{3} \times 13{\cdot}75pft^2\,LI \times 10^{-8} \text{ volts.}$$

If, therefore, this amount be deducted from the total measured volts applied at the terminals, the remainder divided by I indicates at least a minimum value for the true inductance of the armature in 3-phase work.

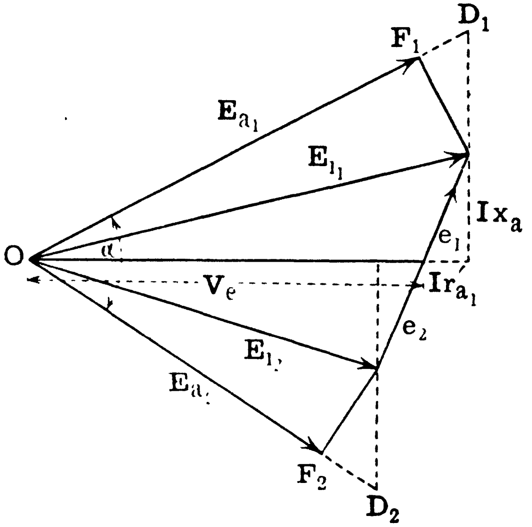

FIG. 620.—Two alternators mechanically coupled together, with external current in phase with terminal voltage.

§ **12. The testing of salient-pole alternators.**—If a pair of similar alternators are available, their efficiency is best measured by a modification of the Hopkinson test, which renders it suitable for alternating currents. The two rotors are coupled rigidly together, but with a slight difference in the positions of the poles relatively to the stator coils in the two cases, so that a difference of phase of the induced E.M.F.'s results. When the combination is now driven by a continuous-current motor of which the efficiency and losses at different speeds are accurately known, the one alternator acts as a motor and the other as a generator; slots in the coupling enable the difference of phase of the two machines to be altered, and thus the input of the one and output of the other are regulated until they reach the required amounts.[2] Then, as in Chapter XXII, § 18, if W_1 be the output of the generator as measured by a wattmeter, and L be the total power supplied from the external source less the loss of power over the two armature resistances, *i.e.* $L = W_1 - I^2 (r_{aD} + r_{aM})$, the efficiency of the generator is similarly to equation (223)

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[1] *Elektrotechnik und Maschinenbau*, Vol. 27, p. 251, abstracted in *Electrician*, 23rd July, 1909.

[2] A complete account of such a test is to be found in *Proc. C.E.*, Vol. 130, p. 247, "Alternating-current Dynamo Tests" (H. F. Parshall). Cp. *Electr. Eng.*, Vol. 37, p. 375.

$$\eta = \frac{W'_1}{W'_1 + I^2 r_{aD} + L/2 + I_x^2 R_x}$$

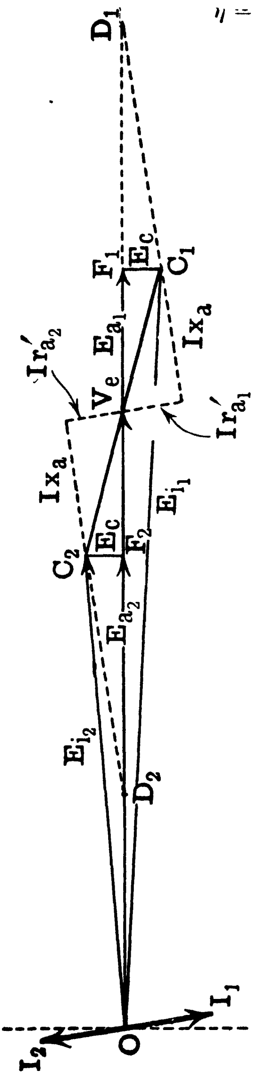

FIG. 621 —Two alternators mechanically coupled together without difference of phase.

With particular values of the two excitations, the current at the terminals will be in phase with the terminal voltage V_e; the vector diagram of the two alternators in parallel will then be as in Fig 620, the generator acting as if working on a non-inductive resistance Hence if a comparison of the readings of a wattmeter with those of an ammeter and voltmeter across the terminals shows that the current is in phase with V_e, by equation (249*a*) the value of $\mathcal{L}_a + \mathcal{L}_c$ can be determined as

$$2\pi f (\mathcal{L}_a + \mathcal{L}_c) = q\left(r_a' + \frac{V_e}{I}\right) \tan\frac{a}{2}$$

where a is the total phase difference between the two machines

Since Ir_a' is small as compared with V_e, it is evident that in Fig 620 D_1D_2 will almost become a straight vertical line, and the armature E M F.'s and fluxes of the two alternators are then very nearly identical. The terminal voltage vector falls midway between E_{a1} and E_{a2}, and since ϕ_0 has the same value in both cases, namely, $\pm \frac{a}{2}$, it results that the excitations are equal, the direct magnetizing ampere-turns of the armatures being in each case demagnetizing; for though the current is oppositely directed in the motor, it has also the opposite phase relation to the induced E M F

If the two alternators are coupled together without any difference of phase, and they are unequally excited (Fig 621), the vector difference $\overline{E_{i1} . E_{i2}}$ between the induced E M F 's, or $C_1 C_2$, is the fall of potential over the impedances of the two armatures in series, corresponding to the resultant current which is displaced by nearly 90° from the E M F 's Since one of the components of the vector C_1C_2, namely, $I(r_a{'}_1 + r_a{'}_2)$ is small, $C_1 C_2$ is nearly equal to $F_1 F_2$, *i e* to the difference between the armature E M F 's $E_{a1} - E_{a2} = 2Ix_a$ Whatever occurs in the generator armature is reversed in the motor armature, so that E_C has the same numerical value in the two machines, and so also has AT_{DM}, but in the generator it is demagnetizing and in the motor forward-magnetizing On the apparent synchronous impedance assumption, therefore, the open circuit E M F of the generator would be $E_{01} = OD_1$, to which would correspond AT_1, while the open circuit E.M F. of the motor would be $E_{02} = OD_2$, to which would correspond AT_2. Knowing the two excitations AT_1 and AT_2, and adopting a single total characteristic of excitation with approximate allowance for the full back turns (the transverse reaction being practically negligible), we can thence check the assumed values of Ix_a and AT_{DM} by the construction of Fig. 622. On either side of the point on the characteristic corresponding to the terminal voltage, mark off two points C_1 and C_2, the one higher than V_e by the same amount as that by which the other is lower, *i.e.* by Ix_a in each case; the differences $AT_1 - OF_1$ and $OF_2 - AT_2$, *i.e.* the amount by which AT_1 exceeds OF_1 and AT_2 falls short of OF_2 must then each be equal to AT_{DM}, demagnetizing in the one case and magnetizing in the other, while $E_{a1} - E_{a2}$ must yield the observed current.

The three phases of two similar 3-phase machines can be similarly coupled, care being taken that the measuring ammeter and wattmeter do not interfere with the balance of the phases. Analogous methods of determining the various factors when only a single machine is available by running it as a synchronous motor have been described by Prof. André Blondel.[1]

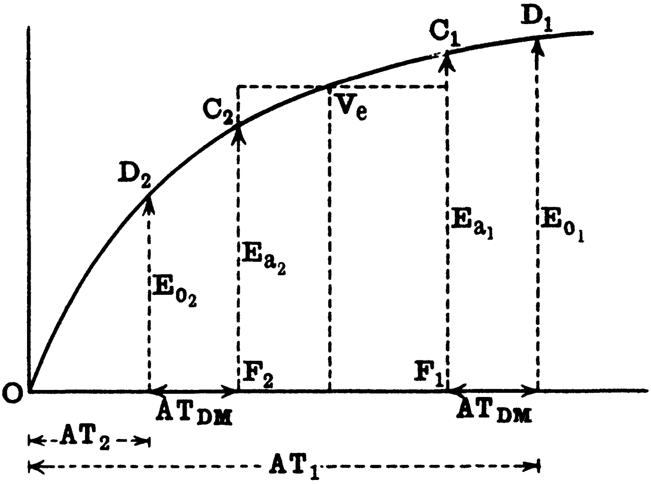

FIG 622 —Determination of AT_{DM} and Ix_a from excitations of two alternators mechanically coupled together

An extension of the Hopkinson method of testing to the case of a single alternator has been given by Mr Mordey [2] By it the armature coils were divided into two unequal groups and connected in opposition, one group acting as a motor, and the larger group as a generator, so that only the difference of their E M F 's was effective in circulating an almost wattless current round the armature, exactly as in Fig 536. With the heteropolar type of field-magnet and numerous poles it is equally easy to divide the poles into two unequal groups and connect these in opposition, two adjacent poles at each end of the groups having the same polarity But the objection to both variations is that the motor fields are powerfully strengthened by the armature current and the generator fields similarly weakened. The flux-densities are, therefore, very different, and the motor group must be reduced in number to allow for its much greater density of flux Unbalanced magnetic pulls are then set up between the rotating and stationary parts, and vibration ensues.

When there are 12 or more poles, this objection is entirely overcome by a modification due to Prof. S. P. Smith ; the rotor winding is opened at 4 places as a minimum, and the pole-pairs thus divided into two diametrically opposite larger groups, each comprising $A/2$ pole-pairs, and two smaller groups, each comprising $B/2$ pole-pairs, which are also diametrically opposite. The full-load exciting current traverses all the field-coils, but its direction in each of the $B/2$ groups being reversed, the former act as generator poles and the latter as motor poles. The magnetic pulls, being symmetrical with respect to the shaft or axis of the machine, are thus in balance. When the numbers of pole-pairs, A and B, to pass the normal armature current over the impedance, z_a, are not conveniently reached with 4 openings, 6 or 8 can be similarly used, and with some loss of complete symmetry two sections may each be given an uneven number of poles[1] At each opening a drum coil acted upon by the two poles of similar polarity only gives an E.M F. proportional to the *difference* in shape and content of their flux waves, and this yields an excess in favour of the motor action, due to the forward-magnetizing AT of the armature increasing the motor flux ; at each of the $2q$ openings, therefore, a pole is virtually lost to the generator sections, and the amount added to the motor sections is practically half of E_r, the volts per pole-pair due to armature reaction. The volts per pole-pair of generator and motor being respectively $E_g = E - \frac{1}{2} E_r$ and $E_m = E + \frac{1}{2} E_r$,

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[1] *Trans. Intern. Elect. Congress, St. Louis*, 1904, Vol. 1, p. 625.

[2] " On Testing and Working Alternators," *Journ. I.E.E.*, Vol. 22, p. 116.

$$xE_g - (y\,E_m + q\,E_r) = Iz_a$$

$$x + y = p - q, \text{ and } A = x + q/2,\ B = y + q/2.$$

Many other methods of making temperature tests within the factory without the expenditure of much power have been devised. The first and best is to run the machine as a synchronous motor without mechanical load at full rated volts and full armature current, leading in phase with practically zero power-factor. The armature reaction being demagnetizing, the field must be over-excited, so that its temperature rise is unduly high, and correction must be made for this To obviate this, the machine may be run alternately for different periods with a leading armature current and field over-excited, and a lagging current with field under-excited. The test in either form requires, however, equivalent kVA generating capacity, and if this is furnished by a duplicate machine, the circulating-energy method may also be available and is preferable.

By the direct-current open-delta method, the three phases of a 3-phase alternator are connected in delta with one corner open, at which point direct current is introduced.

A third method consists in running the machine first on open circuit and then on short-circuit either separately in each case until temperatures are constant, or alternately for different periods But both the latter methods are at best hardly better than makeshifts,[2] owing to their departure from the true working conditions.

The object of all such methods is to do away with the necessity for and cost of a full-load prolonged run at the makers' works in order to determine the heating. With the large alternators of modern times it may not be possible to supply the necessary driving power at full-load at the makers' factory ; hence the development of such compromise tests which only call for driving power to make up the losses. In practice, too, the regulation can be calculated from the open-circuit characteristic and low power-factor or short-circuit tests quite as accurately as it can be taken by direct observation under the disadvantageous conditions of testing at the factory, when it may be difficult to maintain perfectly constant steam pressure and speed[3]

The retardation method of testing the various separate losses under different conditions is especially suitable to the case of alternators, owing to the large moment of inertia of the rotor which allows of the readings being taken very accurately. The results of such a test on a 150 kW alternator are given by M Routin in *L'Éclairage Électrique* (Vol. 9, p 170), and an electrical method of measuring the change of speed as the alternator slows down has been suggested by Dr. Sumpner.[1]

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[1] *Journ I.E E.*, Vol. 42, p 190, with discussion.

[2] For a valuable comparison of these methods, see Mr F D Newbury, Messrs. E. F Collins and W E Holcombe, *Trans Amer. I E E.*, Vol. 32, Part I, p. 649 ; *ibid*, p 667

[3] For suggestions as to testing of alternators in the factory, see R Goldschmidt, " Artificial Loading of Alternating-current Machinery," translated from *E.T.Z.* in *Electr. Eng.*, Vol 28, p. 848 ; S. Senstius, *Trans. Amer. I.E.E.*, " Heat Tests on Alternators," Vol. 35, p. 311, with discussion.

A truly non-reactive circuit for absorbing the output of an alternator on a full-load test is not easily obtained. A water-bath is non-inductive and is the most convenient form, but it may possess some capacity, so that there remains some doubt[2] whether $\cos\phi_e$ is 1. With wire coils or transformers lightly loaded, a wattmeter must be used to determine the true power.

§ 13. Iron losses in armature core and teeth.—It has been shown in Chapter XIV, §§ 8 and 9, that in reality there is no true physical constant of hysteresis η and no one power of B which when multiplied by such a constant yields correct results for the alternating hysteresis loss over any very large range. As a fact, $\eta B^{1\cdot6}$ is only strictly true for low flux-densities from, say, $B = 5000$ to $B = 10{,}000$ For the higher flux-densities in armature cores from, say, $B = 7000$ to $B = 14{,}000$, an equally good, if not better, approximation is given in very many cases by ηB^2 with a different value[3] for η. Of course, upon the assumption of a constant, say, $\eta = 0\cdot003$, for a given curve of hysteresis loss experimentally obtained, the value of $B^x/10^7$ can be worked out and plotted in relation to B; if then another grade of iron be assumed to give a loss curve of analogous shape, the loss can be deduced from the above $B^x/10^7$ curve multiplied by a lower or higher value for η But, strictly, the division into the two quantities η and B^x is unreal, and it would be more sound to assume directly two curves for h, the joules per c cm. per cycle (*cp* Fig. 216) for two grades of iron, one, say, a good sample and one an ordinary sample of poorer quality, and to work from these by interpolation according to the quality which is thought to be in use.

Expressing the hysteresis loss in an alternating field as proportional to B^2, and embodying the expression of Chapter XXI, § 19, for the eddy-current loss also in an alternating field, the loss in a volume of V_c c. cm of iron may be expressed per c cm as

$$W = \left(\alpha_1 B^2 f + \frac{\pi^2 t^2 f^2}{6\rho \times 10^6} B^2\right) \times 10^{-10}$$
$$= (\alpha_1 + f\beta_1)\, B^2 f \times 10^{-10}$$

where α_1 and β_1 are respectively hysteresis and eddy-current coefficients, and $\rho \times 10^6$ is the volume resistivity of the iron in microhms per cm. cube. Practical values for α_1 and β_1 have now to be found for the case of alternator armatures

If w_s is the specific loss in watts per lb of the unworked iron or steel sheet as determined, say, by Epstein's apparatus under the standard conditions of $f = 50$ and $B = 10{,}000$, then analogously to the above expression

$$\frac{w_s}{59} = (\alpha + 50\,\beta) \times 50 \times 10{,}000^2 \times 10^{-10}$$
$$= \tfrac{1}{2}\,(\alpha + 50\beta)$$

1 lb being reckoned as the weight of 59 c cm. of iron. Thence

$$\alpha = \frac{w_s}{29\cdot5} - 50\beta = 0\cdot034\, w_s - 50\beta$$

and if ρ is known, α is found. But even without exact knowledge of ρ, if the sheet is purchased under a guarantee of a maximum w_s, something further can be stated If the discs are 0·05 cm or, say, 0·020 in. thick, and w_s is low, say, 1, ρ must be high; otherwise with this thickness the eddy currents would be so important that the loss would be high. But if the discs are only 0·035 cm or 0·014 in thick, the same low loss must be due mainly to a low value of α itself, since the eddy currents play a much less important part. It cannot, therefore, be far out if for discs 0·035 cm. thick a high value is assumed for ρ, say, 40 microhms, whatever the value of w_s. The error will be on the safe side in so far as it may make a too high. Thence

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[1] *Journ. I.E.E*, Vol. 31, p. 634.

[2] But *cp*. Dr. W. M. Thornton, *Journ. I E E*, Vol 35, p 157; M. B. Field, *Journ. I.E.E.*, Vol. 32, p 651, and K. Wallin, *E T.Z.*, Vol. 27, pp. 739-740.

[3] *Cp*. Fischer Hinnen, *E. u. M.*, Vol. 52, p. 11, which is here followed in part

$$\beta = \frac{1{\cdot}645 \times 0{\cdot}035^2}{40} = 0{\cdot}00005,$$

and therefore
for discs 0·035 cm thick, $a = 0{\cdot}034\, w_s - 0{\cdot}0025$.
But for discs 0·05 cm. thick, it will be more nearly true to make ρ decrease when w_s increases and almost in inverse proportion, *i e* say, $\rho = \frac{41}{w_s}$ microhms.
Thence $\beta = \frac{1{\cdot}645 \times 0{\cdot}05^2 \times w_s}{41} = 0{\cdot}0001\, w_s$, and
for discs 0·05 cm thick, $a = 0{\cdot}034\, w_s - 0{\cdot}005\, w_s = 0{\cdot}029\, w_s$
and as an intermediate case, for discs 0·04 (= 0·016″) thick, $a = 0{\cdot}0308\, w_s$.

To the above figures for a would have to be added, in the case of transformers, some increase to allow for the working up of the discs into the finished article ; and in the case of an alternator for hardening of the disc edges by punching, and for unequal distribution of the flux in the mass of the core Further, in the latter case we really have to deal with rotating-field hysteresis, which for moderate flux-densities, such as are often found in stator cores, gives higher figures for the loss An addition of some 30 per cent. may, therefore, be made, and thence for insertion in the main equation,

$$a_1 = 1{\cdot}3\, a.$$

From Chapter XXI, § 19, the eddy-current loss in the core from a rotating field is $\frac{2{\cdot}78\, t^2}{\rho \times 10^6} f^2 B_c^2\, V_c \times 10^{-10}$ so that $\beta_1 = \frac{2{\cdot}78 t^2}{\rho \times 10^5}$, and for the core as a whole

$$\text{watts} = \left(1{\cdot}3\, a + \frac{2{\cdot}78\, t^2}{\rho \times 10^6} f\right) f\, B_c^2\, V_c \times 10^{-10}$$

If the same expression be also used for the teeth with the substitution of V_t, the volume of the iron in the teeth in cm ^{3}, in place of V_c, the total eddy-current loss so obtained is much lower than the measured value Mr. F. W. Carter[1] shows that by comparison with the ordinary formula $\frac{\pi^2}{6} \cdot \frac{f^2\, t^2\, B^2}{\rho}$ which is based on the assumption that the time-rate of change of flux is sinusoidal, the actual eddy-current loss in the teeth is increased in the ratio practically of $\frac{\text{pole-pitch}}{\text{interpolar space}}$, *i e.* the theoretical loss from eddies in the teeth must be multiplied by about 3 or 4 Fischer Hinnen reaches the conclusion[2] that the value of β_1 for the teeth should be $\frac{\pi^2\, t^2}{6\rho \times 10^6} \times \frac{0{\cdot}5\, D}{2 l_g \cdot p}$, in which the last multiplier has such values as 6 to 9 This has the effect of making by far the larger part of the eddy-current loss occur in the teeth. It is, however, difficult to locate the seat of the eddy-current losses, and it must be remembered that so far no allowance has been made for contact between the discs such as may occur even with careful attention to the insulation and subsequent handling It is, therefore, probably as true to increase empirically the eddy-current coefficient for both core and teeth in the same proportion. A multiplying factor[3] of 4 will, therefore, be adopted, and finally we have

Total iron loss in watts =

$$\left(0{\cdot}13\, a f + \frac{0{\cdot}278 \times 4t^2}{\rho \times 10^5} f^2\right)\left[\left(\frac{B_c}{1000}\right)^2 \frac{V_c}{1000} + \left(\frac{B_t}{1000}\right)^2 \frac{V_t}{1000}\right]$$

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[1] *Electrician*, 21st January, 1916.

[2] *E. u. M.*, Vol 32, p. 32, the argument being based on the assumption that the tooth eddy-current loss occurs only in the teeth between the poles but with a very great rate of flux-variation in those teeth.

[3] The same factor is employed by Pichelmayer, *Dynamobau*, p. 426 ; and Petersen, *Wechselstrommaschinen*, p. 167, gives 3 to 5.

Assuming discs 0·05 cm. thick with a specific loss per lb. of 1·9 watts and $\rho \times 10^6$ = about 20 microhms, this becomes

$$(0{\cdot}0072f + 0{\cdot}000139f^2)\left[\left(\frac{B_c}{1000}\right)^2 \frac{V_c}{1000} + \left(\frac{B_t}{1000}\right)^2 \frac{V_t}{1000}\right]$$

and with $f = 50$

$$(0{\cdot}36 + 0{\cdot}35)\left[\left(\frac{B_c}{1000}\right)^2 \frac{V_c}{1000} + \left(\frac{B_t}{1000}\right)^2 \frac{V_t}{1000}\right]$$

i.e.

$$\text{watts} = 0{\cdot}71 \left\{\left(\frac{B_c}{1000}\right)^2 \times \frac{\text{cm}^3 \text{ of core}}{1000} + \left(\frac{B_t}{1000}\right)^2 \times \frac{\text{cm}^3 \text{ of teeth}}{1000}\right\} \quad . \quad . \quad (277)$$

or reckoning 130 cm.^{3} of iron = 1 kilogramme

$$\text{watts} = 0{\cdot}092 \left\{\left(\frac{B_c}{1000}\right)^2 \times kg \text{ of core} + \left(\frac{B_t}{1000}\right)^2 \times kg \text{ of teeth}\right\}$$

This agrees well with the approximate figures of Messrs. Hobart and Punga,[1] who in the same formula as the last for the no-load core loss give the constant with $f = 50$ and for ordinary discs 0·05 cm. thick as ranging from 0·085 to 0·1. These are, however, maximum values, giving losses that err perhaps on the side of caution.

Taking another case of discs only 0·04 cm. thick with a specific loss 1·7 watts, $a = 0{\cdot}0308 \times 1{\cdot}7 = 0{\cdot}0524$ and $a_1 = 1{\cdot}3a = 0{\cdot}068$. With the same volume-resistivity, $\rho \times 10^6 = 20$ microhms,

$$\beta_1 = \frac{2{\cdot}78 \times 4 \times 0{\cdot}04^2}{20} = 0{\cdot}00089. \text{ Hence}$$

$$(a_1 f + \beta_1 f^2)\left\{\left(\frac{B_c}{1000}\right)^2 \frac{V_c}{1000} + \left(\frac{B_t}{1000}\right)^2 \frac{V_t}{1000}\right\} \times 10^{-1}$$

$$= (0{\cdot}0068f + 0{\cdot}000089f^2)\left\{\left(\frac{B_c}{1000}\right)^2 \frac{V_c}{1000} + \left(\frac{B_t}{1000}\right)^2 \frac{V_t}{1000}\right\}$$

and with $f = 50$,

$$\text{watts} = (0{\cdot}34 + 0{\cdot}2225)\left\{\left(\frac{B_c}{1000}\right)^2 \frac{V_c}{1000} + \left(\frac{B_t}{1000}\right)^2 \frac{V_t}{1000}\right\}$$

$$= 0{\cdot}5625\left\{\left(\frac{B_c}{1000}\right)^2 \frac{V_c}{1000} + \left(\frac{B_t}{1000}\right)^2 \frac{V_t}{1000}\right\}$$

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[1] *Elektrische Kraftbetriebe und Bahnen*, Vol. 5, p. 568.

With thinner discs No. 29 S.W.G. = 0·035 cm. thick of special dynamo steel for turbo rotors with low hysteresis loss, and with care in the building of the core, the loss should not exceed

$$\text{watts} = 0{\cdot}43\left\{\left(\frac{B_c}{1000}\right)^2\frac{V_c}{1000}+\left(\frac{B_t}{1000}\right)^2\frac{V_t}{1000}\right\} \quad . \quad . \quad (277a)$$

Throughout the above B_t is to be reckoned as the apparent density at a point one-third of the height of the slot from the point of maximum density, *i.e.* $B_t = B_{t\,max} \times \dfrac{3\,B_{t\,min}}{2\,B_{t\,min} + B_{t\,max}}$.

When compared with the results of actual experiments on no-load with the stator wound, to the iron losses as calculated above, there must be added a further allowance for eddy-currents in the copper conductors due to lines fringing into the openings of the slots; the constants may then require to be increased 10 per cent. or more. Such a loss will especially occur in large high-voltage turbo-alternators, where the air-gaps are long and the slots relatively wide, $1\frac{1}{2}$ ins. or more, unless the upper conductors are laminated.[1]

A further proviso must also here be added In the above expressions alternators with external stator in which the flux-densities in stator core and teeth are usually moderate, not exceeding, say, 15,000 and 18,000 respectively, have alone been under consideration But if the same form of expression is to be applied to the internal armature of the continuous-current dynamo, the density in the teeth is often higher, and allowance must be made for the fact that the rotating-field hysteresis loss is then less than that in an alternating field. Hence if the density in the teeth exceeds 18,000, and in proportion as it exceeds it, the hysteresis coefficient may be reduced, say, to a instead of 1·3 a when the density rises to 19,000 20,000 The iron losses in core and teeth are then best reckoned separately; and for the latter the first coefficient becomes 0·0052 instead of 0·0068 in the case of either of the two kinds of discs assumed above [2] On the other hand, the eddy-current loss in the copper conductors within the slots near the pole-edges in continuous-current dynamos, as described in Chapter XXI, § 21, becomes much more serious, if the teeth are highly saturated, and will raise the constant for the total no-load loss.

The total loss will from the above expressions be proportional to some power of f intermediate[3] between 1 and 2, its exact value depending on the relative proportions of a_1 and β_1. With the above values for a_1 and β_1, for frequencies between 25 and 60 we have approximately

$$\text{Watts of iron loss} = 0{\cdot}003\,f^{1\cdot4}\left\{\left(\frac{B_c}{1000}\right)^2\times\frac{\text{cm.}^3\text{ of core}}{1000} + \left(\frac{B_t}{1000}\right)^2\times\frac{\text{cm}^3\text{ of teeth}}{1000}\right\}$$

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[1] B. G. Lamme, *Trans. Amer. I E E*, Vol 35, Part I, p 269.

[2] For curves illustrating the same point, *cp* Prof. Miles Walker, " The Predetermination of Dynamo-Electric Machinery," *Journ. I E.E.*, Vol. 54, p. 263; and *Specification and Design of Dynamo-electric Machinery*, pp. 51 and 54.

[3] M. L. Schukkerman gives the 1·3rd power (Abstract 1172, Vol. 17, *Science Abstracts, Electrical Engineering*), for normal excitation rising to the 1·5th power for considerable over-excitation. E. Hughes (*Journ. I.E.E.*, vol. 63, pp. 39, 47), finds for direct-current machines 1·5 as the index of the frequency, with the same constant 0·003 as above.

It is unnecessary again to enlarge upon the uncertain nature of any predetermination of the iron losses, in view of the unknown factors that arise from burring of the disc edges, leakage of flux into the flanks of every air-disc, etc. It need only be added that when both rotor and stator are slotted as in a turbo-alternator, the flux-variations set up as the slots in the two elements occupy different relative positions may cause an appreciable additional loss, although the tendency is largely held in check by the longer air-gaps which are usual in the turbo-generator as contrasted with the induction motor.[1]

§ 14. Eddy-currents in pole-pieces.—Owing to the smaller number of slots and their comparatively large size in the alternator, the eddy current effects in the pole-pieces are rather different in character from those in continuous-current machines; usually with few slots per pole per phase the frequency is lower, and even when the slots are half-closed the ratio of the width of opening to the air-gap is large. The two half-waves of the eddy-current E.M.F. are dissimilar in shape, which may be traced to the effects of hysteresis in the teeth; and the shapes of the complete waves differ considerably in different parts of the pole-face, while further they vary greatly according to the magnitude and nature of the armature load, whether inductive or having capacitance.[2] Experiments emphasize the great difference in the magnitude of the loss from entirely open as compared with closed slots.

In connexion with the quantitative results for laminated poles discussed in Chap. XXI, § 29 (Vol. II), reference should be made to the more recent experimental tests of Messrs. T. Spooner and I. F. Kinnaird, who found[3] that for sheet steel laminations 0·0625 in. thick the rate of surface loss in watts per sq. in. of pole-face might approximately be expressed as a function of the 4 variables—the tooth frequency, the air-gap induction, the ratio of slot opening to air-gap, and tooth-pitch—with indices as follows:—

$$\left(S\frac{N}{60}\right)^{1\cdot 6}\left(\frac{B_{g\ av}}{1000}\right)^{2\cdot 6}\left(\frac{w_3}{l_g}\right)^{2\cdot 2}t_1{}^{1\cdot 3}$$

But it should be noticed that all of the surface losses in the pole—by hysteresis as well as by eddy-currents—are here included, and that the air-gap induction is the average over the whole pole-pitch.

§ 15. "Stray losses," especially in single-phase alternators.—Even when allowance has been made for the additional eddy losses in the stator copper conductors as described in Chapter XXVIII, § 6, the actual loss under load in alternators as in direct-current dynamos (Chapter XXI, § 18) exceeds the sum of the no-load losses *plus* the actual I^2r_a' loss, and this additional loss which may more truly be termed "stray loss," is due to such causes as the following[4]—

(1) Eddy current and hysteresis losses set up by the flux due to

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[1] See T. Spooner, *Journ. Amer. I.E.E.*, Vol. 43, p. 646.

[2] *Cp.* "An Investigation into the Periodic Variations in the Magnetic Field of a Three-phase Generator by means of the Oscillograph," by G. W. Worrall and T. F. Wall, *Journ. I.E.E.*, Vol. 38, p. 148; and "Magnetic Oscillations in Alternators," by G. W. Worrall, *Journ. I.E.E.*, Vol. 39, p. 206, and Vol. 40, p. 413.

[3] "Surface Iron Losses with reference to Laminated Materials," *Journ. Amer. I.E.E.*, Vol. 43, p. 723, which should be consulted for further details as to the values of the constants, etc.

[4] See especially R. Rüdenberg, *E.T.Z.*, Vol. 45, pp. 37 and 59.

the alternating current in the stator coils reaching to constructional parts of the machine of solid metal. The coil-supports and end shields of high-speed turbo-alternators are especially the seat of such losses.

(2) As in continuous-current dynamos, the field distortion under load increases the hysteresis and eddy-current loss in the iron core and teeth.

(3) Apart from constant distortion of the field, there is also some pulsation of it always present. Even in the 3-phase alternator the armature reaction is not strictly constant, but, as previously mentioned, varies periodically; experiment shows that a harmonic of triple or sextuple frequency according to the nature of the winding[1] results therefrom, and the consequent variation of the field sets up eddy-currents in the pole-shoes and fluctuation in the exciting current.

The effect from the pulsation of the armature reaction is especially marked with single-phase alternators, and on account of it their efficiency is appreciably lower than that of 3-phase alternators. In the single-phase alternator, a current in the armature of frequency f sets up eddy-currents in the field-magnet of twice that frequency, as described in Chapter XXVII, § 34.

Expressed in terms of $ac_{''}$, the ampere-conductors per inch of periphery, and of Y, the stator pole-pitch in inches, the stray loss in kilowatts has such values at 50 cycles per second as

$$60 \times ac_{''}^2 \times Y^3 \times p \times 10^{-11}$$

and half this amount at 25 cycles; in each case an extra allowance must be added for pole-face losses when the slots are open and the poles solid.

§ 16. Determination of total losses under load.—In order to obtain some estimate as to the value of the total losses under load, the total losses under short-circuit have been used as a guide, the exciting current being given such a value that the full-load current is reproduced. The conditions of full-load are not really reproduced with any exactness, the low density exaggerating the field distortion, but the results seem to agree fairly well with the values obtained by direct tests[2] in the case of slow-speed machines, although not in turbo-alternators. The effect from non-uniform distribution of current is probably much about the same on load and on short-circuit for the same current, and in slow-speed machines this cause accounts for most of the extra loss.[1] Hence the short-circuit method has been adopted in the Standards of the American I.E.E. (§ 4342(*a*), 1921) as the basis for determining the stray losses required for the " conventional efficiency " of alternators (§ 4335 and Table 402).

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[1] *Cp.* Prof W M Thornton, *Journ I E E*, Vol 35, p. 156; and G. W. Worrall, *Journ. I.E E*, Vol 39, p 215, with discussion thereon.

[2] *Cp.* Messrs. W. J. Foster and E Knowlton, *Trans. Amer. I.E.E*, Vol. 32, Part I, p 502, and especially R. Rüdenberg, *E.T.Z.*, Vol. 45, p. 59, who gives the stray loss in fly-wheel alternators as on the average 0·6 per cent. of the rated full-load, but with possible wide variations.

In thoroughly good 3-phase machines with stranded conductors Messrs. Olin and Henderson[2] from the results of tests suggest that the additional load losses may be taken as increasing the sum of the no-load core loss + normal I^2r_a loss at $\frac{1}{2}$, $\frac{3}{4}$, full-load, and 25 per cent. overload by about 1, 3, 10, and 20 per cent.

It might be thought that the *total* armature, friction, and windage losses could be satisfactorily measured by running the alternator as a synchronous motor without mechanical load at the rated voltage and with the field under- and over-excited to give full current. The input as measured by wattmeters at the two points on the *V*-curve would then be averaged. But the objection is that the readings of the wattmeters at such low power-factors are liable to considerable inaccuracy.[3] The best measurements with duplicate machines are, therefore, obtained by the circulating-energy method described in § 12 with full voltage and current at unity power-factor and a calibrated driving motor.

For settlement of the efficiency of an alternator, the method by summation of the separate losses is often adopted. All I^2R losses being reckoned for a mean temperature of, say, 60° C., the no-load core loss, the losses in any rheostat in the main exciting circuit, in brushes and brush contacts at the slip rings, electrically and by friction, the windage loss, and the friction of bearing or bearings forming part of the electrical unit have to be added, together with an allowance for stray loss under load. There may also have to be taken into account the whole loss in the exciter and its field rheostat, and its windage and friction when it forms an integral part of the machine.

§ 17. Dielectric or insulation tests.—The British and American Standardisation Rules call for the application between the stator winding and the frame for one minute of an alternating test pressure of R.M.S. value equal to 1000 volts *plus* twice the rated pressure. The separately-excited field windings of the rotating field-magnet are to be tested with 10 times the exciter voltage, but in no case with less than 2000 (1500, American Standards) or more than 3500 volts.

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[1] *Cp.* F. K. Brainard, *Trans. Amer. I.E.E.*, Vol. 32, Part I, p. 519.

[2] *Trans. Amer I.E.E.*, Vol. 32, Part I, p. 501.

[3] Messrs. Foster and Knowlton, *Trans. Amer. I.E.E.*, Vol. 32, Part I, p. 517.

When the high-pressure test is applied, the insulation resistance in megohms should not be less than

$$\frac{\text{rated pressure in volts}}{1000 + \text{rated output in kVA}}$$

With very high voltages, the multiplying ratio of the testing transformer may be affected by the charging current which flows into the machine as a condenser. The pressure should then be checked by a spark-gap or crest voltage meter,[1] but it is seldom that such a precaution need be taken in the case of alternators.

§ 18. **Efficiency of alternators.**—The efficiency of polyphase alternators varies but little from that of continuous-current dynamos of the same output and speed, the losses in the exciter not being included.[2] The exciting energy varies from about 3 per cent. of the output in a 30-kilowatt alternator to 2 per cent. in a 500-kilowatt machine and 1 per cent. in machines of 800 kilowatts or over; but great differences exist between different types, and even in the same machine the amount varies greatly with the nature of the load if the regulation is much affected by the presence or absence of external inductance. Indeed, the increased field excitation required for a power-factor of 0·8 as compared with unity will as a general rule lower the efficiency of the whole machine some 1 or 1½ per cent. The copper loss in the armature varies in machines of 500 kilowatts and upwards from 1 to 2·5 per cent. of the output at full-load, and the loss by eddy currents and hysteresis varies from 3 to 4 per cent., so that in general the latter is the greater of the two. The one may, in fact, be reduced at the expense of the other, and if a high efficiency is required at light loads, the copper loss in the armature should be given a higher ratio to the constant loss in the armature iron.

Careful tests carried out by Messrs. H. M. Hobart and F Punga[3] upon a 5000 kW 3-phase alternator[4] built by the Siemens-Schuckert Co. and running at 300 revs. per min. 50 frequency enabled the following approximate allocation of losses to be made.

The stator slots 0·92 in. wide were open, and the bars forming the upper of the two layers of winding were divided into 9 vertical laminations, each 0·2 cm. thick and 1·8 cm. deep; the lower bars 1·8 cm. square were solid. From an approximate calculation of the density of the lines slanting inwards into the walls of the slots through their open mouths, Messrs. Hobart and Punga estimated that the eddy loss in the layers of solid and laminated conductors might probably amount to 8 and 3 kW respectively. The total no-load core loss in the armature of 93 kW was experimentally measured.

The total armature loss when the machine was short-circuited and run with full armature current (730 amp) showed, in addition to the ohmic loss over the armature resistance (0·0123 × 730^2 × 3 = 19·6 kW), a further loss of 23·9 kW. The greater part of this is to be attributed to non-uniform current distribution over the area of the laminated conductors near the openings of the slots, and in a lesser degree over the area of the solid conductors at the bottom of the slots; using the curves of Mr. A. B. Field, the ratio of the actual copper loss to the ohmic loss was calculated to be 1·48 in the case of the lower solid conductors and 5 in the upper laminated conductors. Each of these increases only applies to the portions of the conductors within the slots, *i.e.* to 51/284 = 0·18 of the whole; therefore, the loss factor for the whole of an armature phase works out to 0·64 + 0·18 (1·48 + 5) = 1·8, or an increase of 80 per cent. The end-connexions when carrying the full-current will show an increase from the same cause, and also from eddy-currents which their secondary leakage will set up in neighbouring iron masses.

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[1] See Standards of the American Institute of Electrical Engineers, *Trans. Amer. I.E.E.*, Vol. 40, p. 1559.

[2] *Cp.* R. Goldschmidt, *Journ. I.E.E.*, Vol. 40, pp. 466–468.

[3] *Elektrische Kraftbetriebe u. Bahnen*, Vol. 5, pp. 541 ff.

[4] Fully described in Chap. XXVII, § 7, of Vol. 2 of the fifth edition of the present book.

The armature direct reaction varied about ± 8 per cent. with a sextuple frequency of 300. By analogy from experiments, a loss of about 400 watts at full-load with cos ϕ_e = 1 was indicated as probable from eddies set up in the magnet-cores by this pulsation in the effect of the armature back ampere-turns.

The excitation loss with cos ϕ_e = 1 was (170 amp.)^2 × 0·24 ohm = 7 kW, and with cos ϕ_e = 0·8, 228^2 × 0·24 = 12·5 kW.

The measured loss of 52 kW by friction and windage may be divided on the assumption that the former varied as the 1·5th, and the latter as the 3rd power of the revs. per min.

The efficiency may, therefore, be approximated as follows—

Hysteresis and eddies in armature core	82 kW
Eddy-currents in solid conductors, approx	8 ,,
,, in laminated conductors, approx	3 ,,

	cos ϕ_e = 1	cos ϕ_e = 0·8
Measured no-load core loss	93·0 kW	93·0 kW
Ohmic loss over resistance of armature	19·5 ,,	19·5 ,,
Additional loss in solid conductors within slots	1·68 ,,	1·68 ,,
,, ,, in laminated conductors within slots	14·0 ,,	14·0 ,,
Additional loss from end connexions	7·52 ,,	7·52 ,,
Eddy-currents in magnet-cores from pulsation of field	0·4 ,,	0·4 ,,
Excitation	7·0 ,,	12·5 ,,
Friction of bearings	10·6 ,,	10·6 ,,
Windage	41·4 ,,	41·4 ,,
Total losses	195·1 kW	200·6 kW
Output	5000 ,,	4000 ,,
Input	5195·1 ,,	4200·6 ,,
Efficiency	96·2 per cent	95·2 per cent.

The efficiency of the 1750 kVA alternator shown in Fig. 573 is given[1] as follows at 5200 volts and 0·8 power-factor—

	Load. ½	¾	*Full.*
Iron losses, windage, and friction	53	54	55 kW
Stator current losses	7	16	28 ,,
Rotor current losses	12	15	18 ,,
Total losses	72	85	101 ,,
Output	700	1050	1400 ,,
Input	772	1135	1501 ,,
Efficiency	90·7	92·5	93·3 per cent.

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[1] *Engineering*, 1st October, 1920.

All causes combine to render the loss by eddy-currents and hysteresis greater in the single-phase than in the 3-phase alternator, and on this account the efficiency of the former is appreciably lower than that of the latter, chiefly owing to the pulsation of the armature reaction as already mentioned.

In large single-phase alternators driven by steam-turbines at high speeds and low frequencies such as 25 cycles per sec., the armature reaction is so great owing to the large pole-pitch, and the pulsation of the flux is so considerable that it becomes necessary to reduce the amount of the eddy-currents by the use of copper dampers[1] or bars dovetailed into the pole-faces and short-circuited by copper rings or discs (Chap. XXXI, § 42).

§ 19. Weights of alternators.—According to the frequency, degree of regulation, the fly-wheel effect introduced into the rotor, etc, the weight of an alternator per watt per rev. per min., just as the specific torque coefficient, shows very great variations under different conditions.[2] As a rough approximation, it may be said to be proportional to the square root rather than to the $\frac{2}{3}$rds power as in continuous-current dynamos, and has such values as are given by the relation $W = 1260\left(\frac{VA}{N}\right)^{\frac{1}{2}}$ lb For large slow-speed machines with $\frac{VA}{N} > 10{,}000$ and $D^2_{''}L_{''} > 300{,}000$ and a high dimensional coefficient of, say, 27·7, this becomes $W = 240(D^2_{''}L_{''})^{1/2}$ lb. Owing to the different relation of the diameter and length in such machines as compared with continuous-current machines, the increase in weight with increasing $D^2_{''}L_{''}$ is not so rapid. But with smaller alternators with $\frac{VA}{N}$ between 1000 and 10,000, the two types are more similar, and the weight of the alternator becomes approximately $W = 28(L^2_{''}L_{''})^{2/3}$, except in the case of turbo-alternators, which form a class by themselves.

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[1] For figures as to the serious nature of the loss in the single-phase as compared with the 3-phase alternator and as to the effect of dampers, see W. L. Waters, "Modern Development in Single-phase Generators," *Trans. Amer. I.E.E.*, Vol 27, Part II, p 1069.

[2] *Cp.* T. Schou, *Elec World*, Vol 77, p 1160.

CHAPTER XXX

THE TURBO-ALTERNATOR WITH CYLINDRICAL ROTOR

§ 1. The number of poles.—The best economy of the steam turbine is obtained with a high angular velocity. To meet this requirement, the designer of the turbo-alternator, bound as he is by the rigid necessity for the frequency to be, say, 50 or 25, is perforce driven to employ only a small number of poles. How limited is the choice as to the number of poles and speeds is shown by the following table—

Number of Poles	Revs. per min.		
	60~	50~	25~
2	3600	3000	1500
4	1800	1500	750
6	1200	1000	—

But for small outputs below 1000 kW the advantage of high speed in the turbine (5000 r.p.m. or more) can be combined with ordinary and inexpensive alternator construction, say, at 500 or 750 r.p.m., by the use of single-reduction gearing.

§ 2. The torque coefficient.—If in large machines $B_{g\,max}$ is assumed to be 8600, and the amplitude of the fundamental B_{g1} to be 1·07 $B_{g\,max}$, with $E_i = 1\cdot07\,V_e$

$$\frac{1\cdot07\ \text{kVA}}{N} = 0\cdot72 \times 1\cdot07\,B_{g\,max}\,ac_u \times D^2_u\,L_u \times 10^{-11} \qquad (278)$$

and if $ac_u = 1000$ and 1100 ampere-conductors per inch in two-pole and four-pole machines respectively

$$\frac{VA}{N} = 0\cdot062\,D^2_u\,L_u \text{ in the 2-pole machine}$$

$$= 0\cdot068\,D^2_u\,L_u \text{ ,, 4-pole ,,}$$

The air-gap in large turbo-alternators being from 1 to 2 ins., the diameters of the rotor and of the bore of the stator differ appreciably, so that in the above figures the ampere-conductors and flux-density are both given as referred to the diameter at the centre of the air-gap. D must, therefore, be interpreted as that diameter, while L is the over-all length of the core.

In considering such figures as those given above, which are well within the reach of practice, it must be remembered that, automatic regulators being presupposed, the stator ampere-turns may bear such a high proportion (as much as 0·6) to the rotor ampere-turns in turbo-alternators that the regulation may be 25 per cent. at unity power-factor and 40 per cent. at 80 per cent. power-factor. In the smaller machines it is the possible rotor ampere-turns which limit the torque coefficient. In the larger machines at 1200 or 1000 revs. per min., the limiting factor is the possible depth of stator slot (6–7 ins.) and of stator coils, having regard to eddy-current losses in the latter and the difficulty in constructing them when the depth is great in proportion to their width. Even as much as 1750 ampere-conductors per inch of periphery at the air-gap may be found in very large machines where high armature reaction is not a disadvantage, and to house such large values of $ac_{''}$ the slot of Fig. 652 or 665*a* is beneficial by reason of the greater diameter on which the stator-conductors are disposed.

The curves of Fig. 617 are, therefore, only roughly applicable to the turbo-alternator cooled by forced ventilation, and may even be bettered,[1] although in many machines actually built only a few years ago, the volt-amperes per rev. per min. obtained from each cubic inch of D^2L fall much lower.

§ 3. The limiting peripheral velocity as fixed by the end-rings.—For each of the standard speeds in revs. per min., *e.g.* 3000, 1500, 1000, etc., there is a certain maximum diameter of rotor which it is not desirable to exceed. This limitation arises mainly from considerations of the mechanical strength of the end-rings or end-bells, which must not only support the stress from their own centrifugal force, but also take up the centrifugal force from the massive end-connexions of the rotor windings. The mass of every 1 lb. at a radius of 1 ft., such as might form part of the end-windings of a rotor of 27 ins. diameter, when running at 3600 revs. per. min., exerts a centrifugal force of nearly 2 tons. Further, the actual pressure against the inside of the end-ring not only increases very greatly as we pass axially from the edge of the rotor core to the outer edge of the windings along the centre line of a pole, but also increases slightly as we pass circumferentially away from the centre line of a pole to the interpolar line where the coils bifurcate.[1] The material of the ring is thus subjected to bending stresses in both directions; it then no longer remains circular but becomes elliptical, and additional hoop stress is thereby set up.

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[1] *Cp.* a 14,700 kVA 2-pole alternator at 2400 revs. per min. = 6130 volt amperes per rev. per min., with water-cooled rotor 29 ins. diameter × 105 ins. long (J. Rosen, *Journ. I E.E.*, Vol. 61, p 442), whence $VA/N = 0{\cdot}065D^2_{''}L_{''}$; an 18,000 kVA 5500–6000 volt 2-pole water-cooled alternator at 2400 r.p.m. = 7500 volt-amperes per r.p.m. with stator 35 ins. diam. × 108¼ ins. long, whence $VA/N = 0{\cdot}06D^2_{''}L_{''}$; and a proposed 25,000 kVA 2-pole alternator at 3000 r.p.m. = 8333 volt-amperes per rev. per min with rotor 35½ ins. diam. × 98½ ins. long (H. Rikli, *Schweitz. Bauz.*, Vol. 80, p. 201), whence $VA/N = 0{\cdot}063D^2_{''}L_{''}$, D being in all the above cases reckoned approximately to the centre of the air-gap. A 25,000 kW four-pole 25-cycle turbo-alternator for 0·95 power-factor at 750 revs. per min., constructed by Messrs. C. A. Parsons & Co., Ltd., for the Commonwealth Edison Co. of Chicago, giving $VA/N = 35{,}100$, had a rotor 74 ins. diam. × 91 ins. (see *Engineering*, Vol. 96, p. 513), whence the coefficient = 0·0686.

For the construction of the end-rings, three ductile materials present themselves—manganese bronze, oil-treated carbon steel, and nickel-chrome steel containing about 3·5 per cent. nickel, 0·5 per cent. chromium, and 0·25 per cent. carbon.[2]

	Ultimate strength.	*Yield point.*	*Elongation on 2-in. per cent.*	*Reduction in area per cent.*
	Tons per sq. in.			
Manganese bronze	44	22	20	—
Carbon steel	38	22	22	40
Nickel-chrome steel	50	40	18	50
	65	48	15	50[3]

Taking the permissible stress as 45 per cent. of the yield point in the first case and as 50 per cent. in the two latter cases, Dr. Barclay has in the above-quoted paper given diagrams for 3600 revs. per min. (*i.e.* for a normal speed of 3000 revs. per min. *plus* an overspeed of 20 per cent.) and for rotor diameters of 20, 22, 25, 27 and 31 ins., showing the additional load per inch of axial length that can safely be borne by increasing thicknesses of the three materials, together with the approximate average load per inch of axial length on the end-ring from the rotor coils. The diagrams show clearly that at 25 inches diameter the manganese bronze has ceased to be able to carry the additional load from the centrifugal force of the rotor coils, that at 27 ins. diameter carbon steel has followed suit, and that at 31 ins. diameter only nickel-chrome steel over 1·3 ins. thick remains available. The problem, therefore, is one which calls for the co-operation of the metallurgist, especially as correct heat treatment of such a material as nickel-chrome steel is of vital importance.

At 3000 revs. per min. and allowing an overspeed of 20 per cent. a 32-in. diameter rotor may be taken as the limit, the peripheral velocity at the overspeed being then over 30,000, and at the normal speed 25,100 ft. per min. Similarly for 1500 revs. per min. as the normal speed 60 ins. is found approximately to be the maximum permissible diameter, giving 28,200 ft. per min. at the overspeed, and at 1000 revs. per min. perhaps 80 ins.—giving 25,100 ft. per min. at the overspeed. Rotors of about 27 ins. diameter have been actually tested at overspeeds giving practically 30,000 ft. per min., but for 1500 revs. per min. and higher speeds the maximum working value for the peripheral velocity may be set at about 24,000–25,000 ft. per min. in normal cases. It must be remembered also that for a given number of revs. per min. as the rotor diameter is increased, the centrifugal force from each lb.-mass of copper increases; this may require the tooth-width to be increased so much that in spite of the increased diameter the total slot area and the rotor ampere-turns must be decreased. The most effective diameter has then been passed,[1] and this consideration reinforces the limitation that is primarily due to the question of the mechanical strength of the end-rings. Maximum output in fact calls for a balance to be very carefully made between the rival considerations of "mechanical stresses, rotor ampere-turns and flux."

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[1] But for the actual deformation in a given case, see J. Rosen, "Some Problems in High-speed Alternators and Their Solution," *Journ. I.E.E.*, Vol. 61, p. 443.

[2] Dr. S. F. Barclay, "The Mechanical Design and Specification of the Turbo-alternator Rotor," *Journ. I.E.E.*, Vol. 56, p. 472. *Cp.* also B. A. Behrend, *Trans. Amer. I.E.E.*, Vol. 38, Part II, p. 1549, in discussion.

[3] J. Rosen, *loc. cit. supra.*

§ 4. The limits of output.—The maximum permissible or most effective diameters for different speeds or the cross-sections, as it were, of the machines having been settled in accordance with the facts outlined in § 3, and adopting the coefficients suggested in § 2 for use with the diameters to the centre of the air-gap, the possible kVA outputs become in each case simply proportional to the length of core that is regarded as feasible. Thus the maximum outputs are

$$\begin{aligned} &\text{at } 3000 \text{ r.p.m. kVA} = 210 \times L_u \\ &\text{,, } 1500 \text{ ,, } \quad \text{,, } = 400 \times L_u \end{aligned}$$

Formerly the maximum permissible length was fixed by the consideration that the critical speed of whirling should occur well above the running speed, say, by 20–25 per cent. But it may now be said at least in the case of machines for 3000 or 3600 revs. per min. that it is more usual for the first critical speed to be run through in order to reach the running speed, so that the former must fall 20–25 per cent. below the latter. With large outputs at 3000 r.p.m. even the second critical speed may occur below the running speed. The first limitation has, therefore, been removed for 2-pole machines, and in 4-pole machines the critical speed usually falls above the normal speed. But there still remains the fact that with a long rotor core the difficulty of securing a uniform passage of the cooling air through both rotor and stator, and a *uniform* cooling of the windings increases very greatly, even when the air is fed in from both ends. There is, too, with high temperatures and very long rotors a danger of trouble arising from relative movement between the conductor, its insulation and the iron core, owing to the linear expansion and contraction of the first-mentioned as it heats and cools. The coefficients of linear expansion for copper and steel per 1° C. are 0·0000173 and 0·000012 respectively, and the coefficient for built-up mica and paper insulation is about the same as for steel or even less. Starting from the same cold temperature, if both rise equally by 50° C., the difference between the lengths of a copper bar or strip and of the core in which it is embedded, when both initially were 80 ins. long is 0·0212 in. But this is not the full statement of the case, since there may be a fall of some 25° C. across the insulation ; the difference then amounts to 0·056 in. Under gradual loading the steel and copper heat up together, but under very rapid loading the copper might rise 40° C. above the cold iron and yield nearly the same relative expansion.[1]

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[1] F. D. Newbury, "Present Limits of Speed and Output of Single-shaft Turbo-generators," *Trans. Amer. I E.E.*, Vol. 38, Part II, p. 1537.

Next, as the distance between the bearings increases, the deflection of the rotor shaft increases ; to keep the alternating stress at the neck of the journal constant, larger bearings are necessary, and greater losses therein. Lastly, with very large 4-pole and 6-pole units, the rotor may become too heavy to be transported by rail in its completed state ; the stator casings may be in parts, but even so the core may require to be built up and to be wound on the site.

Taking then 70 ins. and 100 ins. as the lengths of core at the two speeds, the kVA outputs are approximately 15,000 and 40,000 respectively, and a rough indication is obtained of the order of the figures that may still be regarded as conservative limits. For when compared with what has actually been reached, they will be found to have been already exceeded, and exceeded considerably at the higher speed, greater lengths being employed.[2] For 50 cycles per second two-pole machines running at 3000 r.p.m. have been built up to 20,000 kVA, and four-pole machines at 1500 r.p.m. up to 45,000 kVA, while 60,000 kVA alternators now built with 6 poles for 1000 r.p.m. will no doubt in the future be reached with 4 poles at 1500 r.p.m. As showing the size of modern machines and modern central stations, five 4-pole 45,000 kVA turbo-alternators at 1500 revs. per min. have been installed at the power station of the Union d'Electricité at Gennevilliers, of which the stators weigh nearly 100 tons each, and the rotors nearly 50 tons each. The rotors of 60,000 kVA 6-pole alternators of total weight 225–250 tons weigh 105 tons and have a diameter of about 90 ins. (23,600 ft. per min. peripheral velocity).

For the standard frequency of 60 cycles as adopted in the United States of America, 3600-r.p.m. generators (2-pole) are now (1925) being built up to 10,000 kVA at 80 per cent. power-factor and 12,500 kVA ; 1800-r.p.m. generators (4-pole) are being built by the General Electric Co. of Schenectady[3] up to 62,500 kVA, 14,000 volts, and 1200-r.p.m. generators (6-pole) are already in service up to 62,500 kVA at 80 per cent. power-factor. For 25 cycles, the same Company is now building two-pole 1500-r.p.m. machines up to 60,000 kW at 11,000 volts.

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[1] But for tests showing no deterioration of the 13,200-volt insulation when experimental bars and punchings 110 ins. long were subject to many thousands of heating and cooling cycles, see " Repeated Thermal Expansions and Contractions ; their effect on Long Armature Coil Insulations," by T. S. Taylor, *Journ. Amer. I.E.E.*, Vol. 43, p. 1047.

[2] *Cp.* J. Rosen, *Journ. I.E.E.*, Vol. 61, p. 439.

[3] *Cp.* W. J. Foster in discussion on Mr. Newbury's paper, *Trans. Amer. I.E.E.*, Vol. 38, Part II, p. 1554, and *Journ. Amer. I.E.E.*, Vol. 43, p. 931.

The increase in the kW output obtainable per machine pole is in fact the most striking advance that has been made of recent years in the field of design ; from 100 kW per machine pole in former engine-driven slow-speed alternators, it may now at 25 frequency reach 15,000 to 20,000 kW with correspondingly large fluxes and large armature reactions per pole.[1]

Outputs of 60,000 kW have formerly in some cases been obtained by the combination of one high-pressure and two low-pressure steam turbines, each of the three driving its own generator of 20,000 kW at 1800 revs. per min., and each being capable of working alone on high-pressure steam in case of emergency. Probably a single unit of 90,000 kVA could be built at 1000 revs. per min. (50 cycles) or even for 1200 revs. per min. (60 cycles), but it remains doubtful whether such large units will be called for. An accident to such a unit due to an internal short-circuit between the turns of a coil or from copper to earth, when the power of the station is concentrated on it may destroy the whole winding or burn a hole in the core laminations; it will then throw out of action perhaps for several months too high a proportion of the total machinery even of a very large generating station.[2]

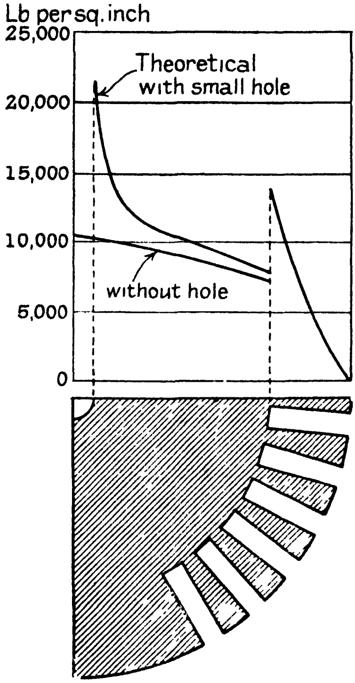

FIG. 623 —Approximate diagram of hoop stress in rotor body and tensile stress on teeth

§ **5. The rotor core.**—For rotors to run at 3000 r.p.m. or higher speeds, the solid-steel forging has proved itself to be thoroughly satisfactory. A material well suited to the purpose is an oil-treated carbon steel, containing about 0·38 per cent. carbon, and possessing such properties as an ultimate strength of 38 tons per sq. in., a yield point of 22 tons per sq. in., an elongation of 22 per cent. on 2 ins., and a contraction of area of 40 per cent.[1] The material should be proved by test pieces, taken not only longitudinally from the shaft, but also peripherally and radially from the body of the core—especially important is the radial tensile strength. From the diameter below the teeth the shaft must be stepped down very gradually to the journal diameter, with a large radius at each change of section when it cannot be simply tapered. While with a solid rotor the maximum hoop stress at the centre may be from 10,000–13,000 lb. per sq. in., with a central hole this is greatly increased, and its theoretical value should then not exceed some 22,000 lb. per sq. in.

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[1] The flux per pole of the 18,000 kW alternator above-cited (p. 356 note) = 156,500 kilolines ; of a 2-pole 20,000 kVA alternator at 25 frequency 200,000 kilolines ; and of the 4-pole 45,000 kVA machines at Gennevilliers about 130,000 kilolines per pole with about 50,000 ampere-conductors in a phase per pole. (E. Roth, *Revue Générale de l'Électricité*, Vol. 13, p. 145.)

[2] *Cp.* F. D. Newbury, *loc. cit.*, and more recently B. G. Lamme, *World Power Conference*, Wembley, 1924 (abstracted in *Engineering*, Vol. 118, p. 108).

A hole is often trepanned axially through the centre of the rotor shaft (Fig. 624*a*) for examination of the walls and testing of the material, or sometimes for water cooling ; theoretically even a very small hole at the centre doubles the hoop stress at the periphery of the hole, but in fact the stresses are rendered more uniform owing to yielding of the metal, so that the practice is not accompanied by the disadvantageous consequence to its full theoretical extent, and is otherwise to be recommended.

Rolled punchings in packets, shrunk on and also keyed to a through shaft (Fig. 624*b*), have been largely used by some makers for machines at 3000 r.p.m. and continue to be so used even at 1500 r.p.m., but not for the largest outputs attainable at that speed. The shaft may then in the 2-pole machine be fluted to give air-ways beneath the core, or in the 4-pole machine may be cruciform. The punchings are firmly compressed between heavy flange plates by bolts running through the laminations from end to end in the regions of the unslotted pole-centres. The shrinkage allowance must be such that the laminations continue to grip the shaft at the overspeed ; although this implies a high initial stress at the inner periphery, it also implies that this stress is but little if at all increased at the overspeed, so that the laminated rotor is differently circumstanced in this respect from the solid rotor.

The solid rotor forging has been employed even up to 35,000 kVA and 4 poles, but when large rotors above 50 ins. diameter are required for speeds of 1800, 1500 or lower revs. per min., it becomes more doubtful whether sufficient work can be done on the ingot under the hammer or under the press to render the solid forging completely sound at the centre. In such cases a hollow steel cylinder can be forged on a mandril, and afterwards shrunk on a shaft running right through it; although the hole through the cylinder increases, as mentioned, the hoop stress at its inner periphery, yet on the other hand the oil-treatment reaches the inside, work is done on the inner bore by the forging on the mandril, and the disadvantage of the known higher [illegible]round the hole is more than counterbalanced by the greate[illegible]ty in the mechanical quality of the material.

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[1] Dr. S. F. Barclay, *Journ. I.E.E.*, Vol. 56, p. 477.

But a [illegible]culty still remains, viz., that after deduction of the slot depth for wedge and coils of, say, 6 ins., and of a further air-slot of, say, 4 ins., only 30 ins. are left out of an original diameter of 50 ins., and this will not allow of a shaft of sufficiently large diameter without causing a very high hoop stress at the inside of the cylinder.

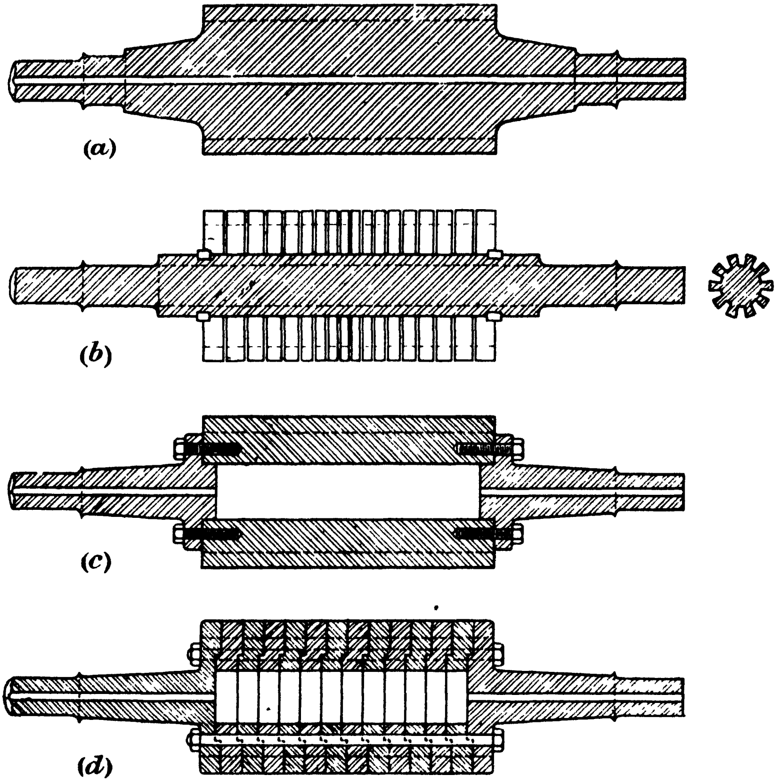

FIG 624.—Rotor constructions.

(a) Solid forging ; (c) Three part rotor for 6 pole machine;
(b) Laminations on through shaft ; (d) Ditto with plate laminations.

To meet these conditions in 4- and 6-pole machines without the use of a through shaft, the hollow rotor cylinder may be shrunk on and bolted to a short stub shaft at each end, upon which is formed the journal for a bearing (Fig. 624*c*) ; the central part alone would with this construction for a 60,000 kVA machine at 1500 revs. per min. weigh in the rough some 50 tons. The whole can be made stiffer than the plate core of Fig. 624*b* with through shaft, and the critical speed is correspondingly increased.

Rolled plates, machined to a thickness of about 2 ins., are also employed, out of which the core is built up.′ Such plates lend themselves to rigorous testing, thorough inspection, and commercially are more easily procurable at low cost. They again may either be mounted on a through shaft, or for larger sizes be slightly rabbeted into one another and into a strong flanged shaft at each end, the whole being firmly compressed by 4 or 6 chrome-nickel steel bolts[1] (Fig. 624*d*).

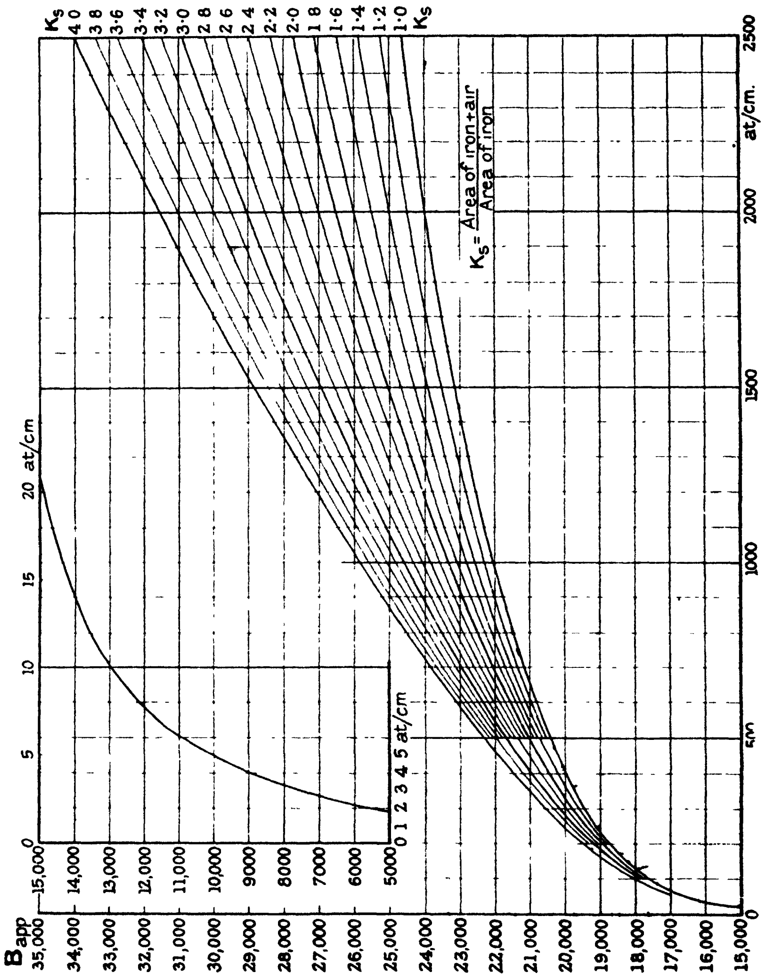

FIG. 625.—Magnetization curves for rotor steel forgings.

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[1] For a full description of such a construction with bolts about 4 ins. in diameter, on a pitch circle over 40 ins. diameter, with illustrations, see A. B. Field, " Some Difficulties of Design of High-speed Generators," *Journ. I.E.E.*, Vol. 54, p. 65 ; and for the pros and cons of the construction, *cp.* also Dr. S. F. Barclay's paper quoted, with the discussion thereon.

The construction employed by the Allgemeine Elektricitäts-Gesellschaft in which laminated teeth holding the exciting coils are forced into dovetailed grooves milled in the surface of the rotor core enables a smaller diameter to be employed for the solid forging which forms the core.

A magnetization curve for forgings of good steel is added in Fig. 625 (from *Papers on the Design of Alternating-current Machinery*).

§ **6. The rotor tooth stress.**—Regarding the wedge as a trapezium of height h_2 ins., having its top of width w_s ins. at the larger radius R ins. and its longer base w_2 ins. at the smaller radius r, the radius to its centre of gravity falls short of its mean radius and is

$$\frac{R+r}{2} - \frac{h_2}{6} \times \frac{w_s - w_2}{w_s + w_2}$$

Its section is $h_2(w_s + w_2)/2$, and if $w =$ its weight in lb. per cubic inch, its weight per inch length is $wh_2(w_s + w_2)/2$. Its centrifugal force per inch run is then in lb.

$$f_w = \frac{0{\cdot}011}{386{\cdot}4} N^2 \frac{w\,h_2}{4} \left\{ (R+r)\,(w_s + w_2) - \frac{h_2}{3}\,(w_s - w_2) \right\}$$

or neglecting the last term which is usually small

$$= 28{\cdot}4 \left(\frac{N}{1000}\right)^2 w\,h_2 \left\{ \left(\frac{R+r}{2}\right) \left(\frac{w_s + w_2}{2}\right) \right\}. \qquad (279)$$

all dimensions being in inches, and w being for gunmetal or phosphor-bronze 0·31 and for steel 0·282.

For the contents of a parallel-sided slot, $r_g =$ practically $(r + r_s)/2$ where r_s is the radius to the bottom of the slot. Taking $w = 0{\cdot}322$ for the weight of a cubic inch of copper and $= 0{\cdot}1$ for mica, and calculating the areas in sq. ins. for each material from the drawing as a_c and a_i, the centrifugal force per inch run is

for the copper $$f_c = 4{\cdot}57 \left(\frac{N}{1000}\right)^2 a_c\,(r + r_s) \qquad (280)$$

and for the insulation $$f_i = 1{\cdot}42 \left(\frac{N}{1000}\right)^2 a_i\,(r + r_s) \qquad (281)$$

The centrifugal force over the core length of L ins. is then in lb. $(f_w + f_c + f_i)L$ per tooth, if each tooth correctly takes the stress from half a slot on each side of it. To this must be added its own centrifugal force. Treating the tooth to the level of the slot depth as approximately a trapezium of height $h_s = R - r_s$ and of widths w_{t1} and w_{t2}, the radius to its centre of gravity is

$$\frac{R + r_s}{2} + \frac{h_s}{6} \times \frac{w_{t1} - w_{t2}}{w_{t1} + w_{t2}}$$

and its weight in steel per inch length is $0{\cdot}28\, h_s(w_{t1} + w_{t2})/2$. Its centrifugal force per inch run is then in lb.

$$f_t = \frac{0{\cdot}011 \times 0{\cdot}28}{32{\cdot}2 \times 12} N^2 \frac{h_s}{4} \left\{ (R + r_s)\,(w_{t1} + w_{t2}) + \frac{h_s}{3}\,(w_{t1} - w_{t2}) \right\}$$

$$= 2\left(\frac{N}{1000}\right)^2 h_s \left\{ (R + r_s)\,(w_{t1} + w_{t2}) + \frac{h_s}{3}\,(w_{t1} - w_{t2}) \right\} \qquad (282)$$

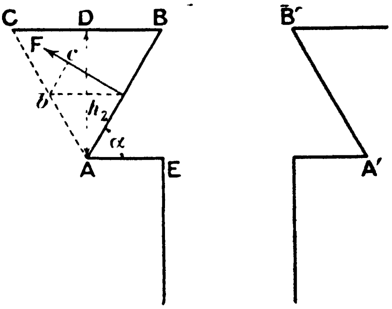

FIG. 626—Stress on rotor tooth-tip.

The value is here slightly over-estimated owing to the metal removed to form the dove-tail of the wedge not having been deducted.

If l = the axial length in ins. of steel after deduction of the radial ventilating ducts, the total centrifugal force is

$$(f_w + f_c + f_t)\, L + f_t l$$

and the section of the metal at the root of the tooth level with the bottom of the slot is $w_{t2} l$. The tensile stress thereat is thus

$$\frac{(f_w + f_c + f_t)\, L}{w'_{t2}\, l} + \frac{f_t}{w_{t2}} \text{ lb. per sq. in.} \qquad . \quad . \quad (283)$$

If there is an axial air-duct below the slot, the calculation for f_t must be extended downwards to the bottom of the air-duct, and the stress again checked with the new value for w'_{t2} inserted.

The outward force on the wedge is divided equally between the two sloping faces AB, $A'B'$ of the dove-tail (Fig. 626). The stress tending to shear off the triangle of the dove-tail along AD or BE in tooth or wedge is thus $f_s = \frac{(f_w + f_c + f_t)\, L}{2\, h_2\, l}$, when the fact that the outward force increases slightly from the bottom to the top is ignored. But the compression stress at right angles to AB, the sloping face of the tooth, as called forth by the outward force and tending to crush the metal at the face, is much greater; if α be the acute angle of the wedge (usually about 60°), the total pressure on the sloping face is $F = \frac{(f_w + f_c + f_t)\, L}{2} \times \frac{1}{\cos \alpha}$. The area of the sloping face of the tooth being $h_2 l \sin \alpha$, the crushing stress on the tooth at its sloping face is

$$\frac{(f_w + f_c + f_t)\, L}{2\, h_2\, l} \times \tan \alpha$$

The bending moment on the tooth-tip is equal to F acting at the end of the arm $b\,c$, and the modulus of the section at AC is $(AC)^2 l/6$ It can then be shown[1] that the maximum stress due to the bending moment is

$$\frac{0{\cdot}75\,(f_w + f_c + f_t)\,L \sin \alpha}{h_2 l\,(1 - \cos \alpha)} \text{ lb. per sq. in.} \quad . \quad . \quad (284)$$

Assuming the yield point of oil-treated carbon steel to be 22 tons per sq. inch, and that the stress is not to exceed 50 per cent. of the yield point, the maximum permissible for any of the above tensile stresses at the overspeed must not exceed 11 tons = 24,600 lb. per sq. in., and if the overspeed is 20 per cent. above the normal, this becomes 17,100 lb. per sq. in. at the running speed. But the best proportions of slot-width to tooth-width to maintain a proper relation between flux and ampere-turns seldom call for a higher stress in the tooth than 14,000 lb. per sq. in., so that, as already mentioned, the limiting speed arises from the end-bells or end-rings rather than from the tooth stresses.

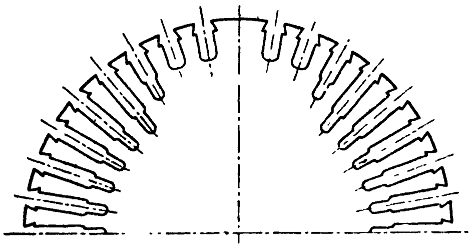

FIG 627 —Rotor slots of lesser depth towards pole-centre.

The rotor slots are usually all of the same depth to receive the same number of conductors, but occasionally they are extended over the pole and are there of reduced depth to give a distribution of flux that is more nearly sinusoidal (Fig. 627).

§ 7. **The retaining wedges.**—Regarded as a beam fixed at both ends and of length equal to its mean width $\frac{1}{2}(w_s + w_2)$ with the load $(f_w + f_c + f_t)L$ concentrated along its longitudinal centre line, the bending moment is

$$\frac{(f_w + f_c + f_t)\,L}{8} \times \frac{(w_s + w_2)}{2}$$

and the modulus of its section being $h_2{}^2 l/6$, the maximum bending stress is

$$f_b = \frac{3}{8} \times \frac{(f_w + f_c + f_t)\,(w_2 + w_s)\,L}{h_2{}^2 l} \quad . \quad . \quad (285)$$

The distinction between L and l is best maintained, since opposite to any radial ventilating ducts the wedge is unsupported. If the load be taken as distributed across the wedge, the stress is, of course, reduced, and an intermediate value is nearer the truth. But on the other hand it is open to doubt whether the ends of the beam can be regarded truly as fixed, and if only supported, the bending stress is increased.

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[1] See *Papers on the Design of Alternating-current Machinery* by Hawkins, Smith, and Neville (Pitman & Sons), p. 128.

The maximum stress when the shearing stress is also taken into account is

$$\tfrac{1}{2}f_b + \sqrt{(\tfrac{1}{2}f_b)^2 + f_s^2}$$

and the greatest shearing stress is

$$\sqrt{(\tfrac{1}{2}f_b)^2 + f_s^2}$$

The materials employed are gunmetal, phosphor-bronze, manganese-bronze, non-magnetic nickel-steel, or magnetic carbon-steel, in the order of increasing strength. Usually the wedges (often inserted in short lengths of about 9 ins.) are non-magnetic, except in the case of any slots in the unwound pole-centre, but if the opening of the rotor slot is wide and the air-gap comparatively small, it is advisable to lessen the non-magnetic width of opening in order to reduce the variation of the flux in the stator teeth as the rotor slots pass them. Even with a strip of thin steel placed above the rotor coil, it is difficult to drive the wedge in over the top of the coil from the end, while at the same time maintaining good pressure on the coil to force it down solidly into the slot. Both points are met by a three-part wedge, devised by Mr. A. B. Field.[1] A bronze centre-piece is pressed down by numerous screws on the coil, and two steel liners are then driven in, one on each side (see also Fig. 652). By the latter the non-magnetic width of opening is reduced.

When driving in the wedges, support must be given to the teeth by temporary wedges in the adjacent slots, to avoid setting up bending strains in the teeth between which the wedge is to be driven home; the rotor coils at the same time must be subjected to strong radial pressure.

§ 8. **The rotor winding.**—Thin copper tape is wound on edge on a former so as to obtain a coil of the exact shape that is required. The span and length of each turn are altered automatically to allow for the different radii as the slots draw radially inwards (Fig. 628). Suspended above the rotor, the turns of the coil are then dropped one by one into the trough insulation of the slot with pure hard mica insulation not less than 0·01 in. thick between the turns. To protect the coil from the action of the ventilating blast when there are axial ducts below the slot, a sheet-steel liner is placed at the bottom of the slots, and if there are also radial ducts, this must be made into a complete sheet-steel cell.

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[1] *Journ. I.E.E.*, Vol. 54, p. 70.

The insulation between the turns in the slot has at once to withstand high temperatures and very great pressure. The latter increases from zero at the bottom of the coil to a maximum at the top, and at the over-speed of 20 per cent. with a coil 3¾ ins. deep on a 31-ins. diameter armature running normally at 3000 r.p.m. this maximum reaches 5000 lb. per sq. in.[1] It is also important that the formed coil should not be liable to shift or " side-slip " at the ends, and a wide strip is favourable in this respect; strip from 1 in. up to as much as 1¾ ins. in width is employed, the greater widths necessitating the use of a small number of wound slots per pole. Under combined baking and pressure the turns of each coil are compacted as far as possible into a solid mass (*cp.* Fig. 629).

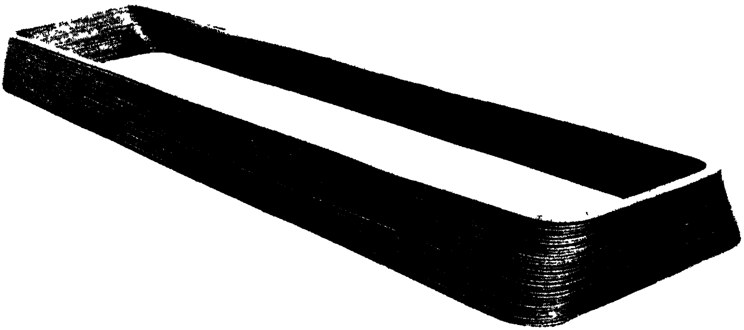

FIG. 628 —Strip-wound rotor coil for 6000 kW turbo-alternator.
(The British Thomson-Houston Co., Ltd.)

To reduce the stress on the end-rings, aluminium has been used for the rotor field-winding, and with success, provided that the rotor cooling ducts are ample. The conductivity of the aluminium is only 60 per cent. of that of copper for an equal section, but by its use the lesser thickness required in the end-ring and the lesser tooth stress enable a deeper slot to be used, and at the same time greater air-space under the windings is gained (Fig. 630).

Fig. 631 shows the turbine end of the rotor of a 35,300 kVA 1500 revs. per min. turbo-alternator built by the General Electric Co. of Schenectady, N.Y.; to secure the field coils in place and prevent any shifting, they are encased in aluminium saddles and packed out with special aluminium spacing blocks.

§ 9. The end-bells or end-rings.—The inside face of the end-connexions of the inner coil stands away from the edge of the rotor core by some 2 to 2½ ins. to allow for the bend of its curved corner. Up to this distance then the centrifugal force acting against the inside of the end-ring is that of the straight projections of the

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[1] Dr. F. S. Barclay, *loc. cit. supra.*

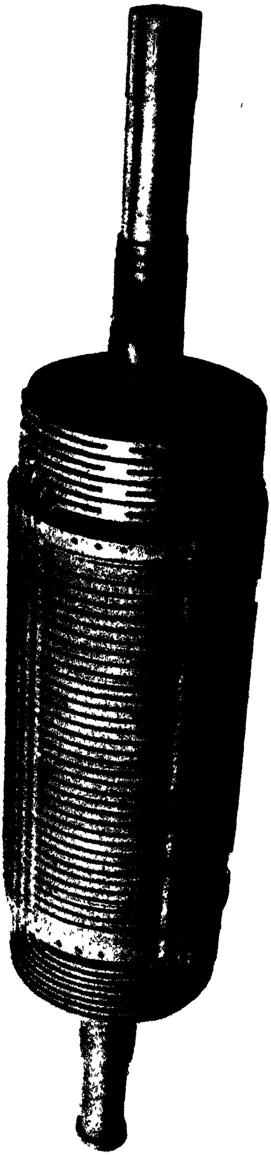

FIG. 629.—Two-pole wound rotor for 3750 kVA, 3000 r.p.m.
(The British Thomson-Houston Co., Ltd.)

rotor coils, pitched at a distance apart equal to the tooth-pitch y_r measured on the inside of the ring. The centrifugal force per inch run of rotor copper in the axial direction may then be reckoned as distributed over $y_r \times 1$ sq. ins. of ring inside, giving a pressure of $(f_c + f_i)/y_r$ lb. per sq. in. In the circumferential direction this ceases opposite to each unwound pole-centre over a length y_p, but the error will be on the safe side if it be assumed to act uniformly all round the ring, like the fluid pressure within a pipe. Since the total number of slots$/2\pi = R/y_r$, the tensile stress[2] due thereto on the single section of the ring will then be

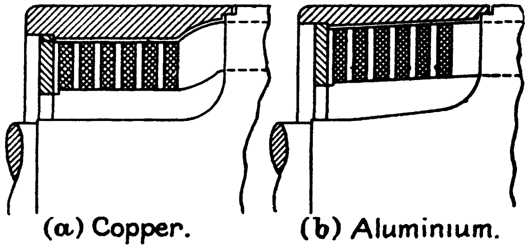

FIG. 630 —Comparison of end-rings with copper and aluminium rotor winding respectively.[1]

$$\frac{f_c + f_i}{y_r} \times \frac{R}{t}$$

where R is the mean radius of the inner edge of the end-ring, and t is its thickness thereat, both in inches.

Moving farther away from the edge of the core and considering one pole-pitch, if c = the number of coils per pole we first have the centrifugal force of the circumferential end of a coil over a circumferential length y_p and of the $(2c - 2)$ axial projections of the remaining coils, followed by the centrifugal force of the circumferential end of another coil over a circumferential length $(y_p + 2y_r)$ and of $(2c - 4)$ axial projections, and so on, until finally at the outer edge we reach the circumferential end of the last coil which practically extends over the pole-pitch Y, and no axial projections enter into the question.

To allow for ventilation between the circumferential ends of the coils, they are spaced out by packing pieces about 1¼ to 2 ins. apart. Let y_a be the pitch of the end-connexions axially. The centrifugal force per inch run of the rotor copper in the circumferential direction may then be reckoned as distributed over $y_a \times 1$ sq. ins. of the interior of the ring, giving a pressure $(f_c + f_i)/y_a$ lb.

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[1] From H. Rikli, " Neuere Entwicklung im Bau von Turbo-generatoren," *Schweiz. Bauzeitung,* Vol. 80, p. 207. *Cp. Schweiz. Elektrot. Verein Bull.* Vol. 15, p. 384.

[2] See Chapter XIII, § 5.

per sq. in. This is practically maintained uniformly round the whole ring, so that the tensile stress due to it is

$$\frac{(f_c + f_i)}{y_a} \times \frac{R}{t}$$

To the above stresses must in each case be added the maximum stress on the inside of the ring due to its own centrifugal force which approximately (equation 62) is

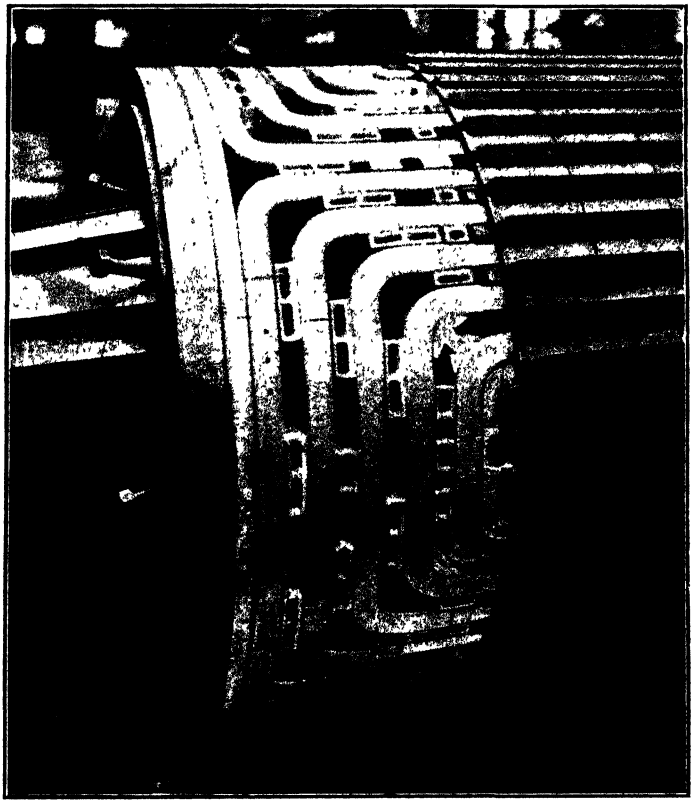

FIG. 631.—End of rotor of 35,300 kVA 1500 r.p.m. turbo-alternator. (The General Electric Co., Schenectady, U.S.A.)

$$2\left(\frac{N}{1000}\right)^2 (3{\cdot}25\, R_o^2 + 0{\cdot}75\, R_i^2) \text{ lb. per sq. in.}$$

if the outer and inner radii of the ring are expressed in inches.[1]

The thickness of the end-ring must be made to increase to a maximum at the outer end, and the difference between its radii here appreciably affects the maximum hoop stress at its inner periphery. But the end-connexions are bent slightly inwards towards the shaft, so that the outer diameter remains nearly the same. In any case, if the air-gap is small, there must be sufficient clearance between the largest diameter of the rotor and the stator bore to allow the rotor to be inserted from one end.

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[1] *Cp.* R. Roberts, *Electr.*, Vol. 80, p. 576 ff.

The end-ring is registered on the rotor core or on projections of the rotor wedges, but must be free to expand elastically; if also registered at the outer end on a supporting disc of steel or bronze with air-holes through it, it may be defined as an " end-ring " in the strict sense of the word (*cp*. Fig. 632). But if integral with the supporting disc, the whole is more appropriately termed an " end-bell." In either case, it must be positively driven by a key, in view of the effect of a sudden short-circuit in retarding the rotor very rapidly. The separate ring is best keyed at its tip to the rotor core by a key in a shallow key-way with good fillets.

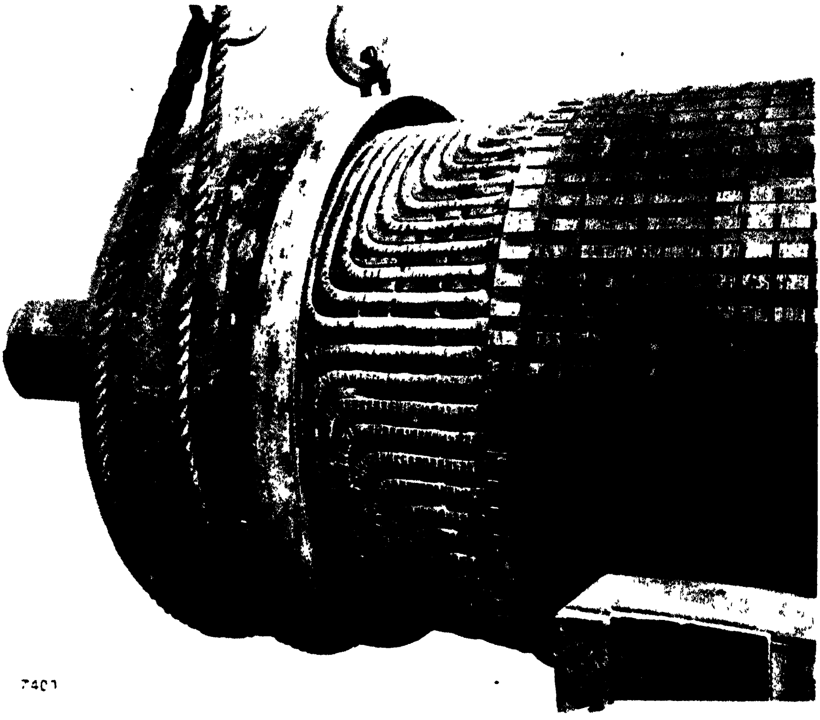

FIG. 632 —Placing end-ring in position on a 2-pole rotor for 9000 kVA 1500 r.p.m. 25-cycle turbo-alternator

(The British Thomson-Houston Co., Ltd)

To prevent the end-ring and its outer supporting disc or the end-bell from being affected by any deflection of the shaft, they have been mounted on a cylindrical ring which projects from the main rotor body, and within which lies the rotor shaft proper.[1] But the possible stiffness of the rotor as a whole is thereby reduced.

To reduce the magnetic leakage from the tooth-tips into the end-rings, composite rings have been employed, a narrow ring next to the rotor core where the stress is least being of manganese bronze.

Under the flux changes caused by short circuit or by breaking the exciting current by a quick-break switch without a discharge resistance, appreciable voltages are induced along the length of the rotor surface and eddy-currents are thereby set up. These may cause arcing[2] from the rotor body or slot-wedges to the end-rings—a defect which must be prevented by bonding key and end-ring together by metal strips extending from the one to the other and pressed against them by centrifugal force, so as to form a complete squirrel-cage. Such strips must be substantial and permit expansion. Fracture of pieces from the inner edge of the end-ring is not unknown, and may be due to causes partly electrical and partly mechanical.

§ **10. The stator core and slots.**—An important point in the design of the stator for a turbo-alternator is sufficient strength in the end-plates and a firmly built core; to secure the latter, especially in large 2-pole machines with great radial depth of discs, through bolts, insulated with mica, and fitting very tightly in holes in a circle not far from the bottom of the stator slots, are sometimes employed. Ventilating ducts are spaced at intervals of 2 ins. or less.

Equally important is the use of strong tooth-supports or spacers of H section, so secured as to be free from any likelihood of vibration under which they work loose and probably end by damaging the winding. Lastly, owing to the high volts per inch length of core, the maintenance of the lamination between the discs right up to the core surface and the elimination of all burrs from the punchings is very necessary to prevent over-heating.[3]

The stator slots may be open, half-closed, or closed, and the winding either concentric or of two-layer diamond type with the first-mentioned, but usually concentric with the two latter. With a two-layer winding and open slots, the coils can be fully insulated before insertion, and the joints at the centre of one end of the coil are well away from stator core and rotor end-ring where they can best be cooled. Tunnels of oval cross-section into which the insulated stator conductors are carefully drawn have much in their favour as compared with open or half-closed slots, and are used by

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[1] G. A. Juhlin, *Journ. I.E.E.*, Vol. 61, p. 877 (Fig. *L*).

[2] *Cp.* Prof. Miles Walker, *Journ. I.E.E.*, Vol 45, p. 304; and J. Rosen, *Journ. I.E.E.*, Vol. 61 p. 453.

[3] See B. G. Lamme, *Trans. Amer. I.E.E.*, Vol. 39, Pt. I, p. 936.

Messrs. C. A. Parsons & Co., Ltd. Apart from the greater freedom which they give from tooth-ripples, the smooth surface of the stator bore lessens any eddy-currents in the body of the rotor due to flux pulsations; in large machines the increased reactance of the stator windings due to the iron bridge is of advantage as reducing the maximum instantaneous short-circuit current. Their usefulness in this respect is, however, somewhat limited by the saturation of the iron bridge which sets in under large stator currents. Hence it is becoming common to employ slots of such a form as is shown in Fig. 652 or 665*a*; the increased height of the neck above the winding helps to postpone saturation under short-circuit, and the air-space is used for ventilation. By the use of such slots the instantaneous short-circuit current of a 45,000 kVA turbo-alternator constructed by the Société Alsacienne de Constructions Mécaniques, Belfort, for Gennevilliers power station is stated to be only some 4 times the full-load current.[1]

If the maximum instantaneous current upon short-circuit $\mathbf{I}_{ssc}$ is not to exceed, say, 10 times full-load current, then roughly speaking what is required is that ten times the normal ampere-turns of armature reaction should be able to pass the whole of the normal flux across the leakage paths, after allowance for any saturation of the iron teeth and slot bridges; the saturation in the latter portions must, therefore, be kept low, so as to call for little expenditure of M.M.F. on them. The main flux is then, as it were, shunted away from the stator coils and across the mouths and sides of the upper parts of the slots by the opposing action of the armature ampere-turns, and can die away gradually in those leakage paths.

§ 11. The stator winding.—To reduce eddy-current losses in the stator conductors, lamination or stranding is necessary. Solid laminations are the most common, twisted at the centre of each bar, or transposed at the coil ends; the rectangular laminations are wrapped round with mica tape which allows of some little sliding relatively to the outer hard-pressed mica insulating covering (the whole measuring 0·1 to 0·2 in. in thickness), and in each case the amount of binding varnish for the mica is reduced to a minimum. Air must for high voltages be excluded. Messrs. C. A. Parsons & Co. Ltd., manufacture by a patented process oval, square, and D-shaped conductors, formed of copper strands separately insulated and helically spiralled without any central core, so that the shape is obtained without any crushing; the over-all insulation, chiefly of mica, is wound when hot tightly round the stranded bars in a special wrapping machine, the moulded tube of mica thus formed having sufficient flexibility to take up the difference in the longitudinal expansions under heat of the copper, insulating tubes and slot-walls. Reliance is thus placed on the elasticity which is definitely given by the greater amount of insulating oils present in the varnish. To withstand the effects of sudden short-circuits, the end-connectors are then of copper strip, to which the stranded bars are united by multiple joints.[1]

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[1] E. Roth, *Revue Générale de l'Électricité*, Vol. 13, p. 143.

For the largest machines, concentric coils have been largely displaced by a two-layer winding with involute end-connexions, usually 2 bars per slot, and each separately insulated. With two poles, the separate half-turns of a loop after insertion are soldered together, and the same practice extends to four poles, although on the four-pole machine it becomes possible to insert the complete loop. The end-connexions either stand out as a cone from the stator core or are bent up in planes almost parallel to the end-plates. To avoid brush discharge on high-tension machines, the insulated ends are sometimes covered with a thin metallic sheath which is earthed.[2] The voltage induced per coil may amount to as much as 250 volts. Fractional pitches (chorded windings) are not infrequent, but necessitate bars of different phases in the same slot where the belts over-lap.

The half-loops are tested for insulation after insertion, and the whole stator winding after completion is tested for resistance to earth ; a 6000 volt machine will thus be tested when hot for one minute with a pressure of 13,000, 15,000, or even 18,000 volts in accordance with the Rules of different bodies.

The interlinked voltage directly generated seldom exceeds 13,000 volts, although 20,000 volts is not beyond what may be anticipated in the future, if the need for it arises.

For heavy currents such as 4000 amperes per phase, when they cannot be divided between several branches in parallel, special precautions must be taken to avoid eddy currents in the conductors. The Brush Electrical Engineering Co., Ltd., have devised and patented for this purpose a split tubular conductor ; a hard-drawn copper tube is split into two halves, but so that the half which is at the bottom along one half of the slot length is integral with the top half in the remainder of the slot length. The two halves fit together with a separating piece of compressed fibre between them, so as to form two conductors in parallel, the whole being then insulated with mica moulded on under heavy pressure at a high temperature.[3] Cooling air is then circulated directly through the conductor.

§ 12. The support of the stator end-connexions.—The bracing of the end-connexions demands the greatest care, their lengths with so small a number of poles rendering this imperative, and some examples are shown in Figs. 656, 657, 664. Strong bronze bolts are employed, often united by one or two complete circular rings.

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[1] See J. Rosen, *Journ. I.E.E.*, Vol. 61, p. 444.

[2] E. Roth, *Revue Générale de l'Électricité*, Vol 13, p. 144.

[3] See description of a double-generator Brush-Ljungström turbo set for 7,140 kVA, 525 volts, 0·7 power-factor, 50 periods, *Electr. Rev.*, Vol. 94, p. 898 ; the current per phase in each of the two alternators being 4000 amperes, it was divided between two leads, making 6 leads in all, and these were so interspaced as to minimize their magnetic effect and prevent any heating of iron parts near which they passed.

Shrinkage of the insulation and packing pieces clamping the end-connexions through prolonged heating must be carefully guarded against, so that there may be no movement of the end-connexions under short circuit with an instantaneous current, perhaps, 10 or more times the normal, and the immunity of the modern turbo-alternator in this respect. after repeated short-circuits bears testimony to the care expended in the design and manufacture of the end-connexion clamps. Especially where the conductors leave the stator slots, efficient supports are required, which may be in the form of a toothed ring of hard insulating material.

External reactances, causing a loss of, say, 5 per cent. of the normal voltage, and composed of concentric turns of copper cable fixed in cement or by strong bolts in air so as to be free from any saturation effects, can only protect the machine against external short-circuits. In the case of an internal short-circuit reliance is placed on the high armature reaction and large leakage paths (Fig. 665*a*).

In connexion with the largest turbo-alternators capable of withstanding dead short-circuits, overload circuit-breakers may become unnecessary. But against internal faults to earth or short-circuits between phases, the stator windings can also be protected by differential relays.

§ 13. The necessity for artificial ventilation.—In order to prove the necessity for the artificial ventilation of the high-speed turbo-alternator as contrasted with the slow-speed alternator with many poles for the same output, it has sometimes been argued that the losses in the former will be at least as great as, if not greater than, in the latter, while the radiating surface of the former is very much less. This statement of the case does not, however, set the facts in their true light, since it ignores the very great influence of the peripheral speed in increasing the cooling effect of the same number of square inches of cooling surface. If the peripheral speed of the turbo-alternator is double that of the engine-driven machine and both are open, much less cooling surface per watt to be dissipated will suffice. Again, if we confine our consideration to a single type of machine it has been argued that while the output and presumably the losses are roughly proportional to the volume which the machine occupies in cubic feet, its radiating surface is only proportional to its superficial area in square feet, and that, therefore, *above a certain size* the radiating surface of the given type of machine bears too small a proportion to the volume to enable the heat loss to be dissipated by natural means. The argument, although true of stationary apparatus such as transformers, is again not universally applicable to rotating machines, as is shown by the case of the slow-speed multipolar alternator in which the cooling conditions become somewhat easier with increasing size; it only becomes nearer the truth when the same peripheral speed is presupposed throughout the range of sizes. The real explanation is that, in the turbo-alternator when a high limiting peripheral speed is approached or reached, the great length of core required by large outputs and the lesser proportion of the end-windings (which are themselves enclosed) to the total length of copper and iron are conditions very unfavourable to cooling. The absolute necessity for artificial ventilation and total enclosure of the machine does not, therefore, arise so quickly as might be supposed. But an additional and powerful incentive to its use in small sizes with medium turbine-speeds, and even in alternators driven by water-wheels, is to be found in the suppression of noise when the machine is totally enclosed, and in some cases also the windage loss is reduced. Yet by the very fact of employing fan draught the turbo-alternator is to a certain extent placed in a worse position, since the air friction and air eddies due to it increase the heat loss which has to be dissipated, and the total losses for the same output become greater than those of the engine-driven alternator.

§ 14. The weight and volume of air required per kilowatt loss.—Depending on the temperature rise which the air is to receive, a certain *mass* of air is required per kilowatt loss, or, to speak more correctly, to carry away heat which is being generated at the rate of 1 kilowatt. The mass may again be expressed in terms of *weight*, but the *volume* of the mass or weight of the air, cold or hot, will be determined by the temperature at which it enters or leaves the machine.

The specific heat of air at 100° F. (= 37·7° C.) is 0·242, and falls to 0·2415 at 40° F. (=4·4° C.); it varies, therefore, but little with temperature,[1] and may be taken as 0·242 for the present purpose. Every gramme of air heated 1° C. will then take up 0·242 calorie, and since 1 kilowatt is equal to 240 calories per second, air supplied at the rate of

$$\frac{240}{0{\cdot}242} \times \frac{1}{453{\cdot}6} \times 60 \times \frac{1}{T_r{}^{\circ}} = \frac{131{\cdot}5}{T_r{}^{\circ}} \text{ lb. per min.} \qquad (286)$$

and raised in temperature T_r degrees Centigrade must be taking up heat at the rate of one kilowatt. Hence if the average rise of temperature $T_r{}^{\circ}$ of all the air passing through is 25° C., 5¼ lb. per minute will serve for each kilowatt of loss.

The volume of this weight of air entering (or issuing from) the machine will depend on its temperature at the inlet (or outlet), and on the height of the barometer H which at high altitudes, as on the Rand, may come into question. One lb. of dry air at 0° C. and a pressure of 760 mm. of mercury has a volume of 12·38 cub. ft., and, at any absolute temperature T° C. or pressure H, of

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[1] See Drs. S. F. Barclay and S. P. Smith, *Journ. I.E.E.*, Vol. 57, p. 303.

$$12{\cdot}38 \times \frac{T^\circ}{273} \times \frac{760}{H} \text{ cub. ft.} \quad . \quad . \quad . \quad . \quad (287)$$

Assuming the height of the barometer to be normal, if the air is supplied at an absolute temperature of T_i° C. at the inlet, the theoretical volume of cool air that is required per kilowatt lost is

$$\frac{131{\cdot}5}{T_r^\circ} \times \frac{12{\cdot}38\,T_i^\circ}{273} = 5{\cdot}97\,\frac{T_i^\circ}{T_r^\circ} \text{ cub. ft. per min.} \quad . \quad (288)$$

If, therefore, the cool air is supplied at 20° C. = 293° abs., and the average temperature of the whole of it is 25° C. higher at exit, 70 cub. ft. of the cool air per min. will suffice for each kilowatt of loss. But usually the volume of the air is more easily measurable at its exit when heated than at its entry, so that the necessary cub. ft. per min. are $5{\cdot}97\,\frac{T_o^\circ}{T_r^\circ}$, where T_o° is the absolute temperature at the outlet; the same weight of air under the conditions above mentioned then becomes when heated 76 cub. ft. per min. per kilowatt of loss.

It is sometimes stated that in order to allow for the air not reaching each part of the machine equally or not being equally heated, in practice as much as, say, 7½ lb. or 100 cub. ft. must be supplied per min. per kilowatt loss, and such an amount may in fact be required. But all that is implied is that in order to reduce the temperature of some parts of the machine to the required figure, the larger total amount per min. may be necessary, but the whole when mixed is not then raised to an average temperature 25° C. higher, as presupposed above. For each kilowatt lost in stator and rotor, bearing losses alone excluded and with allowance made for any gain from cooling by convection from the external housing of the stator or by conduction through the bedplate, the average temperature rise of a given volume of air is a fact determined by the laws of physics.

To give an idea of the magnitude of the problem that the designer has to deal with in the proportioning of the air channels, a 2000 kW alternator may approximately be stated to require at least 830 lb. per min. or 11,000 cub. ft. per min.; a 6000 kW alternator 1350 lb. per min. or 18,000 cub. ft. per min.; a 10,000 kW alternator about 2700 lb. per min. or 36,000 cub. ft. per min.; a 25,000 kVA alternator at 0·8 power-factor 3000 revs. per min. about 49,000 cub. ft per min ; and a 30,000 kW alternator about 5000 lb. per min. or 67,000 cub ft. per min. Or more definitely the case of a 15,000 kVA 25-cycle alternator running at 1500 revs. per min. may be taken for consideration. Its over-all efficiency at 0·8 power-factor will be about 96 per cent., so that its total loss is 500 kW. Allowing 40 kW for bearing friction, out of the remaining 460 kW, being the total loss in stator and rotor, perhaps some 5 per cent.[1] is dissipated by radiation, convection and conduction from the external cast-iron housing. About 440 kW has, therefore to be carried away by the air supplied, and to keep all parts at a reasonable temperature, this may require about 44,000 cub. ft. per minute, raised on an average 17·8° C.

§ 15. **General description of the ventilation system.**—For considerable quantities of air a single fan in an air-chamber formed in the external housing at one end of the stator core is not in general sufficient. Unless, therefore, a separate motor-driven fan is employed, two fans, one at each end of the rotor core, are more usual. These are built on the end-plates of the rotor, and each may blow inwards to the centre of the machine and thence discharge outwards, or one may be confined to blowing or drawing air through ducts in the rotor and the other to the cooling of the stator.

In the simplest case of fans blowing air over the end-rings of the rotor and into the air-gap between rotor and stator from each end, the area for the passage of the air is limited by the fact that the air-gap length does not increase very much in machines for large outputs having long cores. Thus with a one-inch gap and a rotor of 48-ins. diameter, the double area from both ends is only 2·14 sq. ft., so that to supply 44,000 cub. ft. per min., the speed of the air would have to be over 10,000 ft. per min. at the entrance to the core at each end, requiring a high pressure in inches of water. Further the surface of the smooth rotor would not be sufficient to get rid of its heat.

It is evident, therefore, that the passages for the air supply must be increased. In the rotor, the first stage is to provide air-holes in its solid body communicating with radial ducts formed in the core ; the rotor windings are thus exposed to the air at numerous points and at the same time the passages into the air-gap are greatly increased, although the air thus received is partially heated. Next, the axial holes can be replaced by axial ducts formed at the base of the rotor slots (Figs. 633*a* and 634) ; these communicate in the same way with the radial ducts, but the air passing through them is brought into more immediate contact with the thin sheet-steel cells enclosing the rotor coils. The streams of air supplied directly through the air-gap and through the rotor unite to pour through radial ducts in the stator core and thence between the back of the core and the outer casing to issue from the opening at the top of the machine. The rotor air supply is also assisted by passing air

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[1] *Cp.* H. D. Symons and Prof. Miles Walker (*Journ. I.E.E.*, Vol. 48, p. 704).

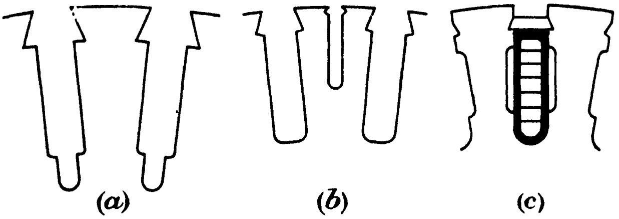

Fig. 633 —Ducts in rotor core.
(After E Roth, *Revue Générale de l'Électricité*, Vol. 13, p 136

through empty slots in the pole-centre (Fig. 642) or in the teeth (Fig. 633*b*) or through ducts on either side of the coils (Fig. 633*c*). If the radial ducts in the solid rotor core are dispensed with, and a sufficient volume of air is drawn through the axial ducts by a suction

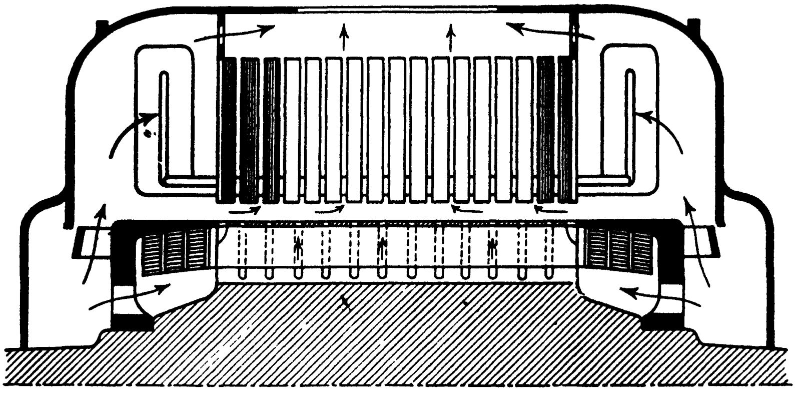

Fig. 634.—Radial ventilation with axial ducts under rotor coils

fan at one end, a rotor core in which the internal system of ventilation is purely axial (Fig. 634*a*) is finally reached.

In the stator, a similar necessity exists for increasing the air supply. Air may be blown circumferentially round the radial ducts in the stator core from a chamber at the bottom of the casing to an outlet at the top, or from two diametrically opposite inlets to two intermediate outlets, but the method is open to the objection that the circumferential stream .more or less interferes with the air passing radially into the ducts from the air-gap. By another

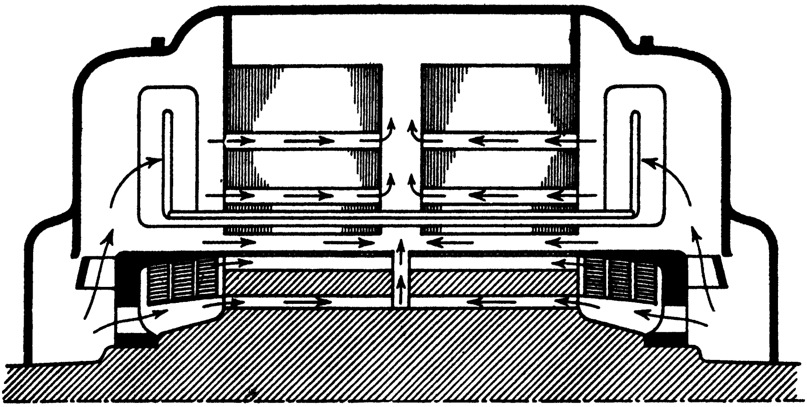

FIG. 634a—Axial ventilation of rotor and stator.

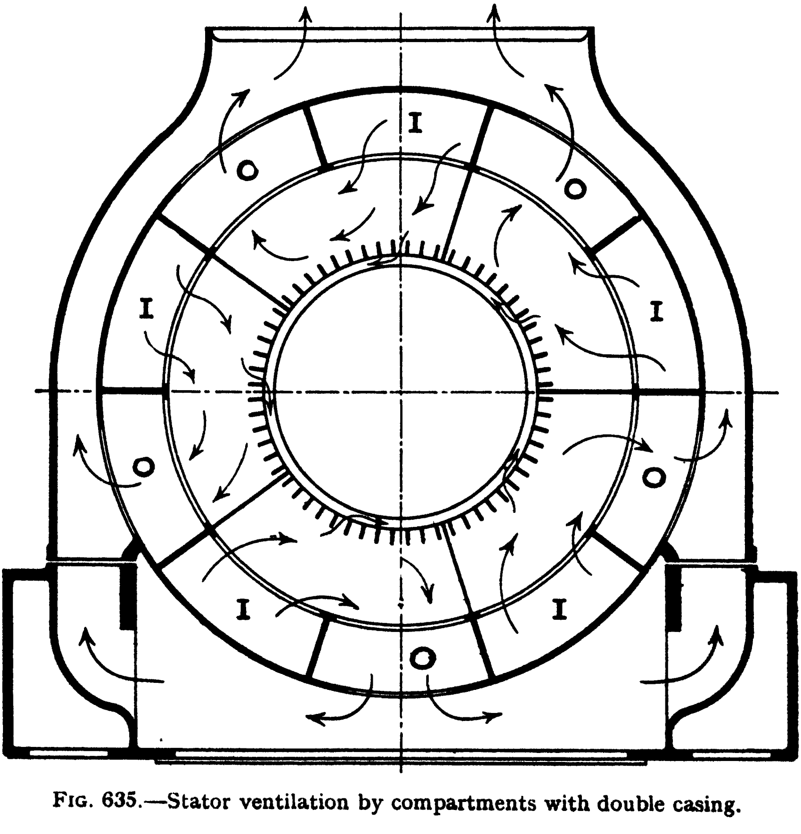

FIG. 635.—Stator ventilation by compartments with double casing.

method (Fig. 635) an inner casing is formed inside the outer casing; the core is divided into pairs of sectors, each pair communicating alternately with the inner and outer chamber. Air is then blown down through the radial ducts in a pair of sectors and guided into the next pair; part, too, enters the air-gap and unites with the air received therefrom to escape outwards up the radial ducts in an adjacent sector. Cool air can thus be introduced into the centre of the machine, which is very beneficial. When there are no radial ducts in the rotor core and no air thrown out therefrom, interference between the two systems of ventilation is largely obviated. But any objection that may be raised on this score is removed if all the radial ducts in the stator core are closed at the air-gap and axial ducts are formed at the bottom of the stator slots and at numerous intervals through the stator core. We thus pass to a purely axial ventilation of the stator (Fig. 634*a*), with air supplied from a fan at one end, or from both ends feeding into a large radial duct in the centre which also carries away the air entering the air-gap. The flow of heat to the surface of the axial duct is good, since its path is in the plane of the laminations and not across them.

Speaking generally, small machines are radially ventilated, and axial ventilation is better suited to large machines. Much has been said and written on the relative advantages and disadvantages of the two systems, but it cannot be said that either possesses undisputed superiority. Combined axial and radial systems (*cp.* Fig. 636) are largely employed; the core is pierced with numerous axial ducts into which the air passes from both end chambers after playing on the stator end-windings, and at the same time the radial duct or ducts with which each axial duct communicates can be chosen so as to secure as far as possible a uniform temperature throughout the entire length of the stator core.[1]

But to secure uniform cooling over the whole length of a long machine ventilation by compartments is now a well-established practice, and this again may be done circumferentially (Fig. 637) or axially. Upon the axial system[2] adopted by Messrs. C. A. Parsons & Co., Ltd., the stator casing is divided internally into separate compartments, pressure compartments into which air is blown alternating with exhaust compartments, and with a pressure compartment at either end; from each pressure compartment the air passes inwards through radial ducts in the stator core and thence makes its way into radial ducts communicating with an exhaust compartment, partly through axial holes behind the stator slots, and partly between the stator teeth and round the conductors into the air-gap where it divides in either direction towards the

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[1] *Cp.* B. G. Lamme, "Ventilation and Temperature in Large Turbo Generators," *Trans. Amer. I.E.E.*, Vol. 39, Pt. I, p. 915.

[2] Illustrated on p. 448 of Mr. Rosen's paper above quoted on p. 375.

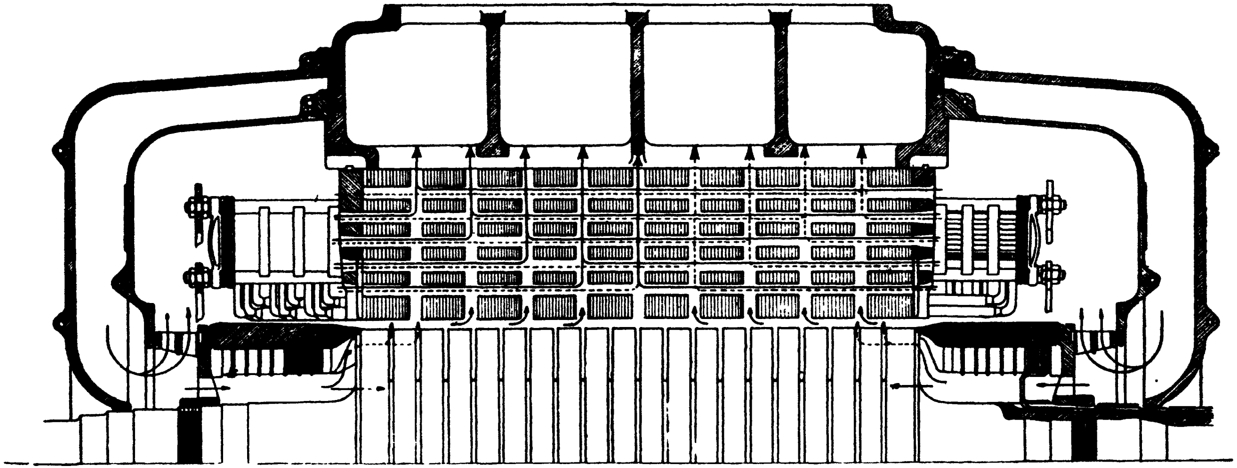

FIG. 636.—Combined radial and axial ventilation for 2-pole turbo-alternator.
(Metropolitan-Vickers Electrical Co. Ltd.)

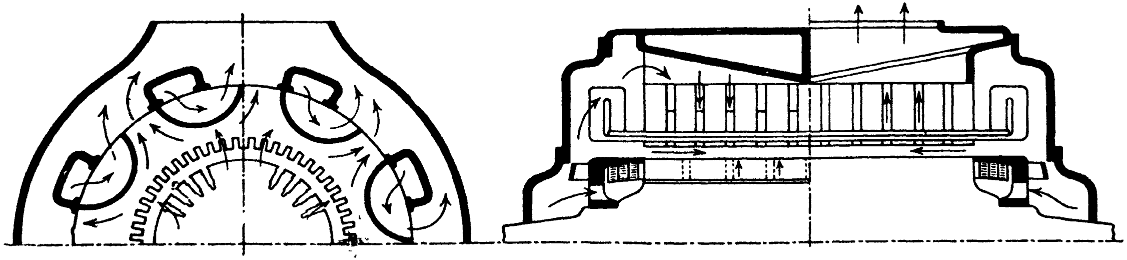

FIG. 637.—Stator ventilation by compartments arranged circumferentially.

exhaust ducts. From the end pressure compartments air is blown through holes in the end-walls of the stator casing on to the end-connexions. In the rotor air is taken in at either end and is forced partly through holes in the end-rings and partly along axial slots machined along the pole centres; in the latter slots, which are purely ventilating, gaps are left in the closing keys opposite each exhaust compartment.

In a large two-pole alternator of Messrs. The British Thomson-Houston Co., Ltd., on the closed-circuit system, cool air enters *via* the end-connexions, also by a central inlet compartment and by others at intervals, and the heated air is drawn out from the intervening compartments through ducts in the foundation by separate fans in the base, whence it passes water-coolers on its way to re-enter the machine.

In the large 35,300 kVA and 62,500 kVA (see § 46) 60-cycle turbo-alternators of the General Electric Co., Schenectady, the multi-path system adopted is as follows.[1] Air is drawn in at each end of the machine through converging ducts formed by the end-bells. At each end is a fan in two sections, which divides the air into two streams. The inner section of the fan feeds directly into the air-gap. The air from the outer section, separated as it issues from the fan by an inner shield attached to the outer end-shield, is carried through sheet iron ducts to a centre compartment in the machine, whence it flows radially inwards to the air-gap, thence axially along the air-gap until it meets the first stream of air from the inner fan. Both streams then flow radially outwards, through the ½-in. air ducts in the stator core. The number of multiple paths is decided by the consideration that many paths mean a high air-duct velocity, low air-gap velocity, poor cooling of rotor and maximum cooling of the stator, and few paths the reverse; the ideal distribution therefore keeps the air-gap full of cool air, and the air-duct velocity about 4000 to 6000 ft. per min.

For the accurate predetermination of the air velocities and pressures needed in passages with changes of direction or section, at each of which there is a loss of head, recourse must be had to experimental tests on models to scale or imitating the conditions of practice. By the use of such models two methods of ventilation have been investigated and the results described in papers by Messrs. C. J. Fechheimer and Donald Bratt;[2] by the one method the air admitted radially inward from the back of the stator core was constrained to pass circumferentially in the air-gap to the outlets, by the other method it passed axially. The experiments showed that the reduction of volume when the rotor revolves, which in the former method was appreciable, was very much smaller in the latter method, and with large air-gaps was practically negligible.

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[1] W. J. Foster, E. H. Freiburghouse and M. A. Savage, *Journ. Amer. I.E.E.*, Vol. 43, p. 927.

[2] "An Experimental Study of Ventilation of Turbo Alternators," by C. J. Fechheimer, *Journ. Amer. I.E.E.*, Vol. 43, p. 416, and "The Multiple-Radial system of Cooling Large Turbo-Generators," by D. Bratt, *Journ. Amer. I.E.E.*, Vol. 43, p. 185; both papers being presented at the Convention of the Amer. I.E.E., Feb., 1924. *Cp.* also C. J. Fechheimer, "Air-Flow in Electrical Machinery," *Elect. Journ.*, Vol. 19, p. 345 ff.

✓ The entrance of even minute quantities of oil into the closed casing of the machine along the shaft or otherwise must be strictly guarded against, since it quickly causes an accumulation of dirt which clogs the ducts, leads to overheating, and being highly combustible may catch fire.

§ 16. The windage loss.—Apart from the actual losses in the fan, as the air passes over the rapidly rotating member, there results a considerable loss from friction, which is expended in raising the temperature of the air. Dr. H. Behn-Eschenberg[1] has given 3¼ watts per sq. in. of surface as roughly the loss by air friction from the cylindrical surface of a rotor broken up by 5 mm. grooves above the slot wedges when running with a peripheral speed of 20,000 ft. per min. This would be reduced over the smooth surface of the end-rings. There has further to be taken into account some loss from eddying of the air, and even in the case of the air passing through the stationary radial or axial ducts of the stator there is loss from its friction with the walls. The total windage loss, therefore, varies greatly in different designs.[2]

§ 17. The fan pressure and fan loss.—The velocity of the air varies with the varying area of the paths presented to it, and ranges from 50 ft. per sec. to 90 or even 120 ft. per sec. in the parts of restricted area. A high velocity of air does much to keep the passages clean, but increases the pressure "head" that is required. This varies greatly according to the nature and area of the passages, and may be from 1½ to 6 ins. or even more of water.

If p = the pressure in inches of water, the pressure in lb. per sq. foot is $5{\cdot}2\,p$, and if Q = the cub. ft. of air supplied per min., the compression work is $5{\cdot}2\,p\,Q$ ft.-lb. per min. At the same time the kinetic energy imparted per min. to the air is $\frac{1}{2}\,\frac{W}{g}\,.\,v^2$ ft.-lb. where W is the weight in lb. corresponding to Q, and v is the velocity at exit from the fan in ft. per sec. that would pass Q cub. ft. per min. Since $W = \frac{Q \times 273}{12{\cdot}38 \times T^\circ}$, the total useful horse-power that must be expended is approximately (the air being assumed initially to be at rest)

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[1] "Charakterische und mechanische Eigenschaften modernen Generatoren inbesondere solcher höherer Tourenzahl," *Intern. Congress at Turin*, 1911. (Vincenzo Bona, 1912.)

[2] The actual windage loss apart from the fan in a 5000-kW turbo-alternator at Carville station (Newcastle-on-Tyne) is given (*Journ. I.E.E.*, Vol. 50, p, 505) as 26 kW or 16 per cent. of the total losses.

$$\frac{5{\cdot}2\,p\,W \times 12{\cdot}38 \times \dfrac{T}{273} + \dfrac{1}{2}\cdot\dfrac{W}{g}\cdot v^2}{33{,}000}$$

or

$$\frac{5{\cdot}2\,p\,Q + \dfrac{1}{2}\cdot\dfrac{Q \times 273}{12{\cdot}38 \times T}\cdot\dfrac{1}{32{\cdot}2}\cdot v^2}{33{,}000}$$

The 100 cub. ft. of air assumed as supplied per min. per kilowatt being at 293° C. and the pressure being assumed as 4 ins. of water and the mean velocity as 60 ft. per sec., the useful power consumed is

$$\frac{5{\cdot}2 \times 4 \times 100 + \dfrac{1}{2}\cdot\dfrac{7{\cdot}55}{32{\cdot}2}\cdot 60^2}{33{,}000} \times 746 = 56{\cdot}5 \text{ watts per 1 kW loss.}$$

When the low efficiency of, say, 25 per cent. for fans fitted on the rotor is taken into account, this becomes 226 watts, and by eddying losses in the rotor and stator it may easily rise to 300–400 watts for every kilowatt lost in the generator.[1]

It is evident that the resistance of the air passages and the fan efficiency require to be carefully studied, since the total ventilation loss may amount to 25 or even 40 per cent. of the total alternator losses.[2] Owing to its better efficiency a separate fan driven by motor at a comparatively low speed has much in its favour. While the efficiency of a fan running at peripheral speeds of 10,000 to 18,000 ft. per min. may be from 10 to 30 per cent., the separately-driven fan can be made to have an efficiency of 40 to 60 per cent., and an actual gain in the over-all efficiency can be secured.[3] The over-all length of the rotor and of the machine as a whole is reduced, and by reason of the higher efficiency of the separate fan, the air is less heated. Further, the separate fan can be run at different speeds and air-pressures to meet different conditions of load or overload. Either the separate or the rotor fan may fail, but failure of the latter will usually involve damage to the stator end-windings, while the former will be run at a moderate speed only.

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[1] A curve of B.H.P. required to drive the fan in relation to the output of the alternator given by Dr. S. F. Barclay (*Journ. I.E.E.*, Vol. 56, p. 513) shows approximately the low value, B.H.P. = 0·006 kW. See also the same paper for the mechanical construction of the fan.

[2] The loss in the separately-driven fan of the 5000 kW turbo-alternator mentioned in the last note but one was 17 kW, raising the total loss by windage and fan to 43 kW or 27 per cent. of the total losses of the alternator.

[3] *Cp.* J. Rosen, *Journ. I.E.E.*, Vol. 61, p. 471.

§ 18. The progressive heat-absorption and rise of temperature of the air.—For the purpose of illustrating in outline the general course of the heat-absorption as the air passes through the machine, an approximate chart is shown in Table XXXVII, in which use has been made of the tests of an 1875 kVA 3000 r.p.m. turbo-alternator described[1] by Prof. Miles Walker and H. D. Symons in their paper on " The Heat Paths in Electrical Machinery."

The machine had a fan at each end, and the recorded results show as follows—

Test B is considered first, since the full aperture was here given to the fan and the air supply was not throttled. The air taken in was about 590 lb. per min., corresponding to $Q = 7900$ cub. ft. per min. at the temperature of entry (21·7° C.); the pressure created was 4¼ ins. of water, and the velocity at exit from the fan which would suffice to pass the required quantity would probably be about 3600 ft. per min. By the previous equations it will then be found that the mechanical power expended in giving useful kinetic energy to the air only amounts to about 800 watts, while the power expended in compression is 4 kW. The whole of the latter appears as heat, and so also does the eddy loss and skin friction to which the inefficiency of the fan is due. The air after passage through the fan at the outset before it begins to do any useful cooling work is, therefore, appreciably higher in temperature than at the intake, and this difference may be as much as 5° or 6° C, In the present case the extra eddy loss is estimated at 11·8 kW, which with the compression heating gives a total of 15·8 kW and an over-all fan efficiency of $\frac{4\cdot 8}{11\cdot 8 + 4 + 0\cdot 8} = 29$ per cent., which is as high as can be expected. But as will be seen in the following Table, the rate of heating the air by fan and windage together is estimated at 29·6 kW. The rate of loss by the machine's own air friction and eddies in its air-channels is $29\cdot 6 - 15\cdot 8 = 13\cdot 8$, which also helps finally to raise the temperature of the air.

Measured total rate of loss from friction, fans, and windage		46 kW
Rate at which heat was carried away		
(*a*) by oil from two bearings	14·6 kW	15·6 ,,
(*b*) by radiation and convection from two bearings, say,	1 kW	
Total rate of loss from fans and windage		30·4 ,,
Rate of imparting useful kinetic energy		0·8 ,,

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[1] *Journ. I.E.E.*, Vol. 48, p. 703; and also in *The Specification and Design of Electrical Machinery*, p. 243.

Rate at which air was heated by air friction and eddies		29·6 kW
Rate of development of heat by excitation of rotor field winding		8·5 ,,
Rate of development of heat by iron loss in the		
stator teeth	4·5	43·5 ,,
back of the teeth	39·0	
Total rate of heat generation		81·6 ,,
Deduct for convection and radiation from cast-iron casing, say,	1 kW	1·5 ,,
Deduct for conduction through supports of frame, say,	½ ,,	
Total rate of heating the air		80·1 ,,

The average rise of temperature of the whole of the air must correspond thereto, and by equation (288)

$$T_r^\circ = \frac{5{\cdot}97 \times 294{\cdot}7}{7900} \times 80{\cdot}1 = 17{\cdot}9^\circ \text{ C}.$$

in agreement with the measured value, as plotted for the outside of the stator discs, and averaged.

The actual distribution of the losses (1) from air eddies, friction, and windage, (2) from electric and magnetic sources together with the corresponding rises in temperature of the air may roughly be allocated as in Table XXXVII, partly from the recorded measurements and partly by guess-work. In test A the air supply was throttled which accounts for the large consumption of energy which has been allotted to the fan; the actual supply of air was about 295 lb. per min. corresponding to 4000 cub. ft. per min. when cold at entry or 4400 at exit when heated, or half that of test B. Half the quantity at double the speed roughly dissipates heat at the same rate but with twice the temperature rises of test B.

Since no current flowed in either test through the stator coils, an appreciable proportion of the heat loss in the core has been assigned to be conducted along the copper and dissipated at the stator end-windings.

§ 19. Air-filters and air-washers.—In towns and industrial districts it is very necessary that the ventilating air should be filtered to free it from dust and dirt. The filters employed may be either dry, the air being drawn through cloth strainers, or wet, with water sprayed on to plates or mixed with the air. Alternatively the air may be forced through holes to impinge on oil or sheets of trickling water. The first-mentioned dry filter has now largely been superseded owing to its size, the inflammable nature of the filter cloths, and the need to clean them at intervals from the accumulated dust.

TABLE XXXVII

APPROXIMATE TEMPERATURE-RISE OF VENTILATING AIR IN 1875 kW TURBO-ALTERNATOR

	TEST B. Full Aperture					TEST A Air Supply Throttled.				
	Air losses in kW	Electric and magnetic losses in kW	Total losses in kW	Temperature rise of air.	Temp. of air	Air losses in kW	Electric and magnetic losses in kW	Total losses in kW	Temp. rise of air.	Temp. of air.
Temperature of air at entry . . .	—	—	—	—	21·7° C	—	—	—	—	21·7° C.
Added during passage through fan by compression, friction and eddies . .	15·8	—	15·8	3·55° C		14·4	—	14·4	6·4° C.	
Added from end-plates and end-connexions of stator, before entry along air-gap .	—	7·0	15·6	3·49° C		—	7·5	15·7	7·0° C	
Added from end-bells of rotor . .	3·1	5·5				2·5	5·7			
Added from rotor along air-gap to centre	7·9	3·0	18·9	4·21° C.		5·0	2·8	15·9	7·1° C.	
Added from internal face of stator bore .	—	8·0				—	8·1			
Added from radial vent ducts in stator .	2·4	25·6	28·0	6·25° C		0·8	22·2	23	10·2° C.	
Added by passage between casing and outside of stator core . . .	0·4	1·4	1·8	0·4° C.		0·1	1·7	1·8	0·8° C.	
Totals	29·6	50·5	80·1	17·9° C.	17·9° C.	22·8	48	70·8	31·5° C.	31·5° C.
Temperature of air at exit . . .					39·6° C.					53·2° C.

Washing the air by a water spray has the further advantage of cooling it. Even if there is no possibility of obtaining cooling water at a temperature below that of the air entering the filter, the latent heat required for any water-particles that are vaporized is taken out of the air, and the output of the machine can thus be maintained in very hot weather. But on the other hand if the velocity of the air through the wet filter is high, there is always a danger of water particles being carried over into the turbo-alternator; the air should, therefore, always be passed over some dry surfaces before it enters the stator casing. In frosty weather the water must be kept from freezing by steam heaters.

In the "Visco" air filter the air is passed through cells filled with numbers of short rings of very thin coppered steel tubing of small diameter; these are covered with a thin film of a special oil which effectively catches and retains a high percentage of the impurities of the air impinging on it. The oil has a very high flash point, and the filter has the advantage that it is unaffected by either frost or heat.

Large stations require now to be laid out with a complete system of air-ducts, and the heated air must be discharged into the outer air through a duct, which also assists in reducing the noise. Dampers are provided in the ducts to cut off the intake or discharge of air in case of an internal fire being started in the machine through failure of the insulation. Steam jets have also been used for the same purpose.

More recently closed-circuit systems have come into favour;[1] in these a small weight of air is continuously circulated through the machine and through an air-cooler placed in the foundation block or close by, with short ducts. The air is initially freed from dust, and the fire risk is reduced owing to the small amount of oxygen present, or the nitrogen content can even be artificially increased, so that it will not support combustion (as proposed for the Gennevilliers station). A 10,000 kW alternator is stated to require only 80 lb. or 1100 cub. ft. of air for circulation.[2] The water circulated through the gilled tubes of the cooler may be supplied from the main circulating pump of the condenser, but the use for this purpose of the condensed steam of the turbine has met with such success in practice that it has been adopted for some of the large 45,000 kVA sets of the Gennevilliers power station. Here the condensate after taking up heat from the air also cools the bearing oil before it is returned to the feed-tanks of the boilers, which results in a gain in efficiency. The Société Alsacienne de Constructions Mécaniques (Belfort) have devised an arrangement to secure a constant amount of cooling water, whatever the load, since otherwise on very light loads there might not be sufficient condensate for the purpose of cooling.[1]

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[1] See J. Rosen, *Journ. I.E.E.*, Vol. 61, p. 449; and *General Electric Review*, Vol. 23, pp. 91 and 99.

[2] J. Rosen, *loc. cit.*, pp. 450, 471.

§ 20. **Water-cooled turbo-alternators.**—Water-cooling of stator and rotor has often been advocated, and methods for applying it have formed the subject of many patents. But in practice it has mostly been used in connexion with rotors. On the closed-water system employed by Messrs. C. A. Parsons & Co., Ltd., water is forced through a rotating tube passing centrally along the shaft from the outer end into a central hole in the rotor; at the coupling end it passes radially outwards through holes in the rotor body, through joints external to the stator end-shields into solid-drawn weldless steel tubes which pass longitudinally through the rotor core immediately below the rotor windings; thence radially inwards through holes into an annular space between the rotating tube and the rotor shaft, and so out into a water box formed in the end pedestal.[2] The pressure of the water on the tubes may amount to 2–3 tons per sq. in. Messrs. The British Thomson-Houston Co., Ltd., employ a number of small holes drilled through the rotor core at some distance below the winding slots, where the stress is comparatively low, and through these the cooling water is passed.

The water employed must be rendered neutral or alkaline to prevent corrosion of the tubes.[3]

§ 21. **Bearings and their forced lubrication.**—The advantage of a small diameter of journal from the point of view of frictional loss has already been emphasized in Chapter XIII, §§ 12 and 13; but in turbine-driven machinery the necessity for a large diameter of shaft at the neck of the journal is all the greater owing to the possibility of vibration and consequent risk of fracture. The greater part of the deflection occurs between the journal and the rotor body, so that a shaft diameter of reasonably large size must be maintained up to and through the bearing to secure rigidity. Apart from any added vibration, the stress is alternating, and the reversals are very rapid, so that to avoid fatigue of the metal the maximum combined torsional and bending stress should not exceed 7·5 to 10 per cent. of the yield point, or, say, 4000 lb per sq. in. Journal speeds up to 6500 ft. per min. are in use. The ratio of length to diameter of journal is from 2½ to 4, and in connection with the pressure per sq. in., which, as already explained in Chapter XIII, § 12, must not squeeze out the oil film, vibration has also to be considered.

In the bearings of high-speed turbo-machines with $p = 70$ lb per sq. inch of projected area, the product of the intensity of pressure and the velocity in ft. per min. reaches as high as 150,000 or 200,000 with a plentiful supply of oil forced through under pressure to cool the bearing artificially. A maximum rarely exceeded would be 350,000 given, say, by 70 lb. per sq. inch at 5000 ft. per min. In order to give viscosity to the bearing so as to absorb and damp out vibration, the bearing bushes are formed of several concentric sleeves with clearances of a few mils between them and also between the inner sleeve and the shaft. It is partly due to the presence of the oil films in the small clearances between the sleeves that in estimating the critical speed (§ 49) the shaft may legitimately be treated as a beam simply supported and not fixed at the bearings.

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[1] See E. Roth, *Revue générale de l'Électricité*, Vol. 13, p. 137, for other details.

[2] J. Rosen, *loc. cit.*, pp. 451 and 476.

[3] For arguments in favour of water-cooling, see J. Shepherd, *Journ. I.E.E.*, Vol. 58, p. 133.

The oil is usually supplied under a pressure of 45 to 60 lb. per square inch, and is cooled in a separate chamber.

Taking the specific heat of the oil as 0·31, each lb. of oil supplied per min. and raised 1° F. in temperature abstracts heat at the rate of

$$0{\cdot}31 \times 778 \text{ ft-lb. per min.} = 0{\cdot}00732 \text{ horse-power}$$

or if raised 1° C at the rate of 0·01315 h p. = 9·85 watts Assuming the specific gravity to be 0·925 (i e the heat coefficient reduced to unit volume = 0·286), 1 lb of oil = 30 cub ins = 0·108 gallon Therefore, each gallon of oil per min. raised 1° C abstracts heat at the rate of 0·122 h p = 91 watts. If, therefore, the whole of the heat is to be abstracted by the oil and no allowance be made for radiation, the number of gallons that must be supplied per min. and per sq inch of projected area of bearing is

$$Q = \frac{\mu p v}{33{,}000 \times 0{\cdot}122 \times \text{rise in °C.}}$$

$$= \frac{\mu p v}{4000 \times \text{rise in °C.}} \text{ gallons per min}$$

where p is in lb per sq inch and v in ft per min.

According to Lasche's experiments with high journal speeds (Chap XIII, § 13, Vol. 1, p 311), if $\mu p T° =$ a constant, we should, with a given temperature of bearing and rise of oil temperature, have

$$Q = \frac{28{\cdot}45v}{4000 \times \text{rise in °C} \times T°}$$

The temperature of the bearing bush should not exceed 75° C, and the oil leaving the bush at 60° C. (*cp* Vol. 1, p 309) has then a temperature as high as is advisable; if higher, it is liable to carbonize and cause a deposit in the oil-pipes and channels Hence if cooled to 45° C,

$$Q = \frac{28{\cdot}45v}{4000 \times 15° \times 75°} = 6{\cdot}33 \times 10^{-6}\, v$$

This minimum amount will require to be increased to allow for unequal distribution of the oil to all parts, and in practice both the temperature of the bush and the degrees through which the oil can be cooled may be so much less that $Q = 20 \times 10^{-6}\, v$, or, say, 0·04 to 0·06 gall per min and per sq inch Or the amount may be calculated from the previous expression on the assumption of a high coefficient of friction $\mu = 0{\cdot}01$.

Circulating currents through rotor shaft and bearings must not be present and sometimes may require to be stopped by insulating the end plummer-block or by earth-connected brushes pressing on a slip-ring on the rotor shaft [1]

§ 22. The thermal conductivities of insulation.—The chief mechanical details of the turbo-alternator having now been touched upon, we return to the consideration of more purely electrical questions and in the first place to the questions of heating and heat transmission.

In large high-voltage turbo-alternators with long cores, coils of several square inches of copper section and slot insulation perhaps 0·3 in. thick, the dissipation of heat from the stator coils is a serious problem, and calls for radical consideration during the process of design. The temperature drop across the slot insulation may easily amount to 40° C., so that the insulation at its hottest spot is 40° C. above the surface temperature as measured by thermometer, and its durability may be impaired, if not heat-resisting. It therefore becomes very advisable to make a calculation, even if only approximate, of the temperature differences required to pass the watts generated to the cooling surfaces, and the same detailed consideration must also be given to the heating and cooling of the rotor exciting coils.

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[1] For shaft currents, their causes and remedies, see Chapter XXVIII, § 29.

In addition to the references[1] given in Chapter XXI, § 13, for the thermal conductivity of copper and various insulating materials, it will only here be necessary to emphasize the great effect of small air spaces in lowering the heat conductivity, which affords another argument in favour of solid impregnation in high-voltage machines. Although solid mica plate has a high thermal conductivity k_t of 0·009 watt per inch cube per 1° C., built-up flexible mica or micanite tubes have a value for k_t of only about 0·0025, and are, therefore, inferior to Empire cloth or oiled cambric for which $k_t = 0{\cdot}003$ to 0·004 when impregnated. In the slots for Empire cloth, mica and tape with minimum air-spaces as for 11,000 volts k_t may be taken $= 0{\cdot}003$ to 0·0035 which will be increased to $k_t = 0{\cdot}004$ or 0·005 when tightly wound and impregnated, but for 6600 volts with more air-spaces k_t is better taken as $= 0{\cdot}0025$ only. In the coil-ends insulated with varnished fabrics, k_t may be taken as 0·0035.

§ 23. The transmission of heat transversely and longitudinally.— In considering the transmission of heat from the windings to the cooling surfaces, two leading cases may be distinguished. Neither of these separately represents the exact conditions of practice, which can only be represented by a combination of them, but the first is the more applicable to rotor coils and the latter to stator coils. Both will, however, first be discussed in relation to stator coils.

In dealing with these, allowance must be made for the additional watts due to non-uniform distribution of the current in large solid conductors, and the amount of this depends on the resistance of the copper. But if I be the useful current in amperes in a conductor and $k_e I^2 R$ are the total watts over any length of resistance R, an assumed current that will give the same total loss in watts is $\sqrt{k_e} \,.\, I$, and $\sqrt{k_e} \,.\, I/a$ may be taken as the current density in amperes per sq. in., a being the sectional area of one conductor.

(1) First, let it be supposed that all the heat generated within any given length of a coil embedded in a slot escapes *transversely* through the slot insulation at the rate at which it is generated.

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[1] *Cp.* also P. Janet, *Rev. Gén. d'Électr.*, Vol. 9, p. 393.

Let r = the resistivity of copper in ohms per inch cube at temperature T

$= 0{\cdot}626 \times 10^{-6} \times T/234{\cdot}5$

where T = its temperature in degrees Centigrade + 234·5, *i.e.* its temperature above the inferred zero of resistivity.

A steady state of constant temperatures having been reached, the rate at which heat is generated in a thin section down the slot of thickness dx in one layer of a 2-layer winding containing z conductors per slot is

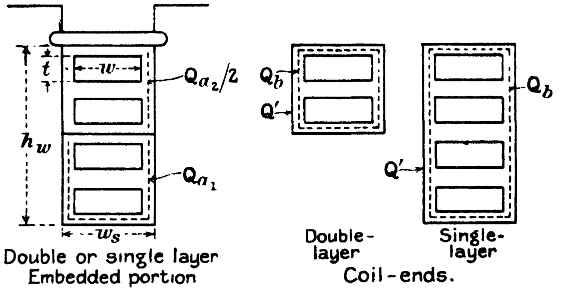

FIG 638 —Transverse heat-paths through slot and coil-end insulation

$$dW = k_e \frac{I^2}{a} r \frac{z}{2} dx$$

Let T_d = the temperature difference in ° C. between the copper and the outer surface of the insulation across the total thickness t_i of the insulation from copper to iron including clearances in inches.

If the mean area of the heat path through the insulation is Q_a sq. ins. per 1 in. axially, the area for the thickness dx is $Q_a\, dx$. The heat passing through this for temperature difference T_d is $\frac{k_t \,.\, T_d\, Q_a\, dx}{t_i}$, and as it is now being assumed that the rate at which it is passing is equal to dW,

$$k_e \frac{I^2}{a} r \frac{z}{2} = \frac{k_t T_d Q_a}{t_i}$$

or

$$T_d = \frac{k_e}{k_t} \frac{I^2}{a} \frac{rzt_i}{2Q_a}$$

Let w = the width across the slot of one conductor, and n_a = the number abreast, if more than one ; let t = the total radial thickness of each conductor whether laminated or not, and n_l = the total number of layers of conductors (not of coils) in the slot (Fig. 638). Then assuming no heat to pass from one layer to the other in the two-layer winding, for the layer at the bottom of the slot the numerical value of the area Q_{a1} for unit length axially is the mean of the lengths $h_w + w_s$ (where h_w = the winding height in the slot, not including the wedge) and $n_l t + n_a w$.

The same also holds for the upper coil-side at the top of the slot, if the wedge be metallic and heat-conducting. But if it be wooden and reckoned as impervious to heat, the area Q_{a2} for the upper coil-side is the mean of the lengths h_w and $n_l t$ only.

For the two layers separately, therefore,[1] with their different eddy-current co-efficients k_{e1} and k_{e2} in the lower and upper layer respectively

$$T_{d1} = \frac{k_{e1}}{k_t}\frac{I^2}{a}\frac{rzt_i}{2Q_{a1}} = \frac{k_{e1}}{k_t}\frac{I^2}{a}\frac{rzt_i}{(h_w + w_s + n_l t + n_a w)}$$

$$T_{d2} = \frac{k_{e2}}{k_t}\frac{I^2}{a}\frac{rzt_i}{2Q_{a1}}$$

or if the wedge is heat-insulating,

$$= \frac{k_{e2}}{k_t}\frac{I^2}{a}\frac{rzt_i}{2Q_{a2}} = \frac{k_{e2}}{k_t}\frac{I^2}{a}\frac{rzt_i}{(h_w + n_l t)}$$

For the slot as a whole if an average value be taken for k_e

$$T_d = \frac{k_e}{k_t}\frac{I^2}{a}\frac{rzt_i}{2Q_{a1}} \text{ or } = \frac{k_e}{k_t}\frac{I^2}{a}\frac{rzt_i}{Q_{a1} + Q_{a2}} \quad . \quad . \quad . \quad (289)$$

and the same will also hold for a single-layer winding when Q_{a1} and Q_{a2} are given the values indicated for the lengths in Fig. 638.

In the test on a 5000 kW turbo-alternator cited by Mr. H. D. Symons and Prof. Miles Walker (*Journ. I.E.E.*, Vol. 48, p. 683), we have t_i = 0·177 in., made up of Empire cloth, 0·07 ; mica, 0·03 , varnish and air, 0·02 ; paper, 0·017 ; linen tape, 0·04. Each conductor was composed of 2 copper straps, each 0·45 in. × 0·2 in., whence a = 0·18 sq. in., and I was 328 amperes. The copper being at a temperature of 62° C., $r = 0{\cdot}792 \times 10^{-6}$ and the estimated value of k_e was about 1·19. Since $z = 4$, the watts per inch length for the whole slot are

$$k_e \frac{I^2}{a} rz = 2{\cdot}26.$$

Some heat was conducted to the outer portions of the winding, but not very much, so that 1·9 watts per inch-length will be assumed as the rate of heat conduction to the slot wall. $Q_{a1} + Q_{a2}$ = 4·5 ins., so that if k_i be reckoned as 0·0036, the difference of temperature between copper and slot wall would be

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[1] *Cp.* R. B. Williamson, " Notes on Internal Heating of Stator Coils," *Trans. Amer. I.E.E.*, Vol. 32, Part I, p. 153.

$$T_d = \frac{1{\cdot}9 \times 0{\cdot}177}{0{\cdot}0036 \times 4{\cdot}5} = 20{\cdot}7\ ^\circ\text{C}.$$

If instead of $1{\cdot}9/4{\cdot}5 = 0{\cdot}423$ watt per sq. in. it was desired to pass 0·6 watt per sq. in. laterally through slot insulation 0·2 in. thick as for 13,200 volts, then the temperature drop between copper and slot wall might be as much as

$$\frac{0{\cdot}6 \times 0{\cdot}2}{0{\cdot}003} = 40^\circ\ \text{C}.$$

This does not, however, present the problem as it occurs during design. It will be noticed that the resistivity of the copper r contains a term dependent on the temperature which is unknown. The watts, therefore, change with the temperature, so that the difference of temperature that is required between copper and slot wall entirely depends on the actual temperature of the copper or the part of the temperature scale at which the fall occurs. To proceed further, therefore, some assumption would have to be made as to the distribution of the temperature T_i of the iron core along any length under consideration, into which the heat flows, and upon this more will be said later.

(2) If the rate of generation of heat per inch length of core in copper and iron is severally such that at every point each reaches of itself the same temperature, no interchange of heat can take place between the two, and the copper conductors are in effect encased in a heat-insulating tube. In these circumstances when a steady state is reached, the heat must flow away entirely *longitudinally* from the hottest spot in either direction towards the ends.

When a steady state of constant temperatures has been reached, reckoning distance from the point of maximum temperature in the core, the rate at which heat is generated over the whole distance from $x = 0$ to $x = x$ is $\int_0^x dW$, and on the present hypothesis the rate W_2 at which heat is passing through a thin section of thickness dx situated at point x must be the same, or $\int_0^x dW = W_2$.

Thus, through the cross-section of the copper distant x inches from the point of maximum temperature the rate at which heat is passing is the sum of all the heat generated in unit time between this point and the point x divided by the sq. ins. of section. To pass this amount increasing with an increase of x, the temperature gradient must continually increase, *i.e.* the curve of the temperature T_c of the copper must fall more and more rapidly. If dT is the temperature difference between the two sides of the thin lamina *dx*, then for the copper area *za* of the slot as a whole of thermal conductivity k_c and temperature gradient dT/dx

$$W_2 = -k_c za \times \frac{dT}{dx} = k_e \frac{I^2}{a} z \times \frac{0{\cdot}626 \times 10^{-6}}{234{\cdot}5} \int_0^x T dx$$

Here T is the actual temperature $T_c + 234{\cdot}5$, and the negative sign is used, since T decreases as x increases.

Again, therefore, the change in the resistivity of the copper as dependent upon the actual temperatures or the part of the temperature scale covered is vital to the problem, since

$$-\frac{dT}{dx} = \frac{k_e}{k_c}\left(\frac{I}{a}\right)^2 \frac{0{\cdot}626 \times 10^{-6}}{234{\cdot}5} \int_0^x T dx$$

can only immediately be used to determine the maximum temperature upon some assumption as to the temperature at which the ends are maintained.

§ 24. **The combination of the two cases.**—Now in the stator the iron teeth themselves become heated, and the conduction of heat laterally away from the coils through the slot insulation is thereby checked; further, the ends of the coils are, as a rule, less thickly insulated, and are exposed on two surfaces to ventilating air. They are, therefore, maintained at a lower temperature, and the heat from the embedded conductors passes partially away to the ends, so that the centre of the core may be expected to be the hottest spot. But it may be the case that the end-windings are so thickly insulated and their ventilation so poor that they are not able to do much more than dissipate their own heat; the flow of heat out of the embedded length must then be mainly through the slot insulation to the iron core, and in a smaller degree through the insulation and wedge to the outer cooling air, so that we approach case (1).

A combination of the two cases thus in general represents the actual facts.[1] A certain number of watts is abstracted laterally, and at the same time there is a fall of temperature from the centre to the ends in the case of the copper or of both copper and iron. The watts, $k_e I^2 R = W$ over any length of the embedded coils may then be divided into two portions, of which W_1 is conducted through the slot insulation, and the remainder W_2 is conducted longitudinally through the thin section of thickness *dx* situated at any point distant *x* ins. from the hottest spot which may usually be assumed to be the centre of the length of the core. The heat W_2 carried forward longitudinally beyond any point *x* as in case (2) is $-k_c za \times \frac{dT}{dx}$, but this is now equal to the total heat generated within the *x* inches *less* the amount conducted away transversely within the same length, *i.e.* $W - W_1$. The latter difference for the slot as a whole is

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[1] *Cp.* B. G. Lamme, *Trans. Amer. I.E.E.*, Vol. 39, Pt. I, p. 922 ff.

$$\int_0^x k_e \frac{I^2}{a} z \times \frac{0{\cdot}626 \times 10^{-6} \times T}{234{\cdot}5}\, dx - \int_0^x \frac{k_t}{t_i} 2Q_{a1}T_d\, dx$$

Therefore

$$\int_0^x k_e \left(\frac{I}{a}\right)^2 \times \frac{0{\cdot}626 \times 10^{-6} \times T}{234{\cdot}5}\, dx - \int_0^x \frac{k_t\, 2Q_{a1}}{t_i\, za} \left\{T - (T_i + 234{\cdot}5)\right\} dx = -k_c \frac{dT}{dx}$$

where T_i is the temperature of the iron teeth in degrees Centigrade.

$$\text{Let } s = \frac{k_t\, 2Q_{a1}}{t_i za}, \text{ and } m = \frac{0{\cdot}626 \times 10^{-6}}{234{\cdot}5} \times \frac{I^2}{a}, \text{ so that}$$

$$\int_0^x \frac{k_e m}{a} T\, dx - \int_0^x s \left\{T - (T_i + 234{\cdot}5)\right\} dx = -k_c \frac{dT}{dx}$$

$$\text{Thence } k_c \frac{d^2T}{dx^2} - \left(s - \frac{k_e m}{a}\right) T + s\,(T_i + 234{\cdot}5) = 0$$

of which the solution is

$$T = (T_i + 234{\cdot}5) \frac{s}{s - \dfrac{k_e m}{a}} + T_2 \cosh \sqrt{A_a}\, x + T_3 \sinh \sqrt{A_a} \,.\, x$$

where $A_a = \dfrac{1}{k_c}\left(s - \dfrac{k_e m}{a}\right) = \dfrac{1}{9}\left(s - \dfrac{k_e m}{a}\right)$, the thermal conductivity for copper, k_c, in watts per 1 in. cube per 1° C. being taken as 9.

$$\text{Let } T_{ao}^\circ + 234{\cdot}5 = T_1 = (T_i^\circ + 234{\cdot}5) \frac{s}{s - (k_e m/a)}.$$

One terminal condition is that when $x = 0$ at the centre of the core where the temperature may be assumed to be a maximum, $dT/dx = 0$ whence T_3 is 0. The other terminal condition is given by the temperature $(T_L^\circ + 234{\cdot}5)$ of the copper where it emerges from the slot when $x = L_a$, half the length of the core, and

$$T_L^\circ + 234{\cdot}5 = T_1 + T_2 \cosh \sqrt{A_a}.\, L_a$$

$$\text{whence } T_2 = -\frac{T_{ao}^\circ - T_L^\circ}{\cosh \sqrt{A_a}.L_a},$$

$$\text{and } T = T_1 - \frac{T_{ao}^\circ - T_L^\circ}{\cosh \sqrt{A_a}.L_a} \cosh \sqrt{A_a}.\, x.$$

If, therefore, it is assumed that the temperature of the iron is uniform along its length at some value T_i°, the maximum copper temperature is defined when x is made $= 0$, as

$$T^\circ_{c\ max} = T^\circ_{max} - 234{\cdot}5 = T_{ao}^\circ - \frac{T_{ao}^\circ - T_L^\circ}{\cosh \sqrt{A_a} \,.\, L_a}$$

and the fall along the copper can be completely predicted.[1]

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[1] *Cp.* Prof. Miles Walker, *Journ. I.E.E.*, Vol. 59, p. 298.

But without such assumptions, the problem is soluble by consideration of the combined effect of the two heat-paths in the embedded length and in the coil-ends. The results of the combination when both embedded lengths and coil-ends are acting simultaneously have been investigated mathematically by Mr. C. J. Fechheimer in a valuable paper[1] on "Longitudinal and Transverse Heat Flow in Slot-wound Armature Coils," which has here been followed.

A steady state of constant temperatures being presupposed, we have over the embedded length

$$\int_0^x dW = \int_0^x dW_1 + W_2$$

Applying the same principles to the coil-ends, if heat is entering the section of the copper at any point distant y from the assumed point of minimum temperature at the rate W_2 and is being generated over this length y at the rate $\int_0^y dW$, both amounts must traverse the insulation of the coil and be dissipated within the length y mainly by convection air-currents due to the fans and also partly by radiation. *I.e.*

$$W_2 + \int_0^y dW = \int_0^y dW_1$$

Let suffixes a, b indicate respectively embedded length and coil-end. Let

$$K_a = \frac{k_{ta}\,Q_a}{t_{ia}},$$

(the value Q_{a1} being used for the lower layer and Q_{a2} for the upper layer when the two differ owing to the wedge being heat-insulating; in this latter case $\frac{1}{2}\,(Q_{a1} + Q_{a2})$ must be used for the slot as a whole, but otherwise Q_{a1} holds also for the slot as a whole)

$$K_b = \frac{k_{tb}\,Q_b}{t_{ib}}$$

and Q' = the surface per unit length of the end-connexion under consideration.

With a two-layer winding Q_b is equal to the mean of the external perimeter Q' of the end and $(n_t t + 2n_a w)$, if the same disposition of the conductors is retained as in the slot. With a single-layer winding it is the mean of the new value of Q' and $(2n_t t + 2n_a w)$.

In the end-connexions if T_b = the temperature of the copper, and T_e = the temperature of the outside surface of the insulation at any point distant y ins. from the point of minimum temperature

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[1] *Trans. Amer. I.E.E.*, Vol. 40, p. 589.

$$dW_1 = \frac{k_{tb}\,(T_b - T_e)\,Q_b\,dy}{t_{tb}}$$

But the temperature T_o of the air as it is blown upon the coil-ends is less than T_e, and

$$T_e - T_o = \frac{1}{\xi' Q'} \times \frac{dW_1}{dy}$$

where Q' is the surface per unit length of end-connexion after taking into account the loss of surface due to bracing clamps, and ξ' is the number of watts that can be taken up by the air per unit of surface and per 1° difference of temperature between air and outside surface.[1]

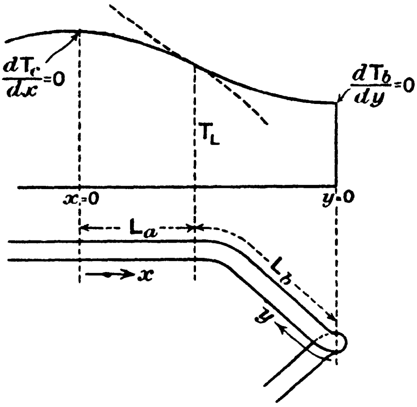

FIG 639.—Temperature-gradients for embedded and end-connexion portions of stator coil passing into each other

Thence if $\gamma = \cfrac{1}{1 + \cfrac{K_b}{\xi' Q'}}$

$$\int_0^y dW_1 = \gamma K_b \int_0^y (T_b - T_o)\,dy$$

At the end of the core the two values for W_2 must be the same, and the temperature gradient or the slope of the two curves of the temperature of the copper at the junction point between embedded length and coil-end must be identical, so that they run into one another (Fig. 639). For the establishment of the resulting formulæ upon the three different assumptions (a) that the temperature of the iron is maintained uniform over the whole length of the core, (b) that it varies linearly from a maximum at the centre to the same minimum at each end, and (c) that it varies as a parabola, the reader must consult the original paper.[2] The solution upon the first and simplest assumption will alone be given here, making use of the symbols adopted above. Let $\frac{0{\cdot}626 \times 10^{-6}}{234{\cdot}5} \times \frac{I^2}{a} = m$,—a term which, when multiplied by the temperature ($T_c + 234{\cdot}5$) and by the appropriate eddy-coefficient, is proportional to the ohmic loss in one conductor ; let k_{ea} be an average value of the eddy coefficient for the embedded length of the upper or lower layer or slot as a whole, whichever is under consideration, and k_{eb} the similar quantity for the coil-end.

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[1] Estimated by Mr. Fechheimer as 0·045 watts per sq. in. per 1° C. from unpublished data. *Cp.* T. S. Taylor, *Amer Soc. Mech. Eng. Journ.*, Vol. 42, pp. 230, 259, April, 1920; and G. E. Luke, *Trans. Amer. I.E.E.*, Vol. 42, p. 649.

[2] Where much valuable information will be found, and additional points are fully discussed.

$$\left.\begin{array}{l} \text{Let } A_a = \dfrac{1}{k_c a}\left(\dfrac{2K_a}{z} - k_{ea}\, m\right) \\[2ex] T_{ao}^{\circ} = \dfrac{T_i^{\circ} + 234{\cdot}5\,\dfrac{k_{ea}\, m}{2K_a/z}}{1 - \dfrac{k_{ea}\, m}{2K_a/z}} \end{array}\right\} \begin{array}{c}\text{for a single-}\\ \text{or double-layer}\\ \text{winding}\end{array}$$

Double-layer[1] *Single-layer*[1]

$$A_b = \frac{1}{k_c a}\left(\frac{2\gamma K_b}{z} - k_{eb}\, m\right) \quad \text{or} \quad \frac{1}{k_c a}\left(\frac{\gamma K_b}{z} - k_{eb}\, m\right)$$

$$T_{bo}^{\circ} = \frac{T_o^{\circ} + 234{\cdot}5\,\dfrac{k_{eb}\, m}{2\gamma K_b/z}}{1 - \dfrac{k_{eb}\, m}{2\gamma K_b/z}} \quad \text{or} \quad \frac{T_o^{\circ} + 234{\cdot}5\,\dfrac{k_{eb}\, m}{\gamma K_b/z}}{1 - \dfrac{k_{eb}\, m}{\gamma K_b/z}}$$

$$\beta = \frac{\sqrt{A_b}\tanh\sqrt{A_b}\,.\,L_b}{\sqrt{A_a}\tanh\sqrt{A_a}\,.\,L_a} \quad (\text{for } L_a \text{ and } L_b\text{, see Fig. 639})$$

then the temperature of the copper at the edge of the stator core is

$$T_L^{\circ} = \frac{T_{ao}^{\circ} + \beta\, T_{bo}^{\circ}}{1 + \beta} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (290)$$

and the maximum temperature of the copper at the centre of the core is

$$T_{c\,max}^{\circ} = T_{ao}^{\circ} - \frac{T_{ao}^{\circ} - T_{bo}^{\circ}}{\cosh\sqrt{A_a}\,.\,L_a} \times \frac{\beta}{1 + \beta} \quad . \quad . \quad . \qquad (291)$$

§ **25. Heat transmission in the air-gap.**—The stream of air passing axially through the air-gap finds itself between the stationary cylinder of the stator bore on the one side and the rotating cylinder of the rotor on the other side, and both impart heat to it, but under different conditions. The value of ξ, *i.e.* the rate in watts at which heat is picked up per sq. in. of cooling surface per 1° C. excess temperature of the stator bore above the temperature of the passing air, would in the first case if the stream of air were not broken up differ but little from that for axial ducts (§ 27), say, $\xi = 0{\cdot}0133\ (v'/1000)$, where v' is the velocity of the air past

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[1] The reason for the necessity of different expressions for a single- or double-layer winding in the case of the coil-ends is that with the same conductors per slot forming a single group at the ends, the new Q' and the new Q_b or K_b are not double the Q', Q_b, or K_b of the arrangement in two layers. The values of γ and K_b are, therefore, different in the two cases for the coil-ends, but for the embedded lengths the same Q_a and K_a is to be used as explained under § 23 (1).

the stator surface in ft. per min. But actually owing to disturbances caused by the air thrown out from the rotor vents this is greatly improved upon, and we have, say, $\xi = 0{\cdot}02\ (v'/1000)$ whence the temperature difference between air and stator bore is

$$t^\circ\,\text{C.} = \frac{1}{\xi}\frac{W}{S_c} = k\frac{W}{S_c} = \frac{W}{S_c \times 0{\cdot}02\ v'/1000} \quad . \quad (292)$$

But near the rotor surface the air clings to it and is carried round with the rotor, so that the heat transmission is not nearly so good,

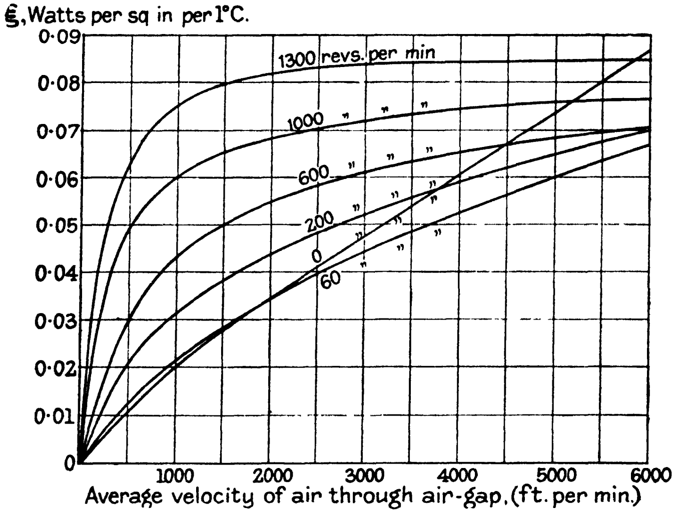

FIG 640—Forced axial ventilation through a ½ in. single air-gap surrounding a rotating rotor 25 ins diam. × 36 ins long.

and the resistance to the axial flow is greatly increased. For the same velocity of air, ξ improves as the peripheral velocity of the rotor is increased, but experiment shows that for any given peripheral velocity of rotor, provided that it is high, ξ is roughly constant, whatever the velocity of the air between a range of 2000 to 6000 ft. per min. (Fig. 640).[1] We return, therefore, to a type of formula for the rotor surface dissipation expressed in terms of V', the peripheral velocity of the rotor and not of v', the velocity of the air, viz.

$$\xi = \frac{1 + 0{\cdot}5\ (V'/1000)}{60}$$

and

$$t^\circ\,\text{C.} = \frac{1}{\xi}\frac{W}{S_c} = k\frac{W}{S_c} = \frac{60}{1 + 0{\cdot}5\left(\dfrac{V'}{1000}\right)} \times \frac{W}{S_c} \quad . \quad (293)$$

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[1] G. E. Luke, *Trans. Amer. I.E.E.*, Vol. 42, p. 646, from whose paper on "The Cooling of Electric Machines" Fig. 640 is reproduced.

where S_c is the surface of the rotor core only, V' its peripheral speed in ft. per min. Thus with $V' = 20{,}000$ ft. per min., and $W/S_c = 4$ to 5 watts per sq. in., the temperature of the rotor surface is about 22° to 28° C. above that of the cooling air passing it. Owing to the greater smoothness of the surface of the rotor as compared with the usual broken surface of the end-windings of the continuous-current barrel armature, it is natural that the index of the power of V is not higher than unity, therein resembling stationary field-coils.

It is found advantageous to groove the surface of the rotor with a coarse thread;[1] this increases the turbulence of the motion of the air in the air-gap and the cooling surface, and also to some extent checks the eddy currents at stator tooth frequency in the skin surface of the rotor, due to the varying permeance of the doubly slotted system combined with the grouping of the stator M.M.F. in its slots. Although the grooving may reduce the volume of air passing, appreciably, yet the temperature rise of the rotor is lowered.

§ 26. Heat transmission in the rotor.—If $\delta\ z$ = the number of conductors per slot, the loss per inch length of a slot is $\left(\frac{I_f}{a}\right) r\, I_f\, z$; the factor k_e is absent, and $I_f z$ is the number of ampere-conductors per slot. If it be assumed that the end-bells can at best only dissipate the heat of the end-connexions, the whole of the above loss must pass through the slot insulation. If $I_f z = 10{,}000$ and the actual temperature of the copper be 75° C., so that $T = 309{\cdot}5$, and if $\frac{I_f}{a} = 1600$ amperes per sq. in., the loss per inch is

$$1600 \times 10{,}000 \times \frac{0{\cdot}626 \times 10^{-6} \times 309{\cdot}5}{234{\cdot}5} = 13{\cdot}25 \text{ watts}$$

The wedge is metallic and heat-conducting, so that for 1 in. of axial length the area Q for the slot as a whole is $2h_w + 2w_s - 4t_i$. If in a typical case, this is $10 + 3 - 0{\cdot}24 = 12{\cdot}76$, the insulation being 0·06 in. thick, the fall of temperature between copper and slot wall would be, say,

$$T_d = \frac{13{\cdot}25 \times 0{\cdot}06}{0{\cdot}0036 \times 12{\cdot}76} = 17{\cdot}3^\circ \text{ C.}$$

But it can be shown that as a general rule owing to the concentration of the copper end-connexions under the end-rings, the cooling surface of the end-rings and of the under side of the rotor coils is not sufficient to dissipate the heat generated therein. The end-rings are themselves the source of some heat due to the alternating leakage field from the ends of the stator core set up by the ampere-conductors in the stator slots. Actually, therefore, the case is a combination of the two effects as described in the preceding section, but reversed. The end-connexions are the hottest part, and some heat is transmitted longitudinally into the rotor core, there to be conducted transversely through the slot insulation, so that T_d usually exceeds 20° C. The amount of this heat is, however, very variable and uncertain, so that only the general principle can be stated without full knowledge of the details of the design.

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[1] W. J. Foster, E. H. Freiburghouse and M. A. Savage, *Journ. Amer. I.E.E.*, Vol. 43, p. 929.

Let it now, therefore, be assumed that the heat conducted from the rotor end-connexions adds 30 per cent. to the heat generated in the slots which has to be dissipated within the slot length; then if

t_1 = rise of temperature of the air before it reaches the air-gap, due to the fan losses and friction, part of the excitation loss dissipated at the end-bells, and the true skin friction loss at the face of the rotor,

t_2 = rise of temperature of the rotor surface above that of the air, and t_3 = the rise of the temperature of the copper above the slot wall,

the rise of the temperature of the rotor copper above that of the cold air supply will be

$$\begin{aligned} & t_1 \quad + \quad t_2 \quad + \quad t_3 \\ &= t_1 + 1{\cdot}3\left(\frac{I_f}{a}\right) r\, I_f z \left\{ \frac{\text{slots per pole}}{\text{rotor pole-pitch}} \times \frac{60}{1 + 0{\cdot}5\,\dfrac{V'}{1000}} \right. \\ &\qquad\qquad \left. + \frac{t_i}{k_t\,(2h_s + 2w_s - 4t_i)} \right\} \end{aligned}$$

Suppose t_1 to have been estimated as about 15° C. and assuming the same slot dimensions and conditions as above, $t_3 = 1{\cdot}3 \times 17{\cdot}3 = 22{\cdot}5°$ C. Let there be 10 slots per pole, and let the pole-pitch be 47 ins. With a peripheral speed of 23,600 ft. per min., then

$$t_2 = 1{\cdot}3 \times 13{\cdot}25 \times \frac{10}{47} \times \frac{60}{1 + 11{\cdot}8} = 17{\cdot}2° \text{ C.}$$

The total rise is then

$$15° + 17{\cdot}2° + 22{\cdot}5° = 55° \text{ approximately.}$$

If, therefore, the air supply is at 20°, the temperature of the copper agrees with the value assumed above.

§ 27. Heat dissipation by the ventilating ducts.—In the case of the ventilating ducts through the stator Messrs. Symons and Walker have shown that the heat that can be picked up per sq. inch of wall per 1° C. difference of temperature between wall and air at any point is proportional to the average speed of the air,[1] and may be expressed as

$$\xi = 0{\cdot}046\,\frac{v'}{1000}$$

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[1] The speed is lower at the walls than at the centre.

where v' is the average speed of the air through the duct in feet per minute.[1] Hence

$$t^\circ \mathrm{C} = \frac{1}{\xi}\frac{W}{S_c} = k\frac{W}{S_c} = \frac{1000}{0{\cdot}046\,v'} \times \frac{W}{S_c} \quad . \quad . \quad (294)$$

where t° C. is the rise of the temperature of the wall surface of the duct above that of the air passing it, after it has picked up heat from eddies in the fan chamber, from the end-plates and from the rotor and stator cylindrical surfaces, not above that of the cool air entering the machine in the first instance.

The lesson to be drawn from the above is that provided the ducts are not too narrow so as to be liable to become clogged with dirt, " for a given quantity of air passed through the machine, narrow ducts will be more effective than wide holes," which " allow the air to pass through without coming in close contact with the iron." " Ventilating ducts from 0·3 in. to 0·4 in. wide having smooth iron walls will keep clean for a great number of years " if the velocity of the air through them be not less than 1000 to 2000 ft. per minute, and there is no oil spray. The ventilating ducts are thus usually ⅜ in. to ½ in. in width.[2] Round axial ventilating holes 2 ins. or 3 ins. in diameter with rough walls unless the air is very well filtered quickly accumulate dirt, especially in rotors where the centrifugal force presses the dust into a solid mass against the surface of the hole farthest from the centre. A layer of coal dust 0·030 in. thick will halve the effect of an axial hole 1¼-in. diameter. But in its clean state with the 1¼ in. hole formed by punchings in the laminations, so that the edges give a rough surface to disturb the air stream, we have[3] roughly in watts picked up per sq. in. per 1° C. rise of temperature

$$\xi = 0{\cdot}015\ \frac{v'}{1000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (295)$$

and for a 2-in. hole about 0·011 ($v'/1000$), although higher results can be obtained if the air flow is disturbed or made to rotate during its passage through the duct.

§ 28. Heat transmission in stator core stampings.—The thermal conductivity of the stator core plates along the plane of the laminations, being that of iron, is about 1·6 watts per inch cube per 1° C.

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[1] Air blown between two flat hot plates about ½ in. apart gives a much lower figure, say, $\xi = 0{\cdot}013\ (v'/1000)$, owing to its straight path not being broken up by irregularly spaced separating fingers and the conductors of the actual machine : *cp*. Fig 15 of the paper by Mr. G. E. Luke on " The Cooling of Electric Machines," *Trans Amer. I.E.E.*, Vol. 42, p. 644

[2] For effects of different widths of packets and ducts, *cp* L. Fleischmann, *E.T.Z.*, Vol. 42, p. 961, abstracted in *Electr.*, Vol. 89, p. 237 ; O. Böhm, *E.T.Z.*, Vols. 42, p. 1388 and 43, p. 810.

[3] G. E. Luke, *Trans. Amer. I.E.E.*, Vol. 42, p. 641 ; and for influence of eddies, see R. Pohl, *Archiv. f. Elektrot*, Vol. 12, p. 361.

difference of temperature. On the other hand the thermal conductivity across the plane of lamination will be very much reduced owing to the intervening insulation of paper or varnish and thin films of air due to roughness of the surface. The transverse conductivity will depend on the proportion of insulation to iron,[1] and it may be only $\frac{1}{60}$th or less of the true conductivity of the metal. Its value in the practical case of the turbo-alternator investigated by Messrs. Symons and Walker was estimated to be 0·044 watt per inch cube per 1° C.,[2] and it was, therefore, only $\frac{1}{30}$th of the conductivity along the laminæ. On account of this wide difference between the two conductivities, it follows that with cooling air playing on the cylindrical surfaces and also on the radial walls of a packet of stator discs, the temperature down the middle of the packet will not show any great difference from centre to top and bottom edges, but that there will be an appreciable drop of temperature between centre and side-walls. The curve of temperature across the packet if all the heat were given off at the walls would be a parabola. The centre of the packet is the hottest spot, and if w = the watts per cubic inch in the iron behind the teeth the difference of temperature in ° C. between centre and wall is

$$t_1 - t_2 = \frac{w}{2 \times 0{\cdot}044} \times x^2$$

where x is the half thickness of the packet in inches. The thickness of a packet seldom exceeds 2 ins. for 50 cycles or 2·75 ins. for 25 cycles, and average figures for w would be 0·7 watt per cub. in. = 2·5 per lb. for 50 cycles or 0·4 watt per cub. in. = 1·44 per lb. for 25 cycles. The maximum difference would then be only 8° C. or 8·6° C. in the two cases, and therefore would be reasonable in amount. But the above shows that narrower packets should accompany high frequencies or discs of lower quality and greater loss.

§ **29. Dissipation from external casing.**—As a guide to an allowance for radiation and convection from the external casing of the stator, Prof. Miles Walker[3] has given as an approximate figure 0·97 watt per sq. in. of the external surface of the stator discs for a temperature rise of 40° C. The surface to be reckoned is here the external circumference of the stator discs multiplied by the gross length of the core, *plus* the area of the end-plates at each end of the core. Dr. S. F. Barclay and Prof. S. P. Smith[1] from careful tests on a 5000 kW 3000 r.p.m. machine found that heat was dissipated by radiation and convection at the rate of 0·011 watt per sq. in. of external casing per 1° C., which will result in less than the above figure. The amount is necessarily somewhat indeterminate, since some heat may pass through the bedplate from turbine to alternator or from the alternator towards the turbine according to circumstances.

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[1] For its calculation, see "The Heat Conductivity of Iron Stampings" by T. M. Barlow, *Journ. I.E.E.*, Vol. 40, p. 612; "The Heat Paths in Electrical Machinery" by H. D. Symons and Miles Walker, *Journ. I.E.E.*, Vol. 48, p. 716; and *The Specification and Design of Dynamo-electric Machinery*, p. 253.

[2] In *The Diagnosing of Troubles in Electrical Machines*, p 52, Prof. Miles Walker gives 0·03 across steel laminations with 10 per cent. paper insulation. *Cp.* also T. S. Taylor, *El. World*, Vol. 76, p. 1159.

[3] *Specification and Design of Dynamo-electric Machinery*, p. 254.

§ 30. Temperature measurement.—Thermocouples or embedded detectors are[2] often built in at different parts of the stator which would otherwise be inaccessible, as *e.g.* between a pair of teeth of adjacent discs of the iron core or in the middle of a slot between the outer coverings of two bars, near to the star point of a 3-phase winding when this is earthed (otherwise their use may be attended with risk) or between the wedge and the winding. By their means the temperature of the insulation can be checked and less directly the temperature of the copper, and the load adjusted accordingly. Allowing a final temperature of, say, 95° C. for class A organic material and 115° C. for heat-resisting mica and mica products as measured by increase of resistance, with a surrounding air-temperature of 40° C. the permissible rises are 55° C. and 75° C. But by thermocouple between two slot layers, which registers nearly the full temperature of the copper, a rise of 80° C. with mica insulation is by some authorities permitted. By thermocouple between coil and wedge or coil and bottom of the slot above 5000 volts, a deduction should on the other hand be made of 1° C. for every 1000 or fraction of 1000 volts owing to the necessary fall in temperature in the covering. In considering the information as to the maximum copper temperature to be obtained from a temperature detector embedded between the two layers of coil-sides in a slot, or between coil and wedge, many factors have to be borne in mind, such as the greater eddy-current loss towards the top of a coil, possible circulation of air about the detector, etc.[3] By thermometer, the rise of temperature of the end windings of the stator should not exceed, say, 65° C.

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[1] *Journ. I E E* , Vol. 57, p. 304. Mr W. F. Dawson (*Gen El. Rev.*, Vol 23, p 158) gives 0·0125 watt per sq. in per 1° C difference of temperature between external surface and surrounding air.

[2] For details as to their construction, see S. L Henderson, *Electr. Journ.*, Vol. 16, p. 193, and Dr. Georg Keinath, *E. U. M.*, Vol. 40, p. 97.

[3] Upon all of which, see F. D. Newbury and C. J. Fechheimer, " Some Practical Experience with Embedded Temperature Detectors in Large Generators," *Trans. Amer. I.E.E.*, Vol. 39, Pt I, p. 971 ; F. D. Newbury, *Trans. Amer. I.E.E.*, Vol. 41, p. 514 ; B. G. Lamme, *Trans. Amer. I.E.E.*, Vol. 39, Pt. I, p. 945 ; " Temperature Limits in Large Machines," by P. Torchio, *Trans. Amer. I.E.E.*, Vol. 41, p. 524, and G. A. Juhlin, *Journ. I.E.E.*, Vol. 59, p. 281.

For a method of estimating the maximum copper temperature from the observed surface temperature and the mean temperature as deduced from resistance measurement, see M. Jakob, *Archiv. f. Elektrot.*, Vol. 10, p. 47.

Apart from any calibration curve, if the cold junction of the external thermocouple is plunged into an oil bath which is slowly heated or allowed to cool while its temperature is watched, the temperature of the hot junction can be ascertained; for at the moment when the reading of the millivoltmeter passes zero, the temperature of the oil bath is that of the hot junction.[1]

§ 31. Stray losses.—The "stray losses" or those which are due to more or less indeterminate causes which are hardly amenable to exact calculation may in the turbo-alternator amount to a considerable item.[2]

Non-uniform distribution of currrent in massive end-connexions of the coils linked with self-induced flux is one source of a loss, which, though its cause is well known, cannot be calculated with the same certainty as the similar loss in the straight conductors.[3] It must be remembered, too, that apart from what may be called the normal eddy-current loss in the stator bars due to the M.M.F. of the current in the slot, there may also be a further loss due to the transverse slot flux when some of the stator teeth are very highly saturated by the main field under full-load.[4] But among the most important of the stray losses is that which arises from eddy-currents in the stator core end-plates, stator end-guards, and rotor end-rings, and especially in the former; with the high values for the armature reaction now permitted, the alternating magnetic field due to the stator end-connexions may set up eddy-currents of sufficient magnitude to cause a large loss in the above-mentioned adjacent metal parts. Especially is this the case if the turbo-alternator is run at full speed with phases short-circuited and excitation reduced to yield the normal full-load current; the end-rings are not saturated as in normal working, and afford an easy path for stator-induced flux, so that the watts obtained from such tests cannot be applied with any certainty as an addition to other known losses to determine the full-load loss and the heating under normal full-load conditions.[5] To avoid a considerable loss by eddy-currents, the bolts and rings bracing the end-connexions to the stator end-plates are usually made of bronze. There are, too, eddy-currents set up in the rotor body due to flux-pulsation when the ratio slot-opening/air-gap is unduly large, or to single-phase armature reaction.

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[1] E. Roth, *Rev Gén. d'Électr*, Vol. 13, p 140.

[2] *Cp.* R Rüdenberg, *Schweiz. Elektrot Ver Bull.* Vol. 14, p. 514, or *E.T.Z.*, Vol. 45, pp. 37 and 59.

[3] For an approximate estimate in a 3-phase case, see Prof. Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, p. 99.

[4] *Cp.* Prof. Miles Walker, *Journ. I E E*, Vol. 59, p. 299.

[5] *Cp.* Dr. S. F. Barclay, *Journ. I.E E.*, Vol 56, p. 483, and Vol. 57, p. 300; and J. A. Kuyser in discussion, p. 477; and for an approximate calculation, Prof. Miles Walker, *loc. cit. supra*, p 103; E. Roth (*Soc. Franc. des Electr.*, 4th series, Vol. 3, p. 504) states that while air-heating tests of the Gennevilliers alternators (§ 44) showed a stray loss of only 90–100 kW, this was increased on short-circuit test to 460 kW, or about 5 times as much.

If the whole stray loss in the 3-phase 2-pole turbo-alternator is expressed as in Chap. XXIX, § 15, in terms of ac_u, and Y in ins., we have on full-load such figures as

$$70 \times ac_u^2 \times Y^3 \times 10^{-11} \text{ kilowatts at 50 cycles}$$
$$\text{and} \quad 40 \times ac_u^2 \times Y^3 \times 10^{-11} \quad \text{,,} \quad 25 \quad \text{,,}$$

while on short-circuit these may be exceeded by some 15 to 20 per cent.; for 2 phases, the loss may be still greater.

Retardation tests enable many of the recognized losses to be separately determined,[1] and may thence be of value as enabling the source of obscure losses to be definitely located.

In order to forecast the probable stray losses in large 31,250 and 62,500 kVA machines (§ 46), and to determine, if possible, their seat, miniature machines, exactly one-third the linear scale, were built, in which by substituting wood for inactive parts, such as shields, flanges, end-fingers, etc., the losses in these various parts could be separately measured. The percentage losses in the miniatures proved in good agreement with the percentages in the large machines.

By tests on the miniature models it was confirmed that, although the fundamental magnetic effect of the end-connexions of the stator winding, rotating at the same speed as the end-rings of the rotor, caused no loss therein, yet high-frequency harmonic fluxes might cause considerable loss in the end-rings. Further, under permanent short-circuit or leading power-factor, *i.e.* with a small number of *AT* on the rotor, the leakage fluxes from the armature *AT*, interlinking with the rotor ends and stator end-shields, cause large losses, whereas under load or lagging power-factor the counter M.M.F. of the larger number of rotor *AT* alters the direction and magnitude of the end flux leakage, and the loss is much reduced.[2]

§ 32. Efficiency and its measurement by air-heating.—The efficiency of large turbo-alternators ranges from about 90 to 91 per cent. at half-load to 95 to 96 per cent. at full-load; in the largest sizes and at unity power-factor it may reach 97 per cent. and even higher.

The enclosed construction of the modern turbo-alternator with forced ventilation lends itself to measurement of the losses in the machine by the increase in the temperature of the air passing through it, and Dr. S. F. Barclay and Prof. S. P. Smith have shown[3] that with due precautions the efficiency of the alternator can be determined thereby with very considerable accuracy. Allowing 1½ per cent. of the air-heating losses as an average amount for radiation and convection from the stator casing, the total loss in the machine is

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[1] *Cp. Journ. I.E.E.*, Vol. 57, pp. 306 and 480.

[2] W. J. Foster, E. H. Freiburghouse and M. A. Savage, *Journ. Amer. I.E.E.*, Vol. 43, p. 928. *Cp.* Fig. 12 in Dr. S. F. Barclay's paper above-cited.

[3] "The Determination of the Efficiency of the Turbo-alternator," *Journ. I.E.E.*, Vol. 57, p. 293.

$$\frac{VT_r^\circ}{5{\cdot}97\,(273 + T_o^\circ)} \times \frac{H}{760} \times \frac{100}{98{\cdot}5} \text{ kilowatts}$$

where T_o° is the temperature of the air at the outlet in degrees Centigrade, V is the volume in cub. ft. per min. also at the outlet, and H is the height of the barometer in mm. of mercury. No correction is practically necessary to take into account the effect of humidity on either the weight of a cub. ft. of the air or its specific heat, since the two changes tend to cancel one another.

A length of trunk must be added to the discharge outlet of the machine, sufficient to render the flow of air fairly uniform over the whole of its area, and baffle plates of expanded metal fitted at the base of the trunk still further assist in the same direction.[1] Even then great care must be taken to ensure that the true average velocity is found, and the same also applies to the finding of the true average temperature of the air by thermometers or thermocouples in parallel. The area of the outlet is divided up into a number of small equal squares, and readings of anemometer and thermometer are taken for each under steady conditions of load, to find the average as correctly as possible.

Provided that the same weight of air passes through the machine per minute under all the conditions in view, it is evident that the temperature rise of the air can be calibrated in terms of kilowatt loss without any knowledge of the actual weight or volume of air passing through. Thus when the rotor is driven by a motor, and the stator circuit is open, let the temperature rises of the cooling air be T_u when the rotor is unexcited, and T_e when the rotor is excited, the measured losses in the rotor winding and in the stator core being in the latter case $(P_c + P_i)$ kilowatts. One degree rise of the temperature of the air above that due to the fan and windage then corresponds to

$$\frac{P_i + P_c}{T_e - T_u} \text{ kilowatts}$$

If, therefore, the temperature rise of the air on load is T_l, the load loss in kilowatts is

$$\frac{P_i + P_c}{T_e - T_u} \times T_l$$

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[1] For the instruments available for measuring the velocity of the air and the temperature, and the precautions necessary in their use, see Drs. S. F. Barclay and S. P. Smith's paper, *loc. cit.* and the discussion thereon; and Prof. Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, p. 108.

The above "calibrated air-temperature method" has, however, the defect that the increase in the volume of air due to the greater total heating on load reduces slightly the weight of air that can be passed through for the same speed; consequently the load loss thus calculated is too high, by perhaps 2 or 3 per cent. in an average case.[1]

Another plan is to pass the discharged air through a tunnel or duct wherein an electric heater supplies additional heat at a known rate of H kilowatts;[2] the rate of loss in kilowatts is then $H\frac{T_o - T_i}{T_f - T_o}$, where T_f is the final temperature of the air after passing the heater and T_o is its temperature at the normal outlet of the turbo-alternator.

§ 33. 3300 kW two-pole turbo-alternator.—In illustration of some of the details of design, a small 3-phase turbo-alternator for 3300 kW at 3000 revs. per min. will be taken.

Output. 3300 kW, *i.e.* 4125 kVA at 0·8 power-factor.
6600 volts interlinked pressure, 360 amperes per phase.
3000 revs. per min., 50 cycles per second.

Induced volt-amperes per rev. per min. at full-load

$$= 1\cdot07 \times 4{,}125{,}000 \div 3000 = 1470$$

$B_{g\ max}$ = say, 7560; $\underset{\sim}{B_{g1}} = 1\cdot07\, B_{g\ max} = 8100$.

ac_u = say, 690

$G = 0\cdot72 \times 1\cdot07\, B_{g\ max}\, ac_u \times 10^{-8} = 0\cdot0402$.

$D^2_u L_u = VA/GN = 1470/0\cdot0402 = 36{,}600$ cub. ins.

Taking a rotor diameter of 27 ins., and $l_g = 1$ in.,

D_u at centre of air-gap = 28 ins.

$L_u = 36{,}600/28^2 = 46\cdot6$, say, 46 ins.

Peripheral velocity $= \pi \times (27/12) \times 3000 = 21{,}200$ ft. per min.

E_a per phase at full-load $= 1\cdot07 \times 6600/\sqrt{3} = 4070$

$\frac{Z}{3} = \frac{\sqrt{2}\, E_a \times 60 \times 10^8}{k_{s1}\, B_{g1}\, N\pi D_u L_u \times 6\cdot45} = 57$, say 60, *i.e.* 5 conductors in each of 12 slots per phase.

Slots per pole per phase, 6

$k_{s1} = 0\cdot5/(6 \sin 5°) = 0\cdot957$

Proceeding now in closer detail, it must be recognized that the radial vents in the rotor and stator render the net iron length and the net axial length of the air-gap appreciably less than the over-all length of core. The effective axial length of the air-gap will thus be, say, 97 per cent. of $L_u = 44\cdot62$ ins., and the diameter of the stator bore is 29 ins. = $1{\cdot}037\,D_u$. The amplitude of the fundamental of the flux-density over the effective axial length *at the stator bore* must then be

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[1] Drs. S. F. Barclay and S. P. Smith, *loc. cit.*, p. 299.

[2] See W. F. Dawson, *General Elect. Review*, Vol. 23, p. 153.

$$B_{g1} = \frac{\sqrt{2}\,E_a \times 10^8}{0{\cdot}957 \times 50 \times \pi \times 29 \times 44{\cdot}62 \times 6{\cdot}45 \times 60} = 7655$$

At the centre of the air-gap, this becomes 7950, and to allow for a small component of transverse leakage, say, 7955.

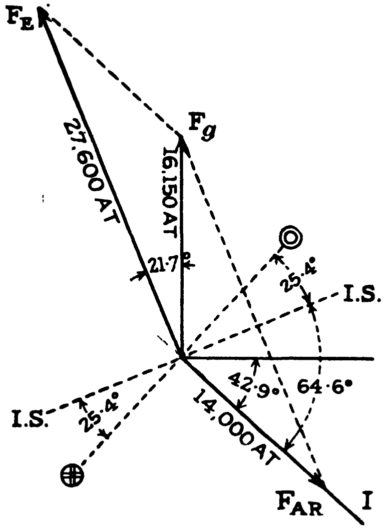

FIG. 641.—Vector diagram for provisional estimate of air-gap *AT* at full-load.

$$F_g = 0{\cdot}8\,B_{g1}\,Kl_g = 0{\cdot}8 \times 1{\cdot}002 \times 2{\cdot}54 \times B_{g1} = 2{\cdot}032\,B_{g1}$$
$$= 2{\cdot}032 \times 7955 = 16{,}150 \checkmark$$

$$k_A = \frac{2\sqrt{2}}{\pi} \times 0{\cdot}957 = 0{\cdot}863$$

$$F_{AR} = k_A\,JZ/4p = 0{\cdot}863 \times 360 \times 180/4 = 14{,}000$$

By Fig. 522, $\sin \phi_i$ = say, 0·68 ✓

$$F_E = \sqrt{16{,}150^2 + 14{,}000^2 + 2 \times 16{,}150 \times 14{,}000 \times 0{\cdot}68} = 27{,}600$$

(Fig. 641).

By equation (241)

$$AT_E = 27{,}600\,\frac{\frac{2}{3} \times \pi^2}{8 \sin 60^\circ} = 27{,}600 \times \frac{6{\cdot}56}{6{\cdot}928} = 26{,}200$$

Allowing, say, 3000 *AT* for rotor core, 2400 *AT* for rotor pole-centre, and 400 *AT* for stator core and teeth, total *AT* per pole at full-load = 32,000.

With certain minor deviations, the slot and end-bell dimensions given in Part III[1] of "The Flux-wave of the Turbo-alternator" will be followed for purposes of comparison, this Paper being frequently referred to later as explaining certain of the processes of design here employed.

Stator slots, 36 as Fig. 644(*a*).

Concentric coils, divided, with end-connexions arranged in 3 planes, as Fig. 643.

Ampere-conductors per slot, 360 × 5 = 1800

Copper conductors, 11/16 in. wide × 0·23 in. thick, with rounded corners.

Area, 0·155 sq. in. ω' at 20° C. = 0·02445 ÷ 0·155 = 0·158 ohm.

Current-density, 360/0·155 = 2320

$$mh = 2{\cdot}42\sqrt{\frac{n_a t}{w_s} \times \frac{f}{234{\cdot}5 + t^\circ}} \times h = 2{\cdot}42\sqrt{\frac{1{\cdot}745 \times 50}{2{\cdot}54 \times 314{\cdot}5}} \times 0{\cdot}585$$

$$= 2{\cdot}42 \times 0{\cdot}33 \times 0{\cdot}585 = 0{\cdot}467$$

By Fig. 560, $K = 1{\cdot}165$.

,, ,, 559, for the 5th layer, $K = 1{\cdot}4$.

Width of stator slot, 1 in.

Micanite wrapping round joint coil,	0·115 × 2	= 0·23	in.
Outside tape wrapping	0·015 × 2	= 0·03	,,
Mica tape round conductor,	0·015 × 2	= 0·03	,,
Slack		= 0·0225	,,
Copper width, 11/16 in.		= 0·6875	,,
		1·0	,,

Depth of stator slot, 2 ins.

Wedge, 1/4 in. + lip, 1/8 in.		= 0·375	in.
Micanite round joint coil,	0·115 × 2	= 0·23	,,
Outside tape wrapping,	0·015 × 3	= 0·045	,,
Mica tape round 5 conductors,	0·015 × 10	= 0·15	,,
Four mica separating strips,	0·01 × 4	= 0·04	,,
Slack		= 0·01	,,
		0·85	,,

Thickness of copper, (2·00 − 0·85)/5 = 0·23.

Mean length of a half-turn = 46 + 82 = 128 ins. = 3·56 yd.

Length of a phase, 3·56 × 60 = 213·6 yds.

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[1] *Papers on the Design of Alternating-current Machinery*, by Hawkins, Smith, and Neville. (Pitman & Sons.)

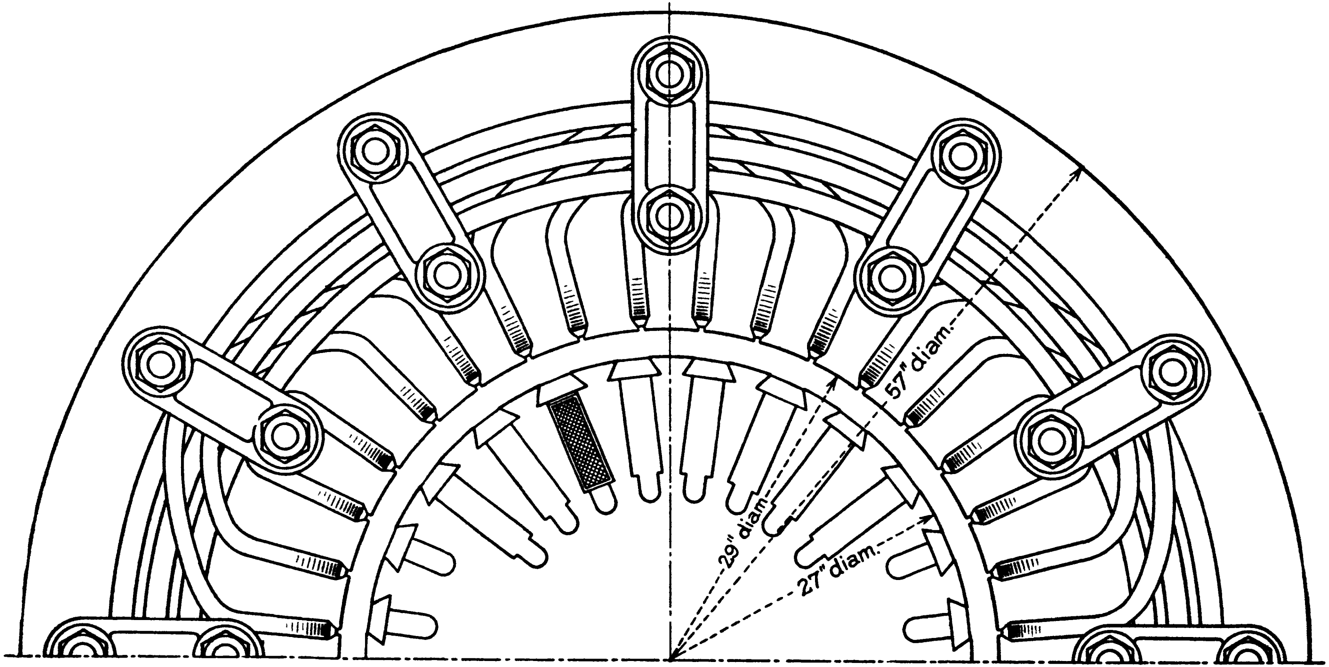

FIG. 642 —Half end-view of 3300 kW turbo-alternator.

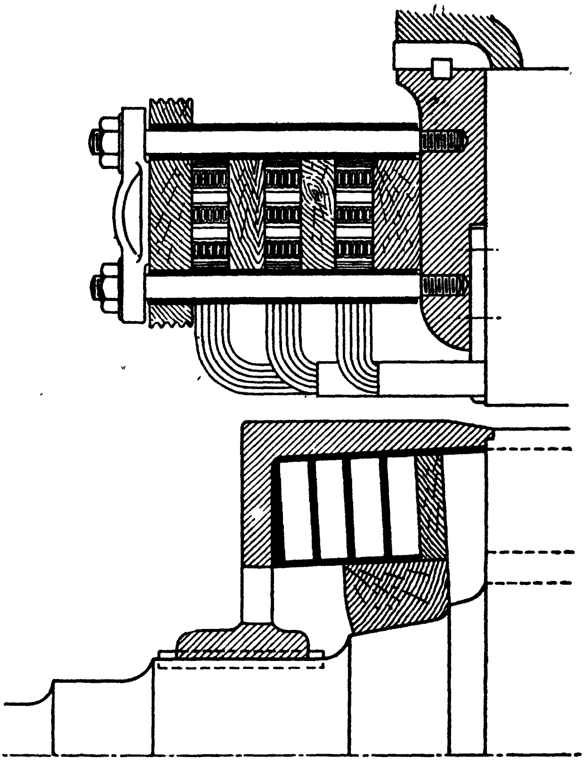

FIG. 643 —Sketch of section of rotor and stator end-windings.

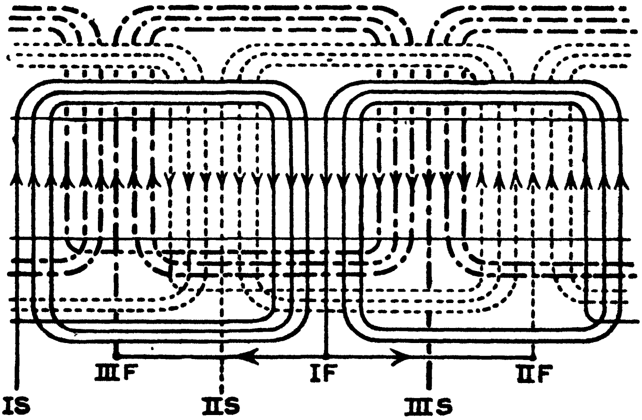

FIG. 643a.—Stator winding.

Resistance of a phase at 20° C. = 0·158 × 213·6/1000 = 0·0338 ohm.

Res. of a phase at 80° C. = 0·0338 × (234·5 + 80)/(234·5 + 20) = 0·0418 ohm.

With allowance for eddies, 0·015 × 1·165 + 0·0268 = 0·0443

Loss of volts per phase = 360 × 0·0443 = 15·95

I^2r_a' for 3 phases, 15·95 × 360 × 3 = 17,200.

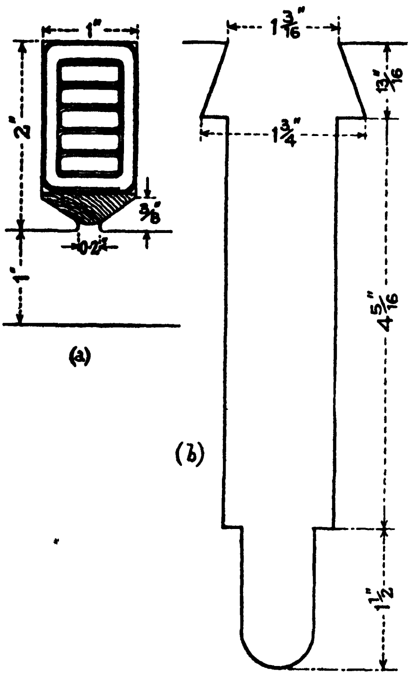

FIG. 644.—Stator and rotor slots.

Rotor slots, 16 as Fig. 644(*b*), pitched as for 24.

Depth below wedge, 4 5/16 ins. Width, 1 3/16 ins.

Brass liner at top of slot		= 0·0425 ins.
Slot insulation doubled over at top,	2 × 0·0625	= 0·125 ,,
36 layers of copper, 0·1 in. thick,	36 × 0·1	= 3·6 ,,
Mica insulation between layers,	35 × 0·01	= 0·35 ,,
Slot insulation at bottom of slot		= 0·0625 ,,
Brass liner at bottom of slot		= 0·0425 ,,
Slack		= 0·09 ,,
		4·3125 ,,

Width of rotor slot, = 1·1875 ins.
Insulation on sides, 2 × 0·0625 = 0·125 „

Copper width = 1·0625 „

Area of copper, $1\frac{1}{16} \times 0{\cdot}1$ = say, 0·106 sq. in.

ω' at 20° C. = 0·02445/0·106 = 0·23 ohm.

Mean length of a half-turn, 46 + 5·5 + 22 + 5·5 = 79 ins. = 2·19 yds.

Total length, 2·19 × 36 × 16 = 1265 yds.

Resistance at 20° C. = 0·23 × 1265/1000 = 0·291 ohm.
at 100° C. = 0·291 × (234·5 + 100)/(234·5 + 20) = 0·383.

Turns per pole = 36 × 4 = 144.

Exciting current, 32,000/144 = 222 amperes.

Slip-ring pressure at full-load, 85 volts.

Ohmic loss in rotor, 18,850 watts.

Current density, 2100 amperes per sq. in.

Stator core, external diameter, 57 ins.

Total radial depth of iron, ½ (57 − 29) = 14 ins.

Depth of iron behind slots, 14 − 2 = 12 ins. ✓

Stator radial vents, 25 each 3/8 in. wide, total 9·36 ins.

Net iron length of stator core, 0·88 (46 − 9·36) = 32 ins. ✓

Net single cross-section of stator iron = 12 × 32 sq. ins. = 2480 sq. cm.

Mean length of path of flux per rotor tooth-pitch referred to the stator core $= \frac{\pi}{24}$ (57 − 12) ins. = 14·96 cm.

Total fundamental stator flux, $7655 \times \frac{2}{\pi} \times \frac{\pi D_a L_a}{2} \times 6{\cdot}45 =$ 7655 × 29 × 44·62 × 6·45 = 63,900,000

Max . B_c = 31,950,000 ÷ 2480 = 12,880 *at* = 7·75

Rotor, solid steel forging.

Radial ducts, 23 each 7/16 in. wide, total width 10·06 ins.

Net iron length of rotor core, 46 − 10·06 = 35·94 ins. = 91 cm.

The air-gap.

$$\frac{\text{Average vent width}}{\text{air-gap}} \text{ for both stator and rotor} = \frac{0{\cdot}406}{1} = 0{\cdot}406$$

By Fig. 236 (Vol. I), K_3 = say, 0·076

Adding both rotor and stator vent openings together,

$$0{\cdot}076 \times (9{\cdot}36 + 10{\cdot}06) = 1{\cdot}475$$

Effective axial width of air-gap, 46 − 1·475 = 44·525 ins.

Total effective gap area at centre of gap $= \pi \times 28 \times 44{\cdot}525 \times 6{\cdot}45$ = 25,250 sq. cm.

Per rotor tooth at centre of gap 25,250/24 = 1050 sq. cm.

Per rotor tooth at stator bore, 1050 × 29/28 = 1088 sq. cm.

Stator slot-pitch at stator bore, $\pi \times 29 \div 36 = 2{\cdot}53$ ins.

$$w_3/l_g = 0{\cdot}2/1 = 0{\cdot}2\,. \qquad k = 0{\cdot}03$$

$$K = \frac{2{\cdot}53}{2{\cdot}53 - 0{\cdot}03 \times 0{\cdot}2} = \frac{2{\cdot}53}{2{\cdot}524} \text{ say } 1{\cdot}002$$

Therefore, for the pole-centre where the wedges are magnetic (except for tooth-divisions 4 and 8, Fig. 646)

$$AT_g = 0{\cdot}8 \times 1{\cdot}002 \times 2{\cdot}54\, B_g = 2{\cdot}032\, B_g$$

$$= \frac{2{\cdot}032}{1050} \times \text{flux of a tooth-division}$$

$$= 1{\cdot}93 \times \text{flux of a tooth-division in kilolines.}$$

Rotor slot-pitch at rotor face $= \pi \times 27 \div 24 = 3{\cdot}53$ ins.

$$w_3/l_g = 1{\cdot}1875/1 = 1{\cdot}1875 \quad k = 0{\cdot}175$$

$$K = \frac{3{\cdot}53}{3{\cdot}53 - 0{\cdot}175 \times 1{\cdot}1875} = \frac{3{\cdot}53}{3{\cdot}322} = 1{\cdot}06$$

For the doubly slotted portion,[1] therefore, over the rotor teeth, the effect of the stator slotting is very small, and $K = 1{\cdot}06$.

$AT_g = 0{\cdot}8 \times 1{\cdot}06 \times 2{\cdot}54\, B_g = 2{\cdot}158\, B_g = \frac{2{\cdot}158}{1050} \times$ flux of a tooth-division

$$= 2{\cdot}05 \times \text{flux of a tooth-division in kilolines.}$$

For tooth-divisions 4 and 8, one at each edge of the pole-centre, a value rather more than the mean of the above, *i.e.* $\frac{1}{2}(1{\cdot}93 + 2{\cdot}05)$, or say 1·98 × flux of a tooth-division may be used, since the rotor tooth-pitch is then virtually slotted on one side only.

---

[1] For an air-gap slotted on either side, if k_1 and k_2 are the extension coefficients for the whole air-gap length for stator and rotor respectively with the opposite face assumed to be unslotted, Mr. F. W. Carter (*Electr.*, Vol. 81, p. 400), gave approximately $K = k_1 k_2$ (rather too high) or alternatively $k_1 + k_2 - 1$. The mean of the two expressions was given in eq. (112). A more complex but more accurate formula has been developed by T. Lehmann, *Rev. Gén. d'Elect.*, Vol. 13, p. 165.

§ 34. The full-load flux curve.—Of chief importance is a preliminary determination of the flux-densities of the pole-centre[1] covering tooth-divisions 4–8.

A provisional calculation shows that the ampere-turns required to pass half of the total stator flux through the single section of the stator core from pole-centre to the plane of zero potential will average for the two sides[2] about 290 AT, since $at \times 2\frac{1}{2}$ rotor tooth-pitches referred to the stator core $= 7{\cdot}75 \times 2{\cdot}5 \times 14{\cdot}96 = 290$.

The point at which the flux bifurcates in the stator core will not be opposite to the middle of tooth-division 6 owing to the distortion of the flux curve, but to one side of it, say, opposite tooth-division 7 if the flux is displaced towards the right and the plane of maximum rotor flux is twisted clockwise, as in Fig. 646. From the bifurcation point the potential of the stator core decreases in either direction, and is written down provisionally (row 1, Table XXXVIII). The ampere-turns AT_{t1} expended over the stator teeth (row 2) do not play any very important part except towards the trailing edge where the saturation is highest; the flux per stator tooth is 24/36 = two-thirds of the flux reaching the stator bore from a rotor tooth-division, and the ampere-turns are obtained from an excitation curve for the teeth of the stator.[3]

The position of the fundamental of the armature reaction or M.M.F. wave of the stator ampere-turns is obtainable from Fig. 519 or Fig. 641; by solution of the triangle, the angle between F_E and F_g is found to be 21·7°, and ϕ_t has been assumed to be 42·9° in view of the given cos ϕ_r. But the angle between F_E and F_g is the same as the angle between IS and E_t; therefore, the angle ϕ_0 between IS and I is $42{\cdot}9 + 21{\cdot}7 = 64{\cdot}6°$. Now the current vector I has been identified in position with the axis of the drum coil of a phase when carrying its maximum current. Consequently the sides of the drum coil of full-pitch are situated at positions $90° - 64{\cdot}6° = 25{\cdot}4°$ on either side of the horizontal diameter dividing the exciting coil into two halves. Reckoning, therefore, the angular distance x from the horizontal diameter, the expression for the ampere-turns of the stator M.M.F., AT_s, is 14,000 sin $(25{\cdot}4° + x)$. Row (3) can then be written down.

Adding (1) and (2) for tooth 7 to AT_E as already calculated, an estimate of AT_P, the potential of the rotor pole-centre face, is reached, *i.e.* $350 + 26{,}200 = 26{,}550$. The method employed for reducing F_E to AT_E yields an underestimate for full-load, so that for AT_P will be assumed 26,720 (row 6). Deducting the sum of (1), (2), and (3) from the value adopted for AT_P, the difference is the available AT_g (row 5), whence the mean air-gap flux per rotor tooth-division is $AT_g/1{\cdot}93$ (or $AT_g/1{\cdot}98$ for tooth-divisions 4 and 8), as given in row 7.

---

[1] See " The Flux Wave-form of the Turbo-alternator ; Part II, Full Load," in *Papers on the Design of Alternating-current Machinery*, pp. 202–219, and especially § 25, p. 216.

[2] The corrected values are here for simplicity inserted at once.

[3] Such as Fig. 33 of Part III of the above-quoted paper.

To continue the process, allowance must be made for the transverse leakage in the air over the pole-centre, by reason of which the flux entering the stator teeth is less than that leaving the rotor tooth-divisions; this is but small until we pass to the outer edges of the pole-centre, where the difference between the potential of the rotor pole-centre and that of the adjacent tooth reinforces the difference between the potentials of the stator tooth-divisions, and is highest at the trailing edge.

The transverse permeance of a rotor tooth-pitch in the air-gap, including the constant $4\pi/10$, is

$$\frac{1}{3{\cdot}66} \times 44{\cdot}52 \times 2{\cdot}54 \times \frac{4\pi}{10} = 38{\cdot}9$$

a rotor tooth-pitch at the centre of the gap being $\pi \times 28/24 = 3{\cdot}66$ ins. This permeance is acted upon by a mean potential difference amounting to about 2800 AT at one edge of the pole-centre

TABLE XXXVIII

POLE-CENTRE FLUXES

Tooth-division	4	5	6	7	8
(1) Potential of stator core in AT	234	266	284	290	285
(2) AT_{t1}	20	22	33	60	140
(3) AT_s from stator armature reaction	13,950	13,750	12,620	10,650	7960
(4) Potential of stator face	14,204	14,038	12,937	11,000	8385
(5) AT_g	12,516	12,682	13,783	15,720	18,335
(6) Potential of rotor pole-centre face, AT_P			26,720		
(7) Mean air-gap flux in kilolines $= AT_g/1{\cdot}93$ or $1{\cdot}98$	6335	6573	7145	8150	9260
(8) ½ of flux leaking transversely in air-gap	58	9	8	7	83
(9) Flux issuing from pole-centre in kilolines	6393	6582	7153	8157	9343
(10) Total flux from pole-centre in kilolines	—	—	37,628	—	—
(11) Flux entering stator in kilolines	6278	6564	7137	8143	9178
(12) Total stator flux from pole-centre	—	—	37,300	—	—
(13) B_{gx} at stator face = (11) ÷ 1088	5770	6040	6555	7490	8445

and to about 5500 AT at the other edge, yielding fluxes abstracted from each tooth as shown in Table XLI. Half of the transverse flux must be added to the mean flux to give the flux issuing from

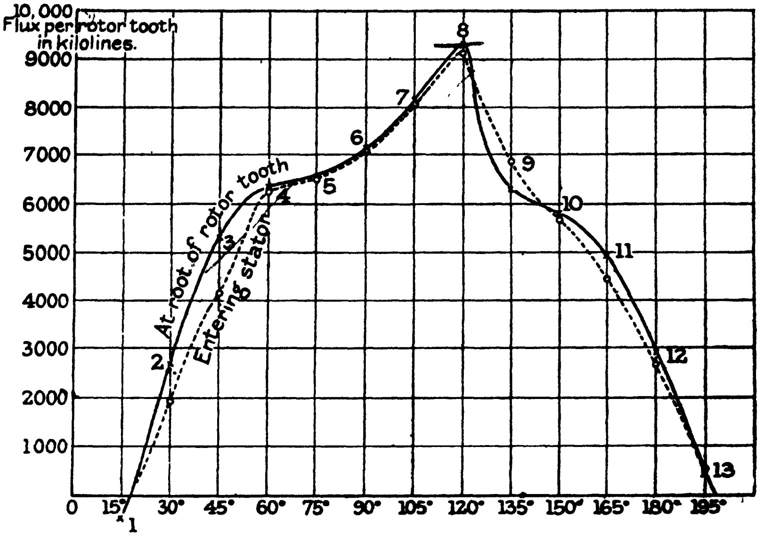

FIG 645 (a) Rotor tooth fluxes.

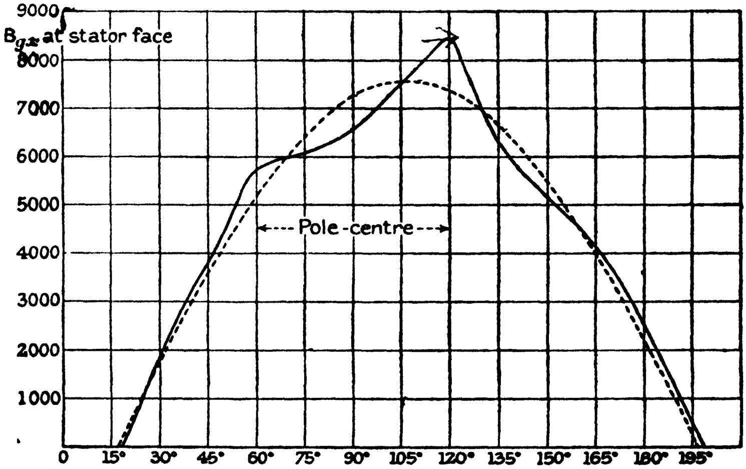

FIG. 645 (b) Flux-density at stator face.

the pole-centre into the stator (rows 9 and 10) and subtracted from the mean flux to give the flux entering the stator from the pole-centre (rows 11 and 12). Adding the end-bell leakage from both sides of the pole-centre (estimated as 5700 kilolines) and the fluxes passing transversely across slots 3–4, and 8–9 from the pole-centre into teeth 3 and 9 (estimated as 4479 kilolines), the total flux passing the root of the pole-centre is

$$37{,}628 + 5700 + 4479 = 47{,}807 \text{ kilolines.}$$

If the ampere-turns of stator and rotor were evenly spread out over the peripheries of the stator and rotor faces, a balance between them would be reached when

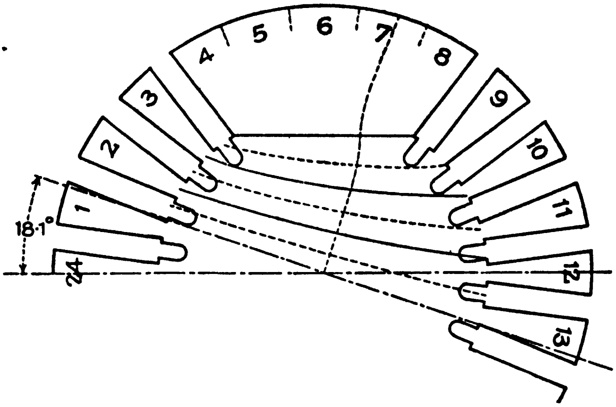

FIG. 646.—Equipotential surfaces in rotor core under full-load

$$8000\,\frac{x^\circ}{15^\circ} = 14{,}000 \sin\,(25{\cdot}4^\circ + x^\circ)$$

$$x^\circ = 18{\cdot}1^\circ, \text{ i.e. more than a tooth-pitch.}$$

By this amount, therefore, approximately the plane of the maximum rotor flux is twisted round aslant to the horizontal plane bisecting the rotor exciting coil (Fig. 646). The magnetic potential of the faces of tooth 1 and tooth 13 farthest from the horizontal plane thus becomes practically zero.

Drawing in the sides of the flux curve to meet the plotted curve of pole-centre fluxes after the fashion of Fig. 645(*a*), and adding end-bell leakages for each tooth, the flux at the root of each rotor tooth can be estimated. This will be higher than the flux leaving the tip to enter stator and end-bell, except in the case of teeth 9 and 10, which are highly saturated and receive flux transversely from the pole-centre.

The AT expended over the rotor core.—Drawing in the equipotential surfaces as assumed in Fig. 646, and the three median dotted lines between them, the average sections of the rotor core along the dotted lines with allowance for air-ducts are calculated approximately as follows—

Section of rotor core.	*Mean length of path.*
24·5 × 117 = 2870 sq. cm.	7·1 cm.
31·2 × 118 = 3680 ,,	5·8 ,,
38·3 × 120 = 4600 ,,	5·3 ,,

and on the zero plane of maximum flux

38·6 × 122 = 4700 sq. cm.	—

The *AT* expended over the rotor core can then be estimated as follows in Table XXXIX.

An accurate calculation of the ampere-turns expended over the rotor teeth can now be made, after determining the *transverse slot leakage as due to the ampere-conductors in each rotor slot* if the iron had zero reluctance.

Dividing up the rotor tooth into the same sections 1–5 as in § 35, Part III of the above-quoted paper,[1] the permeances of the several

TABLE XXXIX

ROTOR FLUXES

	Flux at root in kilolines. Area in sq. cm.	B_{c2}	at per cm × length of path. AT per section.	Sum.
Pole-centre . . .	47,807			
Tooth 9 . . .	6,300			
	54,107 ÷ 2870	18,850	222 × 7·1 = 1575	2740
Tooth 10 . . .	5800			
⅖ths tooth 11 . .	1968			
Tooth 3 . . .	5288			
	67,163 ÷ 3680	18,300	160 × 5·8 = 927	1165
⅗ths tooth 11 . .	2952			
Tooth 12 . . .	2910			
Tooth 2 . . .	2630			
	75,655 ÷ 4600	16,500	45 × 5·3 = 238	238
Tooth 13 . . .	413			
	76,068 ÷ 4700	16,200	35	

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[1] *Papers on the Design of Alternating-current Machinery*, p. 242.

slot sections, including the constant $4\pi/10$, are

$$\text{Top section}\left\{\begin{array}{llr} 1\text{–}2 & \frac{2}{3\cdot5}\times 91\times\frac{4\pi}{10} = & 65\cdot5 \\ 2\text{–}3 & \frac{6\cdot5}{3\cdot02}\times 91\times\frac{4\pi}{10} = & 246\cdot5 \end{array}\right\} 312$$

$$\text{Middle section, } 3\text{–}4 \quad \frac{3}{3\cdot02}\times 91\times\frac{4\pi}{10} = \qquad 114$$

$$\text{Bottom section, } 4\text{–}5 \quad \frac{1\cdot5}{3\cdot02}\times 91\times\frac{4\pi}{10} = \qquad 57$$

The *at* per cm. at the plane of maximum rotor flux has been found to be about 35. Thence it increases, so that for the fall of potential across the bottom of a slot we may roughly assume $40 \times 3 = 120$ ampere-turns. Analogously to § 43 of Part III (p. 249, *loc. cit.*), we then have for the transverse flux due simply to the ampere-conductors in a slot—

Section.

$$\left.\begin{array}{llr} 1\text{–}2 & 8000-120 \qquad\qquad \times\ 65 = & 512\times10^3 \\ 2\text{–}3 & (8000\times\frac{7\cdot75}{11}-120)\times 247 = & 1365\times10^3 \end{array}\right\} = 1877\times10^3$$

$$3\text{–}4 \quad (8000\times\frac{3\cdot0}{11}-120)\times 114 = \qquad 235\times10^3$$

$$4\text{–}5 \quad (8000\times\frac{0\cdot75}{11}-120)\times 57 = \qquad 24\times10^3$$

$$\overline{2136\times10^3}$$

This forms as it were a basic background of transverse slot flux in the several compartments, upon which are superposed local variations due to the different degrees of saturation in the rotor teeth.

§ 35. The AT expended over the rotor teeth.—Table XL (although arranged in a slightly different manner) is then exactly analogous to Table XXI of the above-quoted Part III, and like that Table is worked upwards, the fluxes at the bottom of the rotor teeth in the lowest row being in the first instance assumed. An arrow pointing inwards to a tooth indicates the transverse component flux that would cross the slot due to the fall of potential in the section of the rotor tooth if the opposite wall of the slot were at zero potential; when a pair of arrows pointing in the same direction occur, the higher indicates the component due to the *AT* of the section or sections below, *i.e.* in section 4–3 the *AT* of section 5–4 are multiplied by the permeance of section 4–3, viz. 114, and in section 3–1 the *AT* of sections 5–4 and 4–3 are multiplied by the permeance of section 3–1, viz., 312. A higher row shows the basic transverse flux above calculated, and the algebraic summation of all the component fluxes in a compartment of the slot gives the actual transverse flux. A deduction of the smaller from the greater flux in the slots on either side of a tooth will then give the flux added to or subtracted from

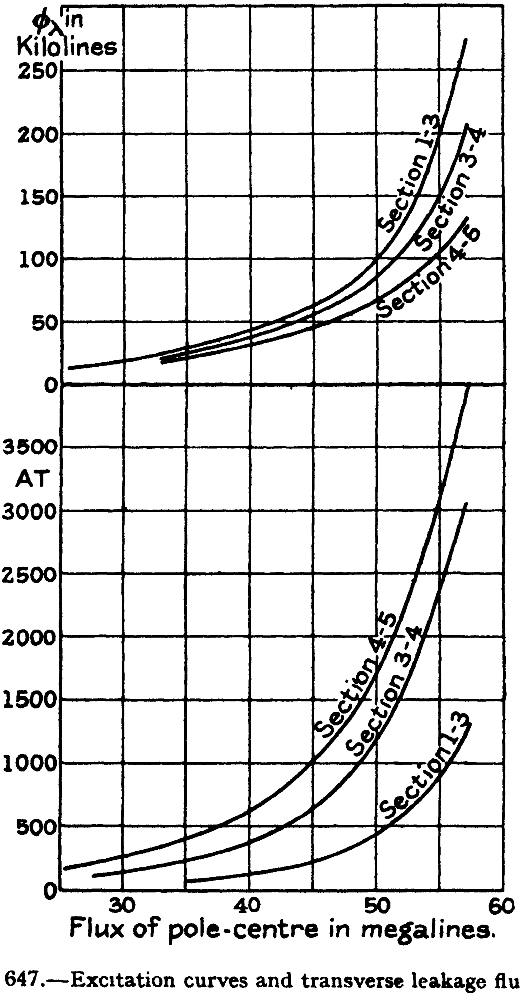

FIG. 647.—Excitation curves and transverse leakage fluxes for pole-centre.

that which enters the lower end of the section of tooth in question. The ampere-turns expended over the pole-centre and rotor teeth and the transverse fluxes corresponding to the ampere-turns over a section for various values of the flux entering the lower end of a section are plotted in Figs. 647–8; for the calculation of such curves, see § 37, Part III, of the above-mentioned paper.

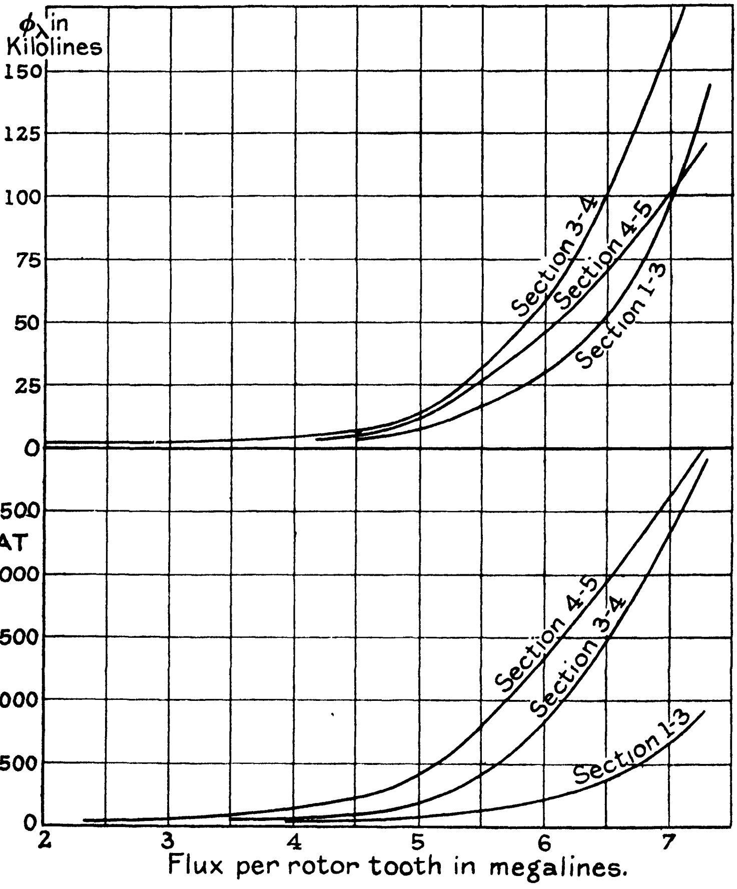

FIG. 648—Excitation curves and transverse leakage fluxes for rotor teeth.

It will be seen that in the top section of slot 8–9 and tooth 9 two fluxes are given. Owing to the high saturation of tooth 9 under the joint action of rotor and stator ampere-turns it will be advisable to employ in slot 8–9 a composite wedge, partly magnetic, or a cast-iron wedge so calculated as to allow about 600 to 700 additional kilolines to pass from the pole-centre into the top of tooth 9. The figure 633 in Table XL represents the additional flux that is thus assumed to be shunted through the by-pass of the wedge into tooth 9.

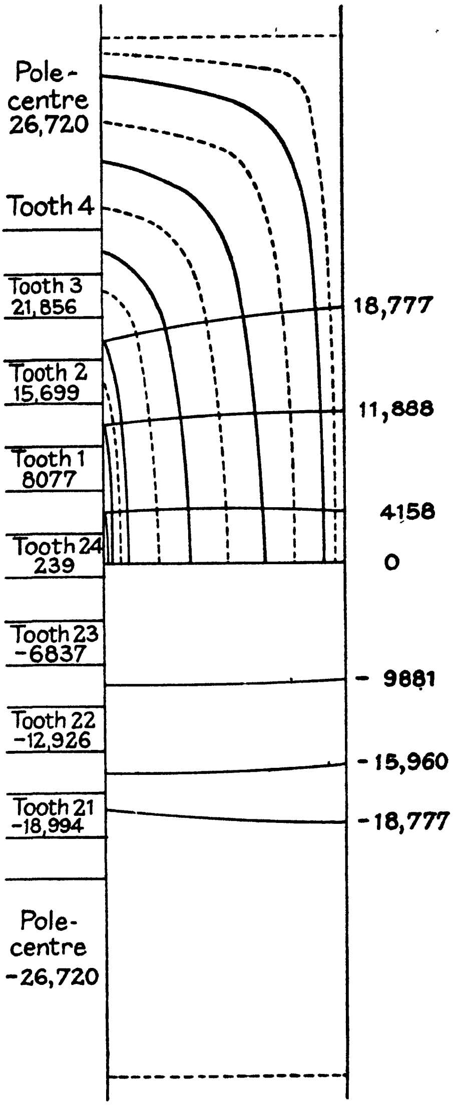

FIG. 649.—The leakage paths and equipotential traces of one end-bell.

§ 36. The transverse fluxes in the air-gap can only be roughly approximated at the outset, and are not of great magnitude. But for the sake of completeness, Table XLI is added to show the nature of the additions to or subtractions from the fluxes issuing from the rotor teeth and pole-centre into the air-gap. The potential differences between the centres of adjacent teeth being calculated at both the stator and rotor faces, the average of the two is taken as acting on the air-gap permeance of a rotor tooth-pitch, viz, 38·9. From the location of the maximum stator ampere-turns, the direction of the potential difference at the stator face changes at tooth 4; consequently this tooth has to supply the fluxes of 112 kilolines to the left and 3 kilolines to the right, or 115 in all. The transverse flux reaches its maximum across the tooth-pitch 12–13; but the rotor flux does not change its direction until after tooth 13. Some portion of the transverse flux thus passes radially into the air-gap over tooth 13, and is shown as additive to the flux issuing from that tooth.

§ 37. End-bell leakage.—Over the tooth-pitch from the centre of tooth 24 to the centre of tooth 23, a potential difference of 239 + 6837 = 7076 *AT* acts. A tooth-pitch at the level of the middle of the end-bell lip measures 8·75 cm. The *at* per cm. is, therefore, 7076/8·75 = 810, corresponding to a density of 21,500. The area of the cross-section of the bell is 93·3 sq. cm., giving a total flux per single cross-section of 21,500 × 93·3 = 2,006,000. Within this space there is a little increase and decrease as lines enter from tooth 24 and emerge into tooth 23. Under the joint action of both the rotor and stator *AT* in combination, the P.D. over a tooth-pitch may exceed or be less than the above-mentioned amount of 7076, and these divergencies are taken into account by the equipotential lines starting from slot-centres opening out or drawing together as

TABLE XLI

TRANSVERSE FLUXES IN AIR-GAP

					Pole-centre.								
Tooth Number . .	2	3	4	5	6	7	8	9	10	11	12	13	14
Potential of stator face in *AT* . .	11,619	13,386	14,204	14,038	12,937	11,000	8385	4994	1371	- 2333	- 587	- 9053	- 11,619
,, rotor ,, . .	15,699	21,856	26,720	26,720	26,720	26,720	26,720	18,994	12,926	6837	- 239	- 8077	- 15,699
P.D.'s between rotor tooth-centres at stator face	2566	1767	818	166	1101	1937	2615	3391	3623	3704	3546	3174	2566
P.D.'s between rotor tooth-centres at rotor face	7622	6157	4864	0	0	0	0	7726	6068	6089	7076	7838	7622
Mean P.D.'s	5094	3962	2841	83	550	968	1307	5558	4845	4896	5311	5506	5094
Transverse flux in kilolines = P.D. × 0·0389 . . .	198	154	112	3	21	37	51	216	188	190	206	214	198
Flux abstracted from or added to flux at rotor face per tooth . .	- 44	- 42	- 115	- 18	- 16	- 14	- 165	+ 28	- 2	- 16	- 8	+ 16	
					328								

they approach the outer edge (Fig. 649). Consequently by making the density rather greater at the outer edge, a total leakage flux of 2,010,000 per single cross-section may be assumed, divided approximately as follows—

KILOLINES.

Part of Tooth	6	275	295	Part of Tooth	6
Tooth	5	570	570	Tooth	7
,,	4	570	570	,,	8
,,	3	425	430	,,	9
,,	2	140	110	,,	10
,,	1	25	35	,,	11
,,	24	5			
		2010	2010		

For the same end-bell at no-load under a total excitation of 32,000 *AT*, the total leakage was 1970 kilolines

Since the leakage flux bifurcates at the pole-centre, the total from a pole-pitch on one side is 4,020,000, and on both sides is 8,040,000. But the whole of this is not lost A peculiarity of full load is that the zero line of maximum flux in the end-bell where the potential of the rotor tooth-tips changes sign is not in line with the plane where the flux through the air-gap into the stator changes its direction and sign. Consequently teeth 1 and 24 or 12 and 13 are peculiarly situated. Some of the flux passing up tooth 11, and entering as leakage into the end-bell from the tip of tooth 11, emerges into the tips of teeth 12 and 13 to the extent of (5 + 25) kilolines; these cannot, however, pass down teeth 12 and 13, but pass into the air-gap through the stator core and so complete their path into the rotor teeth again and through the rotor core. They are, therefore, shown in Table XL as + 10 and + 50 for both sides for teeth 12 and 13, being additive to the flux from them.

The balance sheet for the fluxes is thus—

Total flux entering a pole-pitch of stator = 63,544 kilolines
,, ,, passing through a diameter of rotor core = 76,068 ,,

the latter being made up as follows—

Flux crossing slots 1–2 and 13–14 (1856 + 228 + 20) × 2 = 4,208 ,,
,, ,, air-gap between tooth-pitches 1, 2 and 13, 14, 198 × 2 = 396 ,,
End-bell leakage, 4020 × 2, less leakage 10 + 50 which appears twice in the flux of stator, i e. 8040 - 120 = 7920 ,,
Stator flux = 63,544 ,,
76,068 ,,

The general scheme of the leakage is indicated in Fig 649; although not divided with complete accuracy, a fair approximation is obtained by the method described in § 18 of Part I of the Paper above cited and shown in Tables XLII and XLIII.

§ 38. The fluxes and excitations on full-load.—The completed results are given in Table XLIV, which repeats Table XXXVIII for the pole-centre, and also gives the calculation of the stator core *AT*. For each section of the core between the centres of adjacent rotor teeth the addition of the two halves of row 3 gives the flux added within the section (row 4). The flux opposite the centre of each tooth is thus obtained (row 5), followed by the density and corresponding *at* per cm. (rows 6 and 7). The mean of the *at* for two adjacent teeth is then regarded as holding over the

TABLE XLIII

END-BELL LEAKAGE UNDER FULL-LOAD

	Flux of one end.	P.D. in AT.	Lengths of path, outer and inner	$\frac{AT}{\text{cm.}}$	Density.	Mean density.	Areas in cm.²	
Tooth-pitch 2	1,840,000	6889	— 10·7	644	20,950	21,200	86·84	93·3
			÷ 8·8	783	21,450			
	140,000	3811	÷ 4·4	866	21,650		6·46	
Tooth-pitch 1	1,980,000	7730	÷ 10·4	745	21,260	21,480	92·15	93·3
			÷ 8·75	883	21,700			
	25,000	3919	— 4·375	895	21,750		1·15	
Tooth-pitch 24	2,005,000	4158	— 4·63	900	21,760	21,580	93·067	93·3
			÷ 5·4	770	21,400			
	5,000	239	÷ 0·3	797	21,450		0·233	
	2,010,000	Max. flux at		zero	line	21,600		
Tooth-pitch 23	35,000	6837	— 8·45	811	21,500		1·625	93·3
	1,975,000	9881	÷ 12·1	815	21,510	21,580	91·675	
			— 11·76	840	21,650			
Tooth-pitch 22	110,000	3045	÷ 4·375	696	21,150		5·2	93·3
	1,865,000	6079	÷ 8·7	700	21,150	21,200	88·1	
			÷ 8·33	730	21,250			

tooth-pitch between them. Summing up the AT from tooth 7 in either direction, two different values are obtained; but the numerical value of the potential of the core at each end must be alike, although of different sign. The mean of the two values is, therefore, the potential in AT opposite to tooth 7, and thence by progressive subtractions the values $+69$ and -69 at the two ends of the pole-pitch (row 10) are reached.

After division of the stator fluxes for each tooth by the area of a tooth-pitch at the stator face (1088 sq. cm.), the densities so obtained are plotted in Fig. 645(*b*). The resulting fundamental flux-density is 7545 sin $(-16\ 45° + \theta)$, showing a displacement of 16·45°. The flux-density is less than the preliminary estimate called for, but the next section shows that this was rather an over-estimate, the reactance not being so great as assumed in the value for sin ϕ_1.

§ 39. The reactance of a stator phase.[1]—By equation (254)

$$\lambda_s = \frac{12\cdot57}{6}\left(\frac{1\cdot31}{3\times 1}+\frac{0\cdot17}{1}+\frac{2\times 0\cdot25}{1+0\cdot2}+\frac{0\cdot125}{0\cdot2}+0\cdot2\right) = 3\cdot87$$

$$\lambda_s l = 3\cdot87 \times 36\cdot44'' \times 2\cdot54 \qquad = 359$$

By equation (255a), $\lambda_c = 5 + \frac{9\cdot2}{36}\log\frac{2\cdot53}{0\cdot4} = 5\cdot2$

$$\lambda_c L = 5\cdot2 \times 46'' \times 2\cdot54 \qquad = 608$$

By equation (256), $\lambda_p = 4\cdot6\log\frac{4\cdot04\times 31''}{1\cdot31''+14\cdot5''} = 4\cdot15$

$$\lambda_p\,(l_1 - l) = 4\cdot15 \times 25\cdot56'' \times 2\cdot54 \qquad = 269$$

By p. 201 and eq. (257), $\lambda_e = 1\cdot3\times 2\cdot3\log\frac{1\cdot57\times 16''}{1\cdot31''+3\cdot3''} = 2\cdot21$

$$\lambda_e l_e = 2\cdot21 \times 58'' \times 2\cdot54 \qquad = 326$$

$$\overline{1562}$$

By equation (253)

$x_a = \pi \times 50 \times 60^2 \times 1562 \times 10^{-9} = 0\cdot885$

$Ix_a = 360 \times 0\cdot885 = 318$ volts

$E_a = \sqrt{(0\cdot6 \times 3810 + 318)^2 + (0\cdot8 \times 3810 + 16)^2} = 4010$

B_{g1} at stator face $= 7545$.

§ 40. Losses and efficiency.—Mean diameter of stator core, $57 - 12 = 45$ ins. $= 114\cdot2$ cm.

Volume of stator core, $V_c = 3\cdot14 \times 114\cdot2 \times 2480 = 890{,}000$ c. cm.

Stator slot-pitch, at bore $(\pi \times 29) \div 36 = 2\cdot53$ ins.

,, ,, at root of tooth $(\pi \times 33) \div 36 = 2\cdot88$ ins.

Mean width of stator tooth $= 2\cdot705 - 1 = 1\cdot705$ ins.

Volume of stator teeth,

$$V_t = 1\cdot705 \times 32 \times 2 \times 36 \times 16\cdot38 = 64{,}450 \text{ c. cm.}$$

Maximum flux-density in stator tooth, $\frac{9{,}178{,}000}{1\cdot53 \times 32} = 18{,}750$

Minimum ,, in same tooth, $18{,}750 \times \frac{1\cdot53}{1\cdot88} = 15{,}250$

B_t at one-third height of tooth,

$$18{,}750 \times \frac{3 \times 15{,}250}{2 \times 15{,}250 + 18{,}750} = 17{,}400$$

By eq. (277a) rate of iron loss in stator core and teeth

$$0\cdot43\,(12\cdot8^2 \times 890 + 17\cdot4^2 \times 64\cdot45) \text{ watts} = (62\cdot6 + 8\cdot4) \text{ kW} = 71 \text{ kW}.$$

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[1] For calculation and measurement of end-connexion reactance in turbo-generators, see Dr. R. Pohl, *E.T.Z.*, Vol. 42, p. 1057.

Cylindrical surface of rotor, $\pi \times 27 \times 46 = 3900$ sq. ins.

Windage loss at $3\frac{1}{4}$ watts per sq. in. $= 3900 \times 3{\cdot}25 = 12{\cdot}67$kW.

Cylindrical surface of end-bells, $\pi \times 27\frac{5}{8} \times 10\frac{1}{4} \times 2 = 1785$ sq. ins.

Windage loss at $2\frac{3}{4}$ watts per sq. in. $= 1785 \times 2{\cdot}75 = 4{\cdot}91$ kW.

Total windage loss, $12{\cdot}67 + 4{\cdot}91 = 17{\cdot}6$ kW, say, 18 kW in all.

By summation of losses, the efficiency is estimated to be as follows—

Windage loss	18	kW
Loss from inefficiency of fans, say	30	,,
Stator iron loss	71	,,
Stray loss in core at full-load	14	,,
Stator copper loss, including eddies	17·2	,,
Rotor ,, ,,	18·85	,,
	169·05	,,
Bearing and brush friction	20	,,
	189	,,
Output	3300	,,
Input	3489	,,

$189/3489 = 5{\cdot}42\%$. Efficiency = say, $94{\cdot}5\%$ at full-load.

§ 41. Heating and air calculations.—Owing to the indeterminate nature of so many of the factors, calculations such as the following are liable to many sources of error, and can at best only yield approximate results. They are, however, here introduced for the purpose of indicating the general nature of the calculations, when experimental data from a machine of similar design and not far different proportions are available.

The air has to carry away heat at the rate of 169 kW, radiation and convection from the external casing being neglected. If, therefore, the cooling air is taken in at 25° C. and is raised in temperature 22° C., the weight that must be supplied per minute by eq. (286) is $131{\cdot}5 \times 169/22 = 1015$ lb., of volume by eq. (288) $1015 \times 12{\cdot}38 \times 298^\circ/273^\circ = 13{,}700$ cub. ft. per min. at the inlet temperature of 25° C.

Behind each stator slot is an air-duct of 0·75 sq. in. area, two of which are arranged to communicate with each of the first six radial stator vents nearest the end-plate, three with each of the next five stator vents, and three with each of the three central stator vents; the same also from the opposite ends, so that each of the three central vents is actually fed by 6 air-ducts through the stator core, in addition in every case to the air from the rotor and air-gap.

From both ends combined the area available is 36 × 2 × 0·75 sq. ins. = 0·375 sq. ft.

The sectional area of the air-gap is $\pi \times 28'' \times 1'' = 88$ sq. ins., and for the air entering from both ends the area available is 88 × 2 = 176 sq. ins. = 1·22 sq. ft.

In the rotor, there are

16 axial air-ducts, ¾ in. wide × 1½ ins. deep, with rounded ends
area, 16 × 0·9375 = 15 sq. ins.
8 empty slots in pole-centres,
area 8 × 1″ × 2⅛″ deep with rounded ends = 16 sq. ins.
31 sq. ins.
= 0·215 sq. ft.

or 0·43 for the two ends. The total area of air-gap, stator and rotor ducts is, therefore, 2·025 sq. ft., and if the total weight of air divided between them directly as the areas, the proportionate amounts would be 611 lb., 188 and 216 lb. But owing to the higher temperature and greater volume per lb. of the air entering the air-gap and stator ducts, lesser proportions should be assigned to these paths, say, 600 and 185 lb. respectively, leaving 230 lb. for the air entering the rotor ducts.

The air entering the rotor axial ducts begins by taking up a small amount of heat from the underside of the rotor end-connexions, say, at the rate of

$$W_1 = 1500 \text{ watts.}$$

It has, therefore, risen

$$T_r^\circ = \frac{131{\cdot}5 \times 1{\cdot}5}{230} = 0{\cdot}86^\circ \text{ C.}$$

and its volume is 230 × 12·38 × 298·86/273 = 3100 cub. ft. The speed at entrance to the ducts is then 3100/0·43 = 7200 ft. per min.

The surface of the rotor axial ducts is

$$(16 \times 4{\cdot}5'' + 8 \times 5{\cdot}8'') \times 36'' = 4270 \text{ sq. ins.}$$

If the average temperature of the wall surfaces of the ducts is 16·5° C. above that of the passing air, and the average axial velocity of the air within them is 4000 ft. per min., heat is taken up at the rate

$$W_2 = 16{\cdot}5^\circ \text{ C.} \times 4270 \times 0{\cdot}0133 \times 4000/1000 = 3750 \text{ watts.}$$

The average rise in temperature of the air is then

$$0{\cdot}86 + \frac{131{\cdot}5 \times 3{\cdot}75}{230} = 3^\circ \text{ C.}$$

The cooling area of the 18 radial vents is that of the sides of the teeth and of the coil-sides within the vents, the latter acting as the blades of a fan. The mean width of a tooth being 1·673 ins., the former is 1·673″ × 5·125″ × 16 × 2 × 23 + 2·2″ × 3″ × 4 × 2 × 23 = 6300 + 1210 = 7510 sq. ins. The coil-sides add 4″ × 10·06″ × 2 × 16 = 1285 sq. ins., making a total of 8795 sq. ins. The mean area transverse to the flow is 1·673″ × 10·06″ × 16 + 2·2″ × 10·06″ × 8 = 270 + 177 = 447 sq. ins. = 3·1 sq. ft. The average radial velocity will thus be about 3100/3·1 = 1000 ft. per min. If, therefore, the average temperature of the radial surfaces exceeds that of the passing air by 12° C., heat is taken up by eq. (294) at the rate

$$W_3 = 12^\circ \times 8795 \times 0{\cdot}046 \times \frac{1000}{1000} = 4850 \text{ watts.}$$

and the average rise in the temperature of the air at exit from the rotor vents is

$$3{\cdot}0 + \frac{131{\cdot}5 \times 4{\cdot}85}{230} = 5{\cdot}78^\circ \text{ C.}$$

It will be shown later that heat will be transmitted through the end-bells at the rate $W_4 = 2500$ watts. The sum $W_1 + W_2 + W_3 + W_4 = 12{,}600$ then leaves 18,850 - 12,600 = 6250 watts as the rate at which heat is taken up from the peripheral surface of the rotor core, which after deduction of the vents measures $\pi \times 27'' \times 36'' = 3050$ sq. ins. By equation (293), if its average temperature exceeds that of the passing air by 10·6° C.

$$W_5 = 10{\cdot}6^\circ \times 3050\ (1 + 0{\cdot}5 \times 21{\cdot}2)/60 = 6250 \text{ watts}$$

The surface through which heat passes to the end-bells at the two ends is roughly $\pi \times 25 \times 7\frac{1}{2} \times 2 = 1175$ sq. ins., and the thickness of the mica insulation is $t_t = 5/32$nds of an inch. The temperature difference between copper and the inside of the bell must then be

$$T_d{}^\circ = \frac{W \times t_t}{k_t \times Q_a} = \frac{2500 \times 0{\cdot}15625}{0{\cdot}0075 \times 1175} = 44{\cdot}5^\circ \text{ C.}$$

k_t being taken as 0·0075 for nearly pure mica in a few layers. Heat is generated in the end-connexions at the rate of 18,850 × 33/79 = 7860 watts. After deduction of $W_1 + W_4$, the difference 7860 - (1500 + 2500) = 3860 is the rate at which heat must be conducted into the portion of the winding embedded in the rotor slots. The total rate at which heat must be passed through the slot insulation, 1/16 in. thick and of area transverse to the flow (4·13 + 1·125) × 2 × 46 × 16 = 7750 sq. ins., is thus 18,850 × 46/79 + 3860 = 14,850 watts. The average temperature difference between the slot copper and the rotor core is then

$$T_d{}^\circ = \frac{14{,}850 \times 0{\cdot}0625}{0{\cdot}0035 \times 7750} = 34{\cdot}2^\circ \text{ C.}$$

If the air temperature at entrance and exit to each stage are marked on a sectional sketch (Fig. 649*a*), and the surface differences assumed are added to the means between entrance and exit to give the temperature of the metal at each stage, the rotor core is found to give an average rise of 20° to 21° on its surfaces falling to 16·5° at the surface of the radial ducts, say, 30° in the body, which added to the assumed 34·2° gives a copper rise of 64·2° C.; under the end-bells, $17° + 44·5° = 61·5°$, so that a general rise of 75° C., as measured by resistance, will not be reached.

On leaving the fans, the temperature of the 785 lb. of air has already risen owing to the loss in the fans by the amount $131·5 \times 30/785 = 5·03°$ C. In the stator winding the rate at which heat is generated in the end-connexions is $17·2 \times 0·02585/0·04435 = 10$ kW and in the embedded portions $17·2 \times \dfrac{0·01445 \times 1·28}{0·04435} = 7·2$ kW.

But owing to the excellent cooling of the end-connexions, heat will be conducted into them at the additional rate of, say, 2 kW. After circulating over the end-connexions, therefore, and with some addition from the end-plates, the rise is

$$5·03 + \frac{131·5 \times 12·5}{785} = 7·13°\text{C}.$$

The cooling surface of the end-bells is $\pi \times 27·5'' \times 10·25'' \times 2 = 1770$ sq. ins., and with an average rise of temperature of the surface above that of the air of 7·3° C., the air will take up heat by eq. (293) at the rate of

$$W_4 = 7·3° \times 1770(1 + 0·5 \times 21·2)/60 = 2500 \text{ watts}.$$

The windage loss from the end-bells is at the rate of 4910 watts, so that before entering the air-gap, further heat has been taken up at the rate of $2500 + 4910 = 7·41$ kW.

At entrance to the air-gap, the temperature rise is

$$7·13 + \frac{131·5 \times 7·41}{600} = 8·76° \text{ C}.$$

and the volume is $600 \times 12·38 \times 306·76/273 = 8360$ cub. ft. The speed, which at entrance is $8360/1·22 = 6850$ ft. per min., thence quickly falls as air is drawn off into the radial vents of the stator, although in some measure maintained as air is received from the rotor vents, and heat is added

The internal cylindrical surface of the stator core after deduction of the vent spaces is $\pi \times 29 \times (46 - 9·36) = 3335$ sq. ins. If the average temperature of the stator bore exceeds that of the passing air by 20° C., and the speed of the air is 5000 ft. per min., by eq. (292)

$$W_6 = 20° \times 3335 \times 0·02 \times 5000/1000 = 6670 \text{ watts}.$$

The air in the air-gap and rotor vents also receives heat by friction and eddies at the rate of 12·67 kW, so that when mixed, the volume of air at entry to the stator radial vents has on the average risen

$$\frac{131{\cdot}5 \times \left\{ \left(30 + 12{\cdot}5\right) \frac{600}{785} + 18{\cdot}85 + 17{\cdot}6 + 6{\cdot}67 \right\}}{830} = 12^\circ \text{ C.}$$

and its volume is $830 \times 12{\cdot}38 \times \dfrac{298 + 12}{273} = 11{,}670$ cub. ft.

The cooling surface of the stator radial ducts within the region of the slots, including also the sides of the stator coils within the ducts and one spacer $\frac{1}{8}$ in. thick per tooth is

$$1{\cdot}705'' \times 2'' \times 36 \times 2 \times 25 + 1{\cdot}5'' \times 9{\cdot}36'' \times 2 \times 36$$
$$+ 2'' \times 9{\cdot}36'' \times 2 \times 36 = 6150 + 1010 + 1350 = 8510 \text{ sq. ins.}$$

The mean area transverse to the flow is

$$(1{\cdot}705'' - 0{\cdot}125'') \times 9{\cdot}36'' \times 36 = 533 \text{ sq. ins.} = 3{\cdot}7 \text{ sq. ft.}$$

The mean velocity is thus about 11,750/3·7 = 3180 ft. per min., and if the temperature of the surfaces above that of the passing air is 12° C.,

$$W_7 = 12^\circ \times 8510 \times 0{\cdot}046 \times 3{\cdot}18 = 14{,}950 \text{ watts}$$

The wall periphery of each stator axial duct being $(1 + 0{\cdot}75) \times 2 = 3\frac{1}{2}$ ins., and the average length through which the air passes from one end being 9 ins., the cooling surface from both ends is $3\frac{1}{2}'' \times 9'' \times 36 \times 2 = 2268$ sq. ins. At entrance the volume of the air supplied per minute is

$$185 \times 12{\cdot}38 \times \frac{298 + 7{\cdot}13}{273} = 2560 \text{ cub. ft.}$$

and its speed 2560/0·375 = 6830 ft. per min. The rate of taking up heat, if the temperature of the walls is 20° C. above that of the air, is

$$W_8 = 20^\circ \text{ C.} \times 2268 \times 0{\cdot}0133 \times 6830/1000 = 4{,}120 \text{ watts}$$

All the air has now re-united, and its average temperature rise is

$$131{\cdot}5 \times \frac{104{\cdot}66}{1015} = 13{\cdot}55^\circ \text{ C.}$$

and its volume

$$1015 \times 12{\cdot}38 \times \frac{298 + 13{\cdot}55}{273} = 14{,}330 \text{ cub. ft.}$$

It remains then to secure the taking up of heat at the rate of (169 - 104·66) = 64·34 kW by the remaining length of the stator vents. The area of cooling surface, including two spacers $\frac{1}{8}$ in. thick per tooth, is

$$(57^2 - 33^2) \times 0{\cdot}7854 \times 2 \times 25 + 12'' \times 9{\cdot}36'' \times 2 \times 72$$
$$= 85{,}000 + 16{,}150 = 101{,}150 \text{ sq. ins.}$$

The mean area transverse to the flow is

$$(45'' \times 3{\cdot}14 - 72 \times 0{\cdot}125'') \times 9{\cdot}36 = 1235 \text{ sq. ins.} = 8{\cdot}58 \text{ sq. ft.}$$

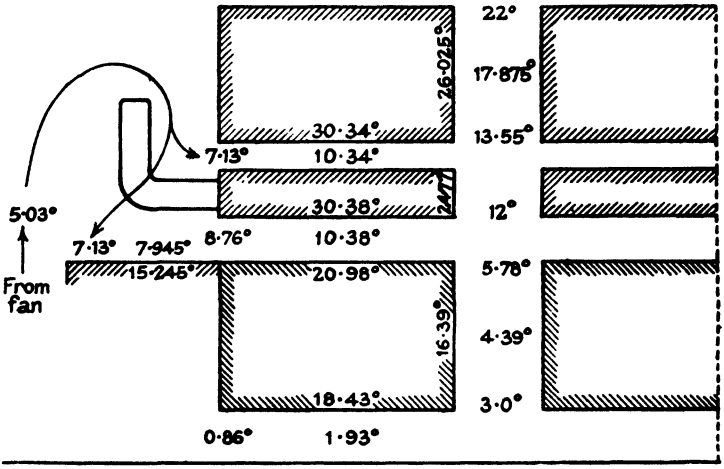

FIG. 649*a*.—Temperature rises of air and average temperature rises of surfaces (all reckoned above 25° C.).

The average velocity is thus approximately 14,500/8·58 = 1690 ft. per min., and if the temperature of the surface exceeds that of the passing air by 8·15° C.

$$W_9 = 8{\cdot}15^\circ \times 101{,}150 \times 0{\cdot}046 \times 1{\cdot}69 = 64{,}340 \text{ watts.}$$

The volume at exit is 1015 × 12·38 × 320/273 = 14,700 cub. ft. per min.

Marking the rises in temperature of air and surfaces on a sketch (Fig. 649*a*) it is found that the rise for the stator core surfaces ranges from about 31° C. at the bore to 27° C. at the exterior. To pass heat at the rate of 5200 watts from the embedded portions of the stator winding through the insulation of thickness 5/32nds of an inch, and of area transverse to the flow (1·375″ + 0·845″) × 2 × 46″ × 36 = 7350 sq. ins., the average temperature difference between the slot copper and the stator core must be

$$T_d^\circ = \frac{5200 \times 0{\cdot}156}{0{\cdot}0035 \times 7350} = 31{\cdot}6^\circ \text{ C.}$$

and the total average rise of the slot copper 31·6 + 31 = 62·6° C. But from the lesser heating of the end-connexions which have greater length than the embedded portions, the average rise in temperature of the whole winding will not exceed the assumed 55° C.

§ 42. Tensile stress on tooth at the level of the bottom of the slot.—By equation (279),

$$f_w = 28{\cdot}4 \times 3^2 \times 0{\cdot}31 \times \frac{13}{16} \times \frac{13{\cdot}5 + 12{\cdot}6875}{2} \times \frac{1{\cdot}1875 + 1{\cdot}75}{2}$$

$$= 1235 \text{ lb. per inch run}$$

Area of copper in rotor slot, $a_c = 0{\cdot}106 \times 36 = 3{\cdot}82 + 0{\cdot}05$ for brass liner $= 3{\cdot}87$ sq. ins.

Area of insulation in rotor slot, a_i

$$= 1{\cdot}0625(0{\cdot}35 + 0{\cdot}1875) + 4{\cdot}3125 \times 0{\cdot}125$$
$$= 1{\cdot}12 \text{ sq. ins.}$$

By equations (280) and (281),

$$f_c = 4{\cdot}57 \times 3^2 \times 3{\cdot}87(12{\cdot}6875 + 8{\cdot}375) = 3350 \text{ lb. per inch run.}$$

$$f_i = 1{\cdot}42 \times 3^2 \times 1{\cdot}12(12{\cdot}6875 + 8{\cdot}375) = 302 \text{ ,, } \quad \text{,,}$$

$$(f_w + f_c + f_i)L = 4887 \times 46 = 224{,}800 \text{ lb.}$$

By equation (282),

$$f_t = 2 \times 3^2 \times 5{\cdot}125 \left\{ \left(13{\cdot}5 + 8{\cdot}375\right)\left(2{\cdot}3425 + 1{\cdot}004\right) + \frac{5{\cdot}125}{3}\left(1{\cdot}3385\right) \right\}$$

$$= 92{\cdot}4\ (73{\cdot}4 + 2{\cdot}29) = 7000 \text{ lb. per inch run.}$$

By equation (283),

$$\frac{(f_w + f_c + f_i)\,L}{w_{t2}\,l} + \frac{f_t}{w_{t2}} = \frac{224{,}800}{1{\cdot}004\ (46 - 10{\cdot}06)} + \frac{7000}{1{\cdot}004}$$

$$= 6250 + 7000 = 13{,}250 \text{ lb. per sq. in.}$$

§ 43. 17,000 kW turbo-alternator.—A 17,000 kW three-phase turbo alternator, constructed by Messrs. The Metropolitan-Vickers Electrical Co., Ltd., for the Stuart Street station of the Manchester Corporation, is shown in Figs. 650–651. The leading constructional data of the machine[1] are as follows—

17,000 kW at 0·85 power-factor = 20,000 kVA.
Frequency, 50. No. of poles, 4.
Revs. per min., 1500.
Interlinked voltage, 6600–6700.
Amperes per phase, 1750. Voltage per phase, 3810–3870.
Stator core : outside diameter, 967 ins.
inside ,, 54·75 ,,
Rotor core, diameter 53 ,,
Diameter to centre of air-gap, 53·875 ,,
Over-all length of stator or rotor (including ducts), 75 ins.
Cubic inches of $D^2{}_{a}\,L_{a} = 217{,}687$.

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[1] Kindly furnished to the author through the courtesy of the makers, which is hereby acknowledged.

At 20,000 kVA, $VA/N = 13{,}333$; and the specific torque coefficient, $G = VA/D^2_a\, L_a N = 0{\cdot}0615$.

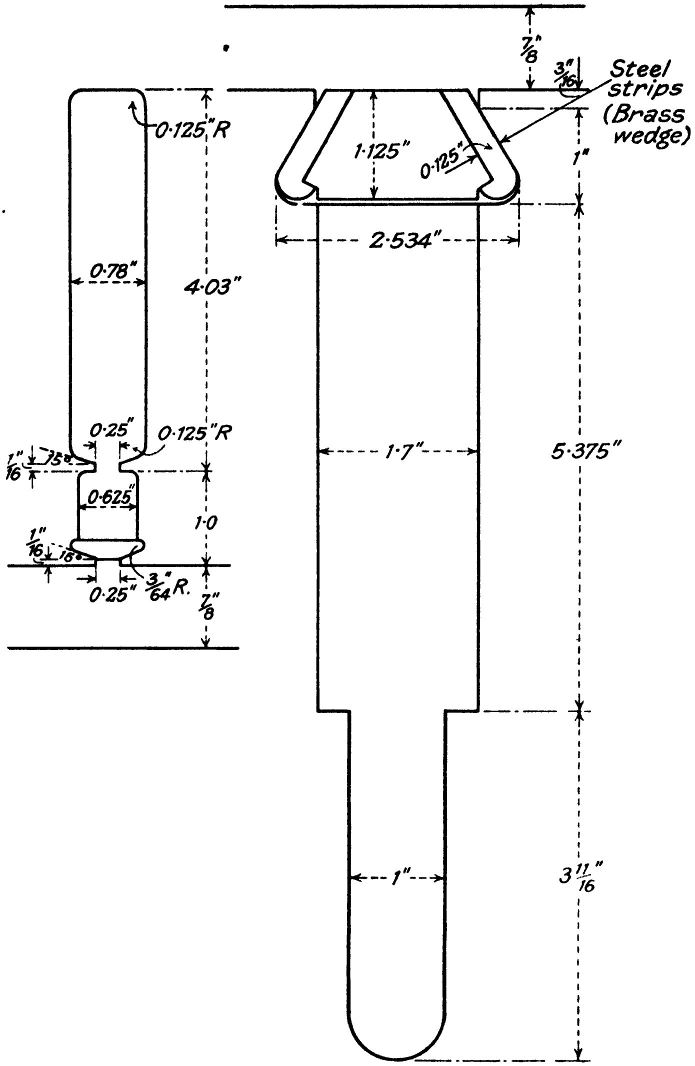

FIG. 652.—Stator and rotor slots of 17,000 kW turbo-alternator.

Number of stator slots, 96, dimensioned as shown in Fig. 652.

Slots per pole per phase, 96/12 = 8.

Stator winding: concentric coils, 4 conductors per slot, all in parallel, and transposed between poles; two micanite slot-cells with two conductors in each.

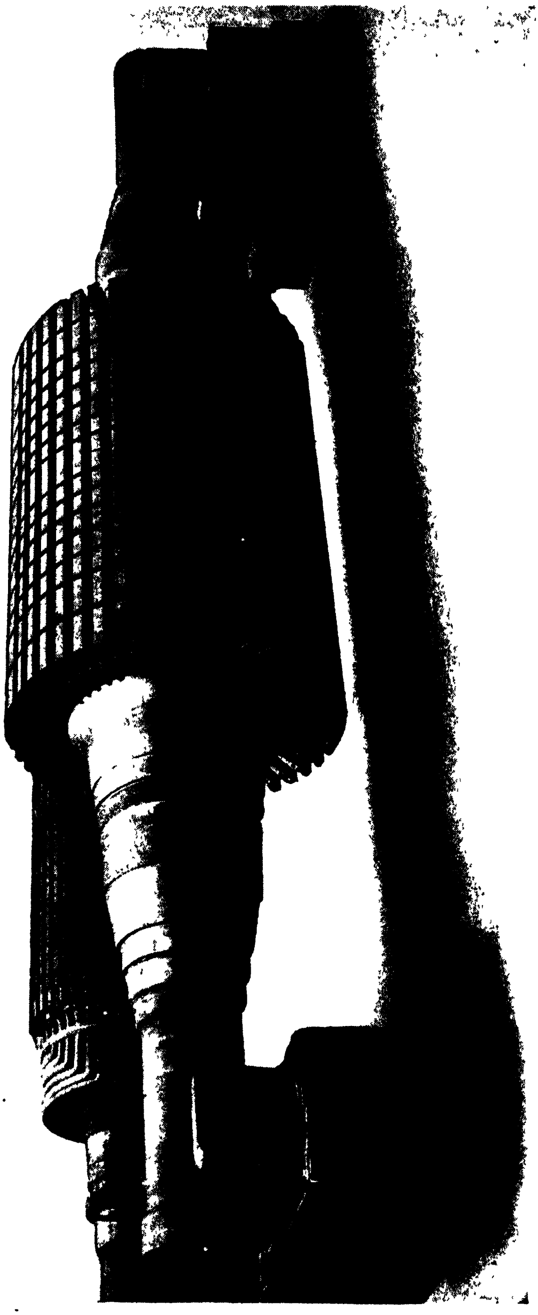

Fig. 653.—Rotor of 17,000 kW turbo-alternator before winding.

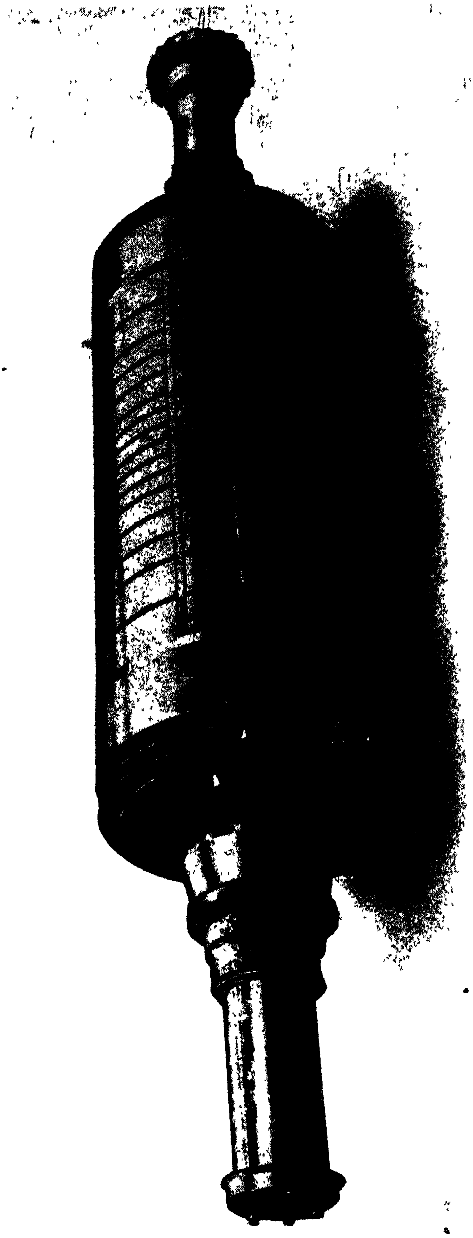

FIG 654.—Wound rotor of 17,000 kW turbo-alternator with fans.

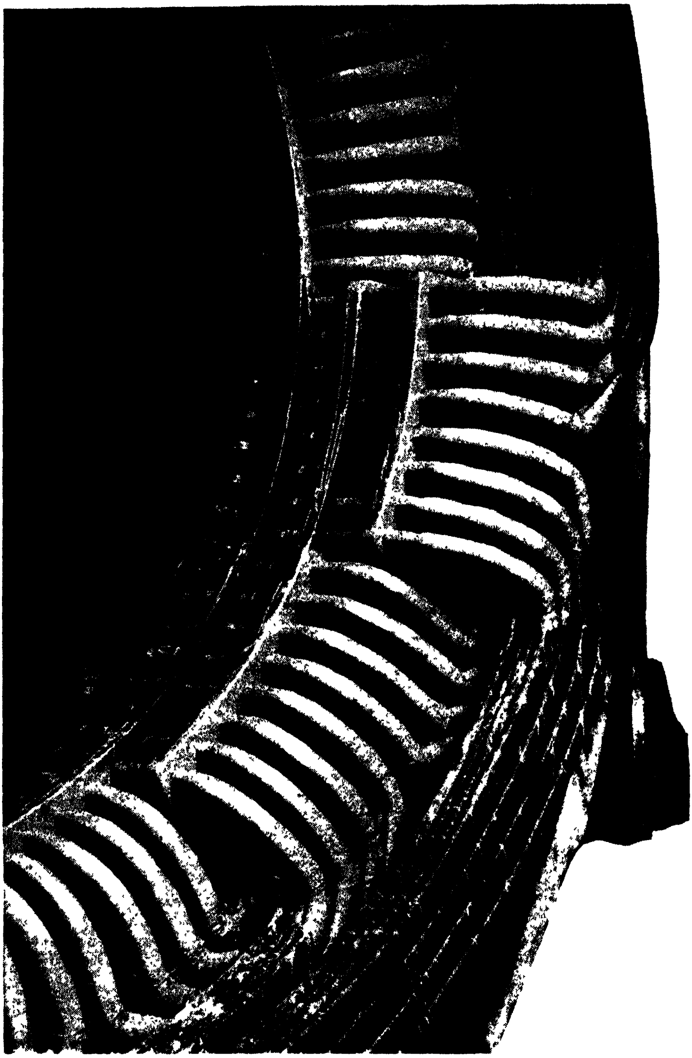

FIG. 655.—Stator end-connexions of 17,000 kW turbo-alternator.

Stator conductors: top bar, 0·45 in. × 4 laminations, each 0·18 in. thick, separately insulated and twisted in the slot.

2nd and 3rd bars, each 0·45 in. × 3 laminations, each 0·225 in. thick, separately insulated and twisted in the slot.

Bottom bar: 0·45 in. × 0·6875 in. solid.

End-connexions: 0·4 in. × 0·8 in. solid.

Rotor slots, 32 pitched as for 47: dimensions as shown in Fig. 652.

Rotor winding, 36 conductors per slot, 1·5 ins. × 0·125 in. in slots, and 2 ins. × 0·125 in. at the ends.

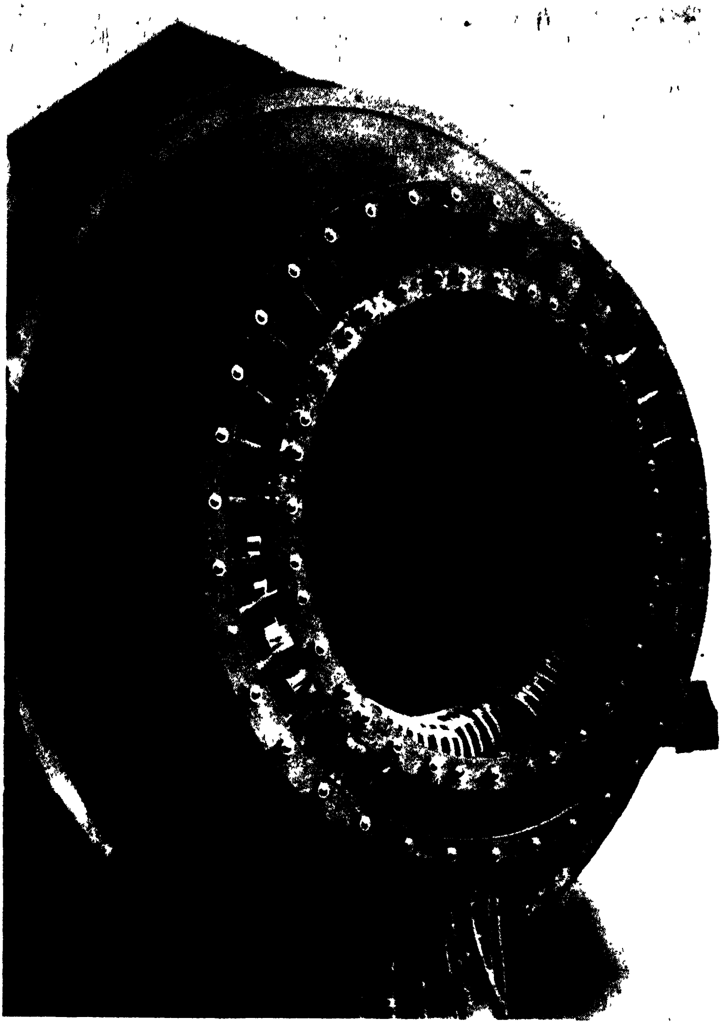

FIG. 656.—Bracing of stator end-connexions (exciter end).

The machine is radially and axially ventilated as follows—

Stator : Nine radial ducts, each 1·125 ins. wide.
Two circles of 96 axial holes, 1·08 in. diam.
One circle of 48 ,, ,, ,,
Three circles of 72 ,, 1·4 ,,

Rotor : 15 radial ducts, each 0·75 in. wide, communicating with 24 axial ducts below the slots ;
2 axial holes, 0·75 in. and 0·625 in. diam., in each tooth at ends only.

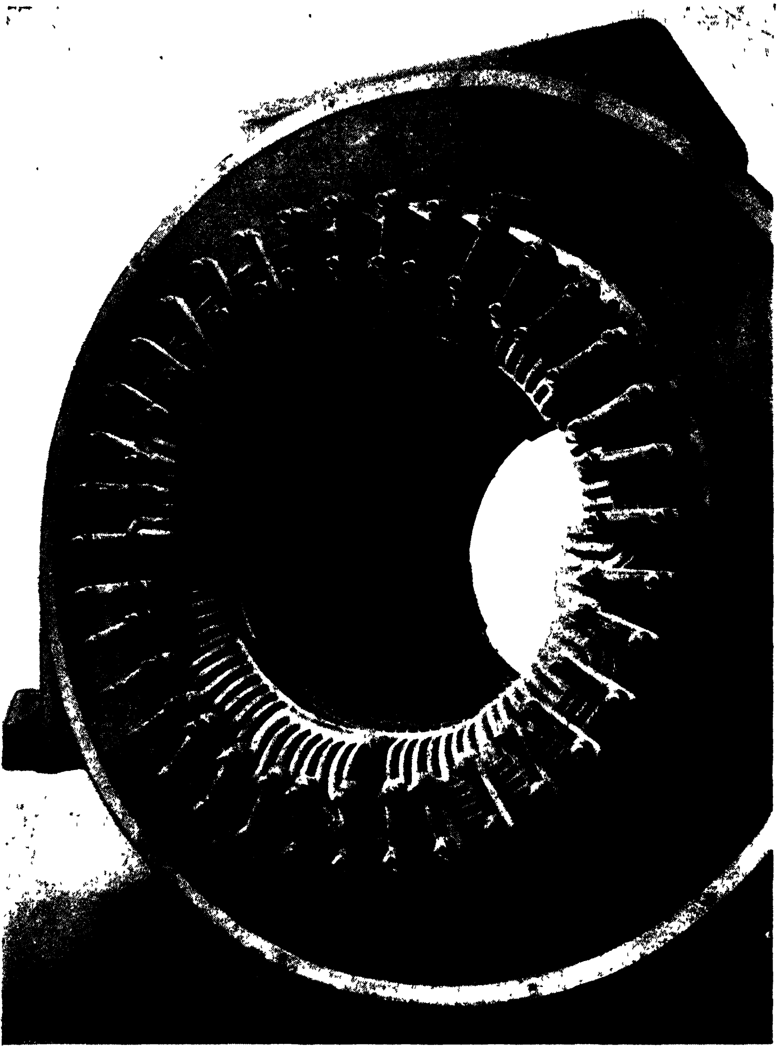

FIG. 657.—Bracing of stator end-connexions (turbine end).

Air pressure in stator end-guards, 11·3 ins. water gauge.

Volume of air at outlet with alternator unexcited, 60,000 cub. ft. per min.

Critical speed (calculated), 2450 revs. per min.

Weight of	stator without end-guards,	56	tons.
,,	rotor, rough turned forging,	29	,,
,,	,, complete	30	,,
,,	exciter	1½	,,
,,	machine complete	93	,,

Mean length of stator turn, 333 ins.
Resistance of one stator phase, cold at 11·5° C., 0·00305 ohm.
Mean length of rotor turn, 225 ins.
Resistance of rotor winding, cold at 11·5° C., 0·464 ohm.

FIG. 658.—17,000 kW turbo-alternator erected.

A large 2-pole turbo-alternator for 20,600 kVA, 0·8 power-factor, 3000 r.p.m. 50 frequency, 6600 volts three-phase, also constructed by The Metropolitan-Vickers Electrical Co., Ltd., is shown in part section in Fig. 660, with its stator end-connexions in Fig. 661; while Fig. 662 shows half of a 4-pole alternator by the same Company to give 31,250 kVA, 0·8 power-factor, 1500 r.p.m. 50 frequency, 6600 volts, three-phase.

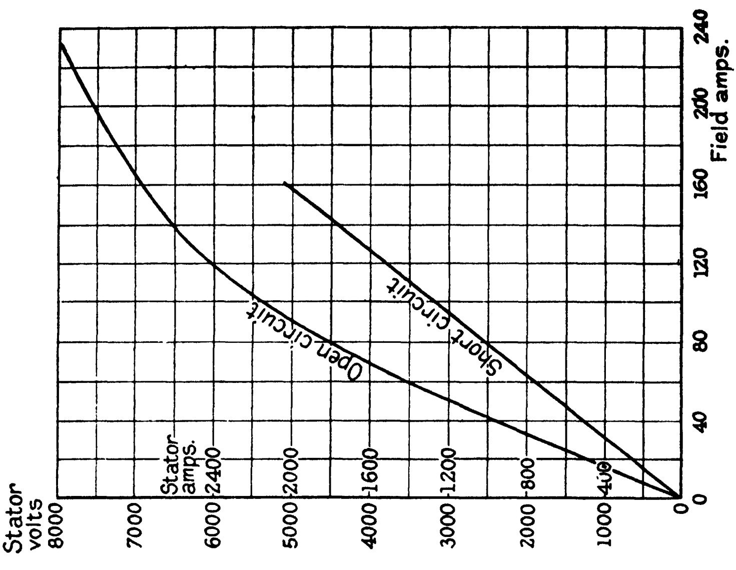

FIG. 659.—Open- and short-circuit curves of 17,000 kW turbo-alternator.

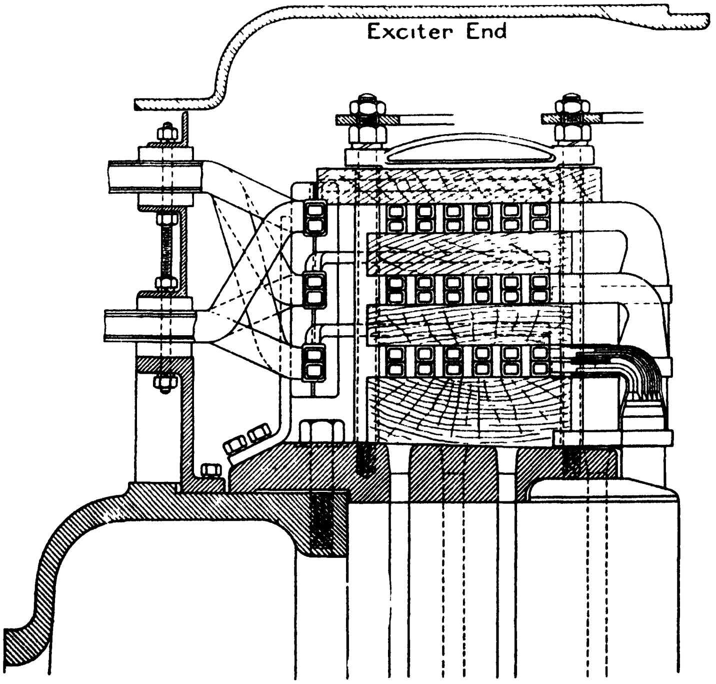

FIG. 661.—Bracing of stator end-connexions for 20,600 kVA 2-pole alternator.

§ 44. 40,000 kW turbo-alternators of the Société Alsacienne de Constructions Mécaniques, Belfort.—Of the five 40,000 kW turbo-alternators installed in the Gennevilliers power-station of the Union d'Electricité for the supply of Paris, three were constructed by the Société Alsacienne de Constructions Mécaniques of Belfort. One of these is shown completed and in course of erection in Figs. 663, 664, and 665,[1] and in further detail in Figs. 666 *a* and *b*.

The normal output of the machine which is 4-pole running at 1500 revs. per min., 50 frequency, is 40,000 kW at 0·9 power-factor, 5500 to 6400 volts, three-phase ; but as the result of tests, the rating can be raised to 55,000 kVA. The rotor core is built up of steel discs, 0·157 in. thick, with a breaking strength of 38 to 41 tons per sq. in. and an elongation of 18 to 15 per cent., as shown by careful tests for mechanical strength, to which all parts of the rotor have been subjected. The discs are riveted together in packets, which are shrunk on to the rotor shaft and clamped between steel end-plates; the rivet-heads keep the packets apart and thus form ventilating ducts.

Forty-eight slots are stamped in the discs, *i.e.* 12 per pole, in each of which are 23 conductors, carrying 470 amperes of exciting current on load ; there are, therefore, 23 × 6 = 138 turns and 65,000 ampere-turns per pole. Two additional empty slots are formed in each pole-centre to limit the flux therefrom. The field resistance, hot, being 0·48 ohm, the exciting pressure is then 225 volts. The rotor slots are closed by bronze wedges, connected at each end to a copper plate to form a complete squirrel-cage damper. The end-rings are of nickel-chrome steel, with a breaking strength of about 57 to 63 tons per sq. inch and an elongation of 16 per cent. ; they are centred on one side on the rotor end-plates and on the other side on a forged steel end-plate, carrying the fan and fixed on the shaft by a bayonet joint. Air is supplied to the rotor at the rate of 27,600 cub. ft. per min., part entering through the slots milled in the shaft, and part passing through holes drilled in the fan side-plate into the end-connexion space and thence escaping through slits in the core end-plates. The total weight of the rotor is about 50 tons, and its critical speed is well above its running speed.

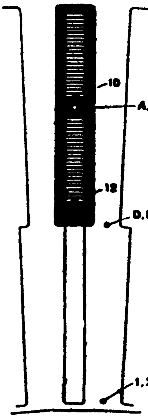

FIG. 665*a*.—Stator slot with positions of thermocouples.

The stator core, 109½ ins. long, of discs with a specific loss of 1·18 watts per lb. at 50 cycles per sec. and $B = 10{,}000$, was carefully built up to avoid any subsequent filing. There are 144 slots, but the two conductors in each slot are soldered together at one end, so as to form a single electrical conductor. At the opposite end the end-connexions are crossed, so that a conductor in the bottom layer is united in series to a conductor in the top layer ; as a further precaution against eddy currents, the insulated strips forming each conductor (Fig. 665*a*) are inclined to the longitudinal axis of the slot, so that each bears the same position relatively to the slot-field.

The bars are encased in a stout mica wrapping, and the involute end-connexions are riveted and soldered to the ends of the bars, and insulated with layers of mica tape ; at the ends of the iron core, and between the two ranges of the end-connexions, toothed annular plates of insulating material assist to hold the bars firmly. The clamping plates, which fasten the end-connexions by bronze bolts, are themselves joined up by a strong end-ring of bronze (*cp.* Fig. 664). The winding being divided into two halves in parallel, there are 24 conductors in series per phase, corresponding at 6000 volts to $\Phi_a = 138{\cdot}5$ megalines per pole. The resistance of a phase when hot is 0·0014 ohm.

A special feature of the stator slot is its long head, forming a leakage path for flux under short-circuit. By its use the instantaneous value of the short-

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[1] From photographs kindly supplied by the makers.

Fig. 663.—40,000 kW turbo-alternator constructed by the Société Alsacienne de Constructions Mécaniques in Gennevilliers power-station.

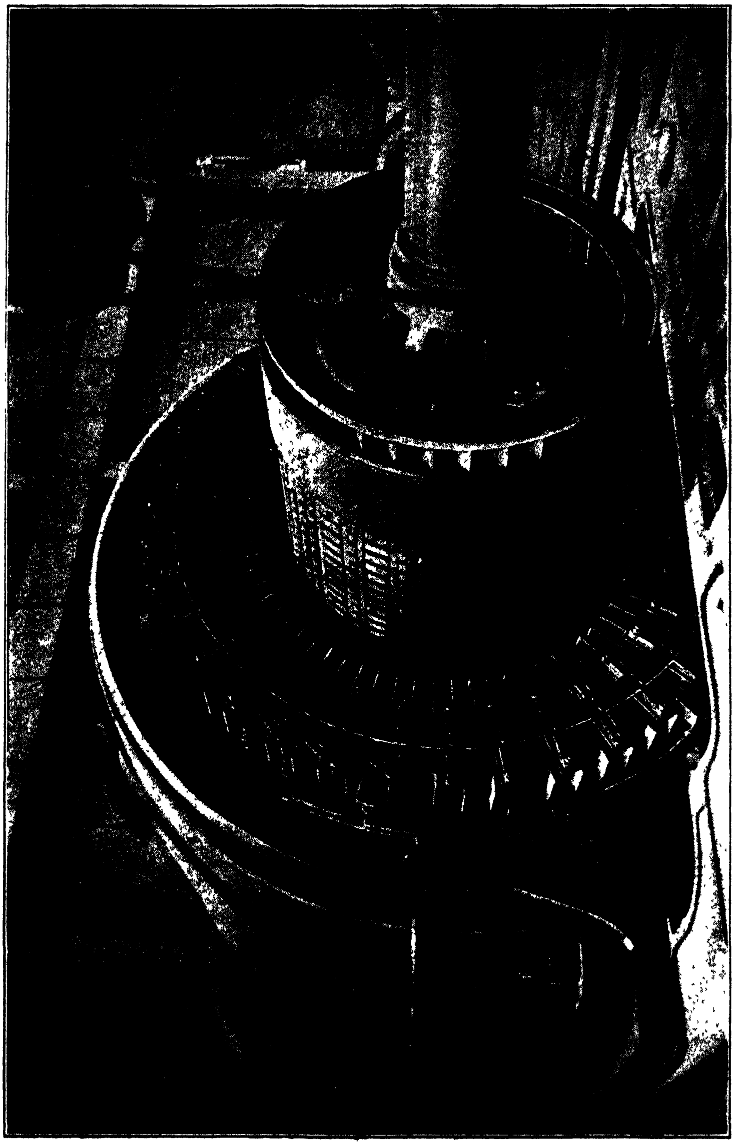

FIG. 664.—The same in course of erection.

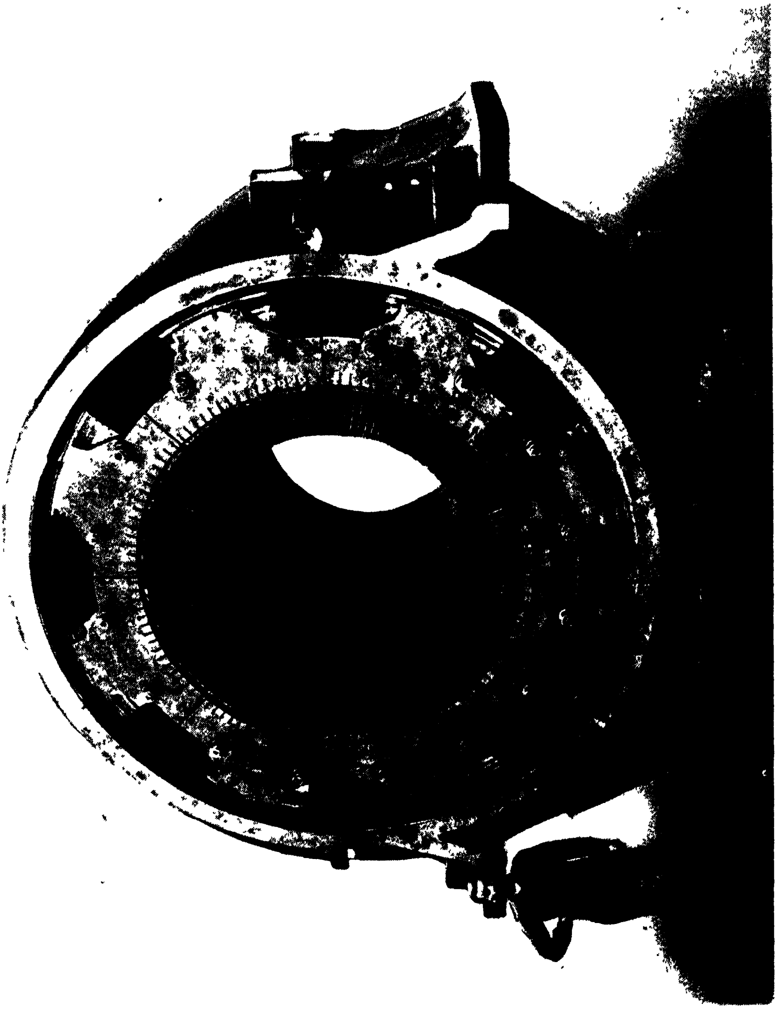

FIG 665 —Stator core and casing for 40,000 kW turbo-alternator.

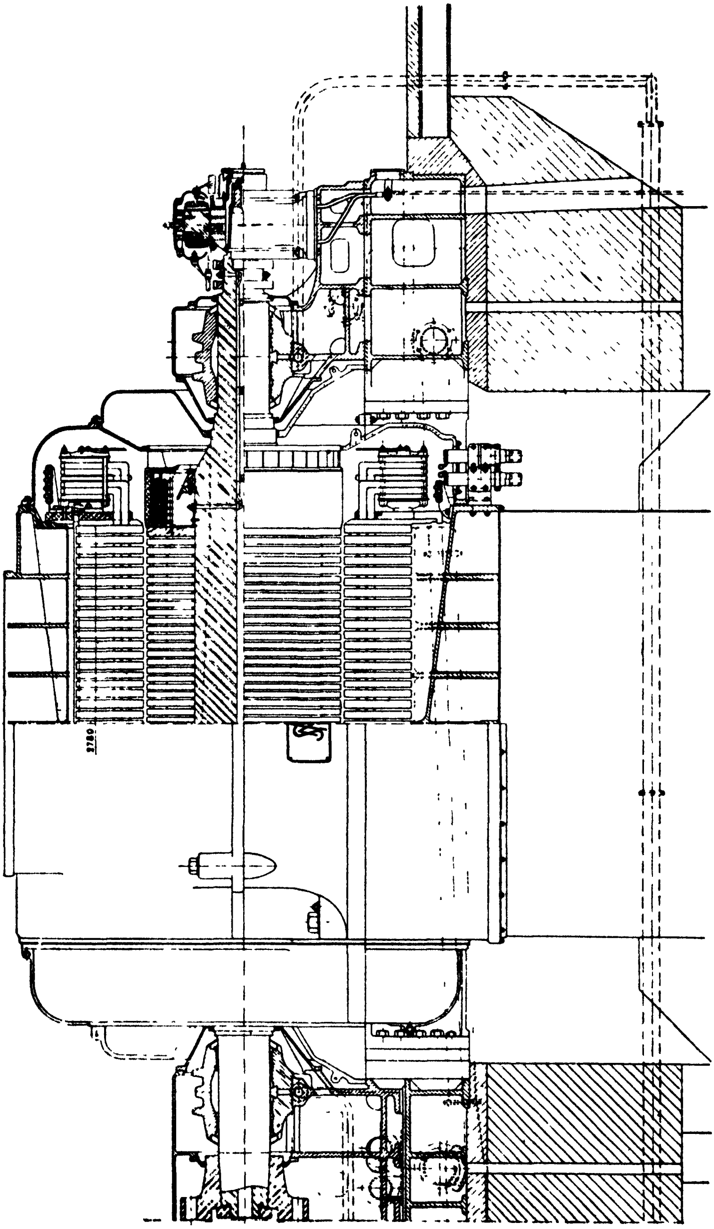

FIG. 666a.—Side elevation of 40,000 kW turbo-alternator.

circuit current is not more than 4 to 5 times the full-load current. The inductive drop due to leakage being, with normal current, 24 per cent. of the normal pressure, the corresponding value of a symmetrical short-circuit current is 4·17 times the normal current. Of the 24 per cent., 10·2 per cent. is due to the slot leakage paths and 9·5 per cent. only to the end-connexion fluxes, so that the forces on these are greatly lessened. The slot channels are also used for axial circulation of cooling air. Except at the ends of the stator core, the slots are closed by a thin bridge.

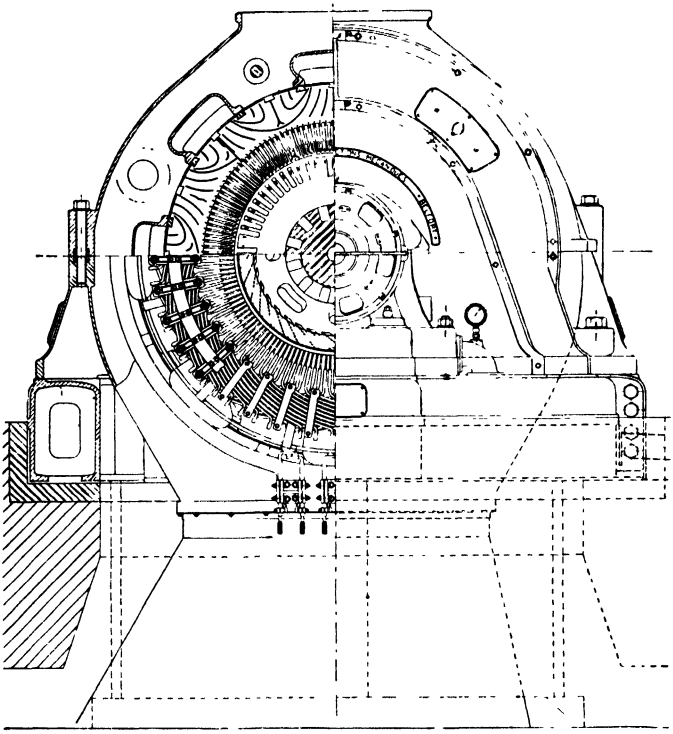

FIG. 666*b*.—End elevation of 40.000 kW turbo-alternator.

Each of the two fans at the ends of the rotor delivers 38,000 cub. ft. of air per min. into the end-guards, where it circulates in the spaces between the planes of the involute end-connexions; thence it enters channels in the frame and is distributed along the whole length of the stator core. Some of the stator ducts are reserved for the purpose of carrying away the air from the rotor, so that the rotor and stator air-circuits are entirely separate.

The closed system of air circulation and the means for cooling the air have already been alluded to in § 19.

The total weight of the machine, complete with exciter, is about 155 tons.

The rise of temperature of the rotor with 470 amps., as measured by increase of resistance, was about 47° C.

When run un-excited, a total of 104,000 cub. ft. of air per minute was raised 8·9° C., so that the losses must then have been 480 kW; when excited to give 6000 volts at no-load (rotor exciting current, 136 amps.), the same amount of air was raised 16·7° C., corresponding to total losses at the rate of 900 kW.

§ 45. 60,000 kVA turbo-alternators.—Two 6-pole 60,000 kVA sets, 6600 volts, 5250 amps, 1000 r.p.m., have been put into operation at the Goldenberg Works near Cologne. Of these, one constructed by the Siemens-Schuckert Werke[1] had 26 steel discs 90·5 ins. diameter shrunk on a forging of about 43·5 ins. diameter; each disc was 4¾ ins. thick nearly, and the total core length 123 ins. Forty-eight slots received the rotor coils, insulated with mica, the copper weighing about 10¾ tons, and nickel-chrome steel end-rings, registered on the teeth, secured the coil-ends. Air enters along grooves cut in the shaft and thence into radial ducts between the discs. The through shaft, of total length 28½ ft., was pierced with a hole of 11·8 ins. diam. At each end of the rotor a steel slip ring carries the exciting current of 1000 amperes at 220 volts.

The stator had an outside diameter of 138 ins and a core length of 118 ins.; the cast-iron stator frame weighed 44 tons. Air enters at each end through 400 axial holes, and thence through radial ducts over the back of the core to the outlet at the bottom. The stator is cooled by 85,000 cub. ft. of air per min., the separate fan which is employed having a capacity of 127,000 cub. ft. per min. at 10 ins. water pressure. The rotor weighed about 102 tons, and the whole machine 245 tons.

The second set, constructed by the Allgemeine Elektricitäts Gesellschaft, follows the usual construction of the Company. The rotor coils are wound and baked at 180° C., and pressed to the exact shape required at about three times the running pressure until they are practically solid. After assemblage on the core, packets of tooth punchings, riveted together and spaced out by the projecting rivetheads, are threaded into dovetails round the solid core or in steel discs on the inner solid forging Heavy bronze wedges, sliding upon a slightly wedge-shaped liner, close the tops of the slots formed by the tooth-packets. Steel wire bands, wound on under a predetermined and recorded tension, hold down the end-connexions of the rotor coils.

For a certain length at each end, the ventilation of the stator is purely axial; it then becomes radial over the central portion. Involute end-connexions are employed, with a single bar per slot.

The maximum instantaneous short-circuit current is given as 15 times the permanent I_{psc}, and the latter as 1·7 times I_n.

§ 46. 62,500 kVA turbo-alternators.—Several large 6-pole 60-cycle turbo-alternators running at 1200 revs per min. and giving 62,500 kVA at 0·8 power-factor have been built in the United States of America, the first of these by the General Electric Co., Schenectady, N.Y., having been put into operation in 1924 at the Hudson Avenue Power House of the Brooklyn Edison Company.[2] An idea of the size of these units is given by the following approximate weights: rotor, 91½ tons; stator, 91½ tons; shields, etc., 11 tons; total, not including base, pedestals or bearings, 194 tons.

The rotor is formed of 3 forgings, the middle one a hollow cylinder, and is 35 ft. long. The field coils of heavy copper strip were wound on edge in a special machine which changes the pitch at every turn to allow for the slope of the radial slots. The retaining wedges are of magnetic steel in the slots next to the pole-centres and of non-magnetic steel in the outer slots of each pole. The end-rings of nickel steel were expanded by heat for shrinking on. The end-connexions are encased in aluminium saddles with spacing blocks, also of aluminium (*cp.* Fig. 631).

The stator frame is built up of 6 cast rings of I section, to the outer flanges of which are riveted heavy boiler plates (Fig 667). Twenty-eight rectangular steel bars, bolted on the inside of the I rings to stiffen the frame, are slotted to receive the dovetail keys of the discs, and also serve as clamping bolts for the end-flanges, their ends being turned down and threaded for this purpose.

The 126 coils of the stator winding are each of three turns with a fractional pitch of 85·8 per cent. Each turn consists of 16 asbestos-insulated strips, 0·21 in. by 0·085 in., giving a total of 0·285 sq. in. The turns and coils were insulated by hand with mica tape over their whole length, and were subjected more than once for many hours to vacuum treatment followed each time by impregnation by a hot sealing compound under pressure; high thermal conductivity and a solid insulation free from entrapped air were thus secured.

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[1] *Z.V.D.I.*, 27th August, 1921, and *Electr.*, Vol. 87, p. 709.

[2] See "Large Steam Turbine Generators," *Journ. Amer. I.E E.*, Vol 43, p. 923, by W. J. Foster, E. H. Freiburghouse and M. A. Savage, of the General Electric Co., to the first of whom the writer is indebted for the photographs from which Figs. 631, 667 have been reproduced.

The interlinked pressure of the machine is 13,800 volts, and in order to meet a guaranteed high-potential test of 40,000 volts for one minute, the total thickness of insulation from copper to slot wall was made 0·28 in. Under this pressure test at 60 cycles the charging current was 5·6 amperes per phase, corresponding to 224 kVA.

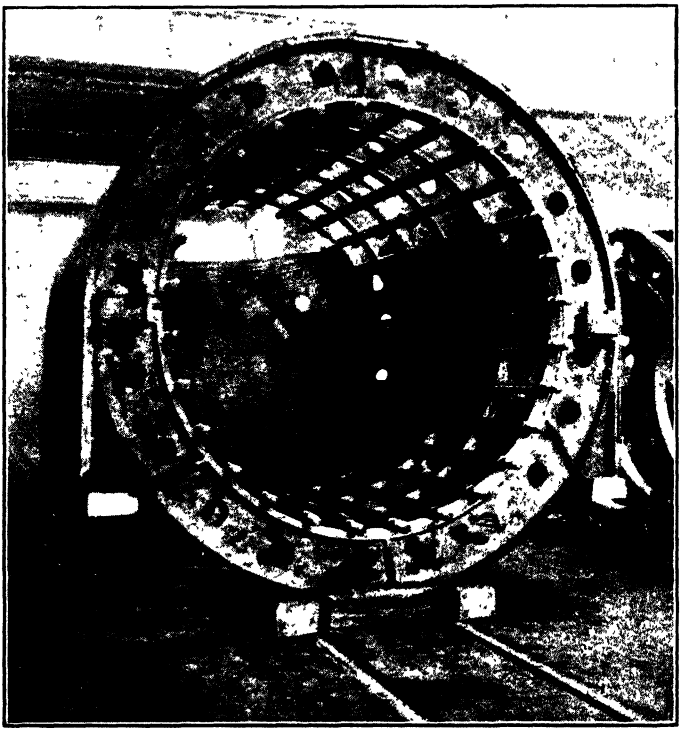

FIG. 667.—Stator frame for 62,500 kVA 1200 r.p.m. turbo-alternator.
(The General Electric Co., Schenectady, U.S.A.)

Per phase there are 6 circuits, and the full-load current at 13,800 volts being 2610 amperes, the current density is 2610/6 × 0·285 = 1521 amperes per sq. in. The 6 circuits per phase are grouped into two sets, from one end of each of which a separate lead is brought to the terminal board, with a common lead to the neutral point; this arrangement enables a differential current transformer to be inserted at the line end of each phase, in addition to the current transformers used with a balanced relay system of generator protection. Normally equal currents from each half of the phase flow through the double primary coils of the differential current transformer, but at any desired degree of unbalance the generator is tripped out.

The efficiencies as determined under actual load by measurement of the temperature rise and quantity of heated air were approximately as follows—

	Percentage efficiency.			
Power-factor.	¼-*load.*	½-*load.*	¾-*load.*	*full-load.*
Unity	93·0	96·2	97·3	97·7
0·9	92·1	95·6	97·0	97·5
0·8	91·4	95·3	96·5	97·1
0·7	90·3	94·5	96·0	96·6
0·6	89·0	93·8	95·4	96
0·5	87·4	92·7	94·5	95·5

The quantity of air was 112,500 cub. ft. per minute.

The temperature rise of the cooling air at full load was 25° C., of the rotor winding measured by change of resistance 57° C., and the maximum temperature rise of the armature winding by resistance temperature detector 50° C.

The normal field-current at full rated load being about 800 amperes, the exciting currents for 13,800 volts at other power-factors and 62,500 kVA were approximately: at 0·5 p f., 850 amperes; at 0·9 p.f., 750 amperes; at unity power-factor, 620 amperes, and on open circuit, about 440 amperes.

§ 47. The deflection of an unbalanced concentrated mass with increasing speed.—In Fig. 668 let C represent the axis of figure passing through the geometrical centre of the rotor; G, the centre of its mass (assumed for the present to be concentrated) which, in the general case of a rotor that is not perfectly balanced, will not quite coincide with C, the distance CG between them being the mass-eccentricity e; and let R represent the axis about which the mass is revolved at any given speed, and passing through the centres of the bearings, so that RC is the deflection, d

In the ideal case of no damping, for which alone the expressions at the head of page 297 (Vol. 1) hold true, the three centres, R, C, and G, remain in the same straight line, so that RC which was there symbolized as r could be immediately added algebraically to CG, which was symbolized as r_0, to give the distance between the running centre and the mass-centre, *i e.* $RG = r + r_0$. In this case, as soon as $M\omega$ exceeds c/ω, r becomes negative, and $r + r_0$ is arithmetically less than the deflection r. Further, it becomes progressively smaller as the speed is increased; and in the limit when $M\omega^2$ is so great that in comparison with it c may be neglected, $r + r_0$ becomes zero, so that r then is $= -r_0$, and R coincides with G.

But, with damping, the three centres no longer remain in the same straight line, and the symbols r and r_0 for deflection and mass-eccentricity are better replaced by d and e.

The advantage of the treatment outlined in the latter part of Chapter XIII, § 10, whereby the motions and forces are resolved first along a horizontal and then along a vertical plane, is that the usual differential equations for linear oscillations become applicable, and their results can be afterwards recombined. Let time in seconds be reckoned from the moment when G is vertically above C, and at this moment let the relative positions of R, C, and G be as shown on the right-hand side of Fig. 668. Now the line GC revolves round C as rotation proceeds, so that the distance of G to the left of C on the horizontal plane is at any moment $e \sin \omega t$, where ω is in radians per second, and its height upwards above C on the vertical plane is at any moment $e \cos \omega t$. Let x and y be at any moment respectively the distance of C to the left of R on the horizontal plane, and its height above R on the vertical plane, as shown on the left-hand side of Fig. 668; either may be positive or negative, as *e.g.* at zero time where x, being from left to right, is negative. The distance of G projected on to the horizontal plane (measured as positive when to the left of R) from R is thus at any instant $x + e \sin \omega t$. The horizontal acceleration of G is $d^2(x + e \sin \omega t)/dt^2$, and the horizontal force that must be acting on the mass M concentrated at G is

$$M \frac{d^2 (x + e \sin \omega t)}{dt^2} = M \frac{d^2 x}{dt^2} - Me\omega^2 \sin \omega t$$

The remaining forces are the horizontal damping force, assumed proportional to the velocity, *i.e.* $b\,dx/dt$, and the controlling or restoring horizontal force, assumed proportional to the deflection, *i.e.* cx. Thus

$$M\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = Me\omega^2 \sin\omega t$$

which is precisely analogous to equation (20) (Vol. 1, p. 117) and of which the solution by (20*a*) is

$$x = \frac{M\omega^2 e}{\sqrt{b^2\omega^2 + (c - M\omega^2)^2}} \sin\left(\omega t - \tan^{-1}\frac{b}{c/\omega - M\omega}\right)$$

Analogously, the distance of G when projected on to the vertical plane (measured as positive when above R) from R is at any instant $y + e\cos\omega t$.

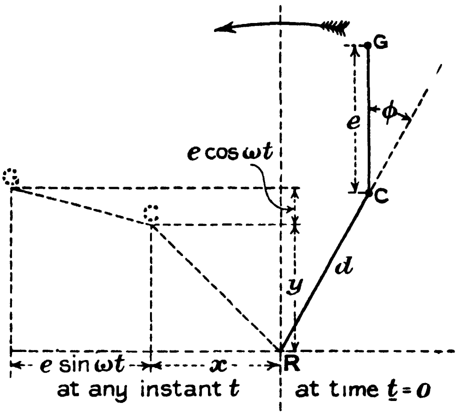

FIG. 668.—The 3 centres R, C and G when damping is present.

The vertical acceleration of G is $d^2(y + e\cos\omega t)/dt^2$, and the vertical force acting on the mass M concentrated at G is

$$M\frac{d^2(y + e\cos\omega t)}{dt^2} = M\frac{d^2y}{dt^2} - Me\omega^2\cos\omega t$$

Thence with addition of the vertical damping and controlling forces,

$$M\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = Me\omega^2\cos\omega t$$

and

$$y = \frac{M\omega^2 e}{\sqrt{b^2\omega^2 + (c - M\omega^2)^2}} \cos\left(\omega t - \tan^{-1}\frac{b}{c/\omega - M\omega}\right)$$

Combining the two component deflections which are at right angles, the actual resultant deflection RC at any given speed[1] is a constant vector of length

$$d = \frac{Me\omega^2}{\sqrt{b^2\omega^2 + (c - M\omega^2)^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (296)$$

revolving with a constant angle of lag $\phi = \tan^{-1}\dfrac{b}{c/\omega - M\omega}$ behind the vector $CG = e$. The two centres, C and G, thus revolve with a fixed orientation to R dependent upon the speed.

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[1] Prof. Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 123–124. See also "The Whirling of Shafts," by J. Frith and F. Buckingham, *Journ. I.E.E.*, Vol. 62, p. 107.

The results thus reached by the combination of Chapter VI, § 23, and Chapter XIII, § 10, are summarized in Fig. 669, which illustrates to an exaggerated scale the effect of increasing speed for a particular value of the damping factor $a = b/b_{crit} = 0{\cdot}15$.

Starting with the mass revolving at a very slow speed (thin line circle), *R* and *C* practically coincide ; at a certain distance therefrom is *G*. But as soon as the rotor is running, the unbalanced centrifugal force comes into action and begins by tending to draw the rotor over in the direction *CG*, so that *C* diverges from *R*. As the speed is increased, the deflection *RC* gradually falls back behind the direction of the unbalanced force by an angle of lag in accordance with Fig. 48. So long as *k* which is now the ratio of the actual speed ω to the particular speed $\omega_{crit} = \sqrt{c/M}$, is well below unity, the effect is not very marked, but begins to be appreciable, *e.g.* under the amount of damping assumed in Fig 669 when $k = 0{\cdot}75$ (small dash circle), and the angle of lag approaches 30°. When $k = 1$ and resonance occurs, the deflection has not quite reached its maximum (small dot circle), and lags 90° behind *CG* or is at right angles to it. When *k* is above unity and the maximum has been passed, the reduction of the deflection at first is fairly rapid and then very slow, until the process ends theoretically at an infinitely high speed when the angle of lag reaches 180°, *R* coincides with *G*, and the deflection is *e*, the rotor revolving about its mass-centre.

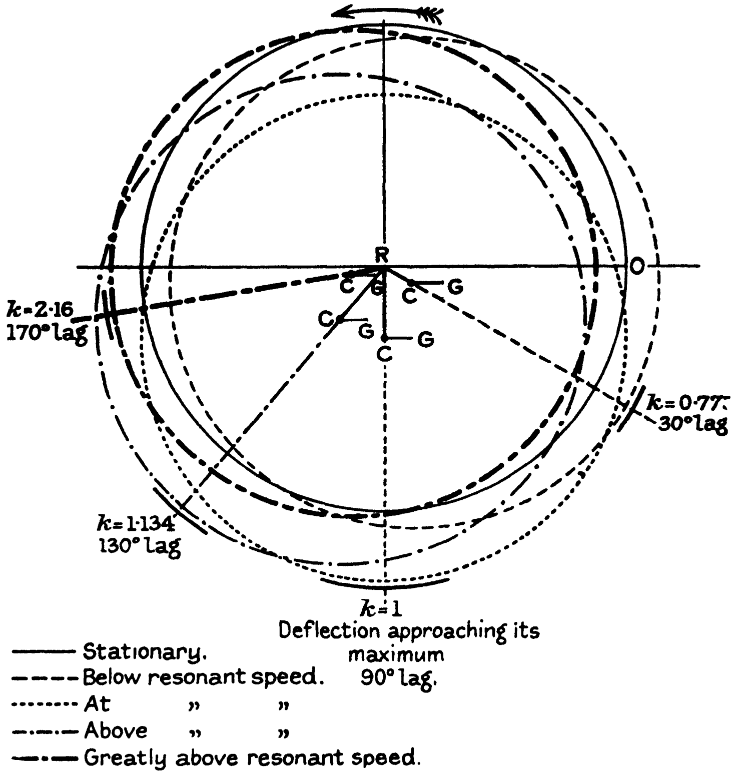
R
O
C
G
C
G
C
G
C
G
k=2·16
170° lag
k=0·775
30° lag
k=1·134
130° lag
k=1
Deflection approaching its maximum 90° lag.
——— Stationary.
– – – – Below resonant speed.
·········· At " "
—·—·— Above " "
—•— Greatly above resonant speed.

FIG. 669.—The three centres, *R*, *C*, and *G*, at speeds below, at, and above the critical speed, with $a = 0{\cdot}15$.

Inserting $a \times b_{crit} = 2a\sqrt{cM}$ for *b* in the above equation, $k\omega_{crit}$ for ω

and ω_{crit}^2 for c/M, the ratio of d, the deflection, to the original eccentricity e is found to be

$$\frac{d}{e} = \frac{k^2}{\sqrt{(1-k^2)^2 + 4a^2k^2}}$$

from which Fig. 670 is plotted. The above expression applicable when the original disturbing force increases as the square of the angular speed should be contrasted with equation (21*c*), where the applied force is of constant amplitude. It will be found that in contrast to the latter case, the maximum deflection is reached *after* the critical speed has been passed, *i.e.* when $k > 1$. The whole sequence of changes is again shown to a scale still further enlarged in Fig. 670, from which the rapidity with which the angle of lag alters in the resonant zone can be judged. Between $k = 0{\cdot}95$ and $k = 1{\cdot}05$, *i.e.* 5 per cent. below and above the speed of resonance, the angle of lag with $a = 0{\cdot}15$ passes from 71° to 108°, covering a zone of 37°; with a lesser degree of damping or $a = 0{\cdot}1$, from 63° to 116°, or 53°; and, in practice, such or even less may be the damping furnished by air friction and oil friction in the bearings.

§ 48. The balancing of rotors.—Now if the bearings of the rotor could be regarded as rigidly fixed during balancing and an independently held pencil were brought up so as just to touch lightly the surface of the rotor, the markings would be similar to those shown on Fig. 669 by the short lines drawn outside their respective circles, and by running the rotor in opposite directions, the line of the heavy side could be discovered. Below resonance the bisector of two markings for precisely the same speed in each direction would indicate the heavy side; above resonance, it would indicate the light side. Further, it would be a matter of indifference at what part of the circumference of the rotor the pencil were held; the same markings would result. But, unfortunately, the conditions during the actual operation cannot be rendered so simple. The marking must be made on some turned portion of the shaft at the ends near the journals, and the bearings vibrate, even if not specially arranged to do so; it is, therefore, rather the oscillation of the bearings that determines the marking. In an ordinary pedestal bearing the freedom to vibrate would be greater in the horizontal than in the vertical plane, and the movement of the shaft end would be elliptical. If the pedestal is mounted on balls and spring-controlled, the movement would approach straight-line motion in the horizontal plane; the horizontal displacement would then be as in the expression for x (p. 456), and the angle of lag ϕ could not be determined.

In the Lawaczeck-Heymann balancing machine[1] the journal is carried in a bearing mounted on the top of a thick flat vertical steel plate, held at the bottom and free to vibrate as a spring; the rotor is run up above the speed at which the spring resonates, and a clutch is disengaged. As the rotor runs down a pencil draws a spiral diagram on a sheet of paper on the end of the shaft. With reversed direction of rotation, a similar spiral diagram is obtained,

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[1] *Cp. Engineer*, Vol. 134, p. 201. For other machines, see H. Heymann, *E.T.Z.*, Vol. 40, p. 234 ff., H. D. Fisher, *Power*, Vol. 57, p. 480 (Akimoff machine), J. Bellisson, *Rev. Gen. d'Él.* Vol. 13, p. 693.

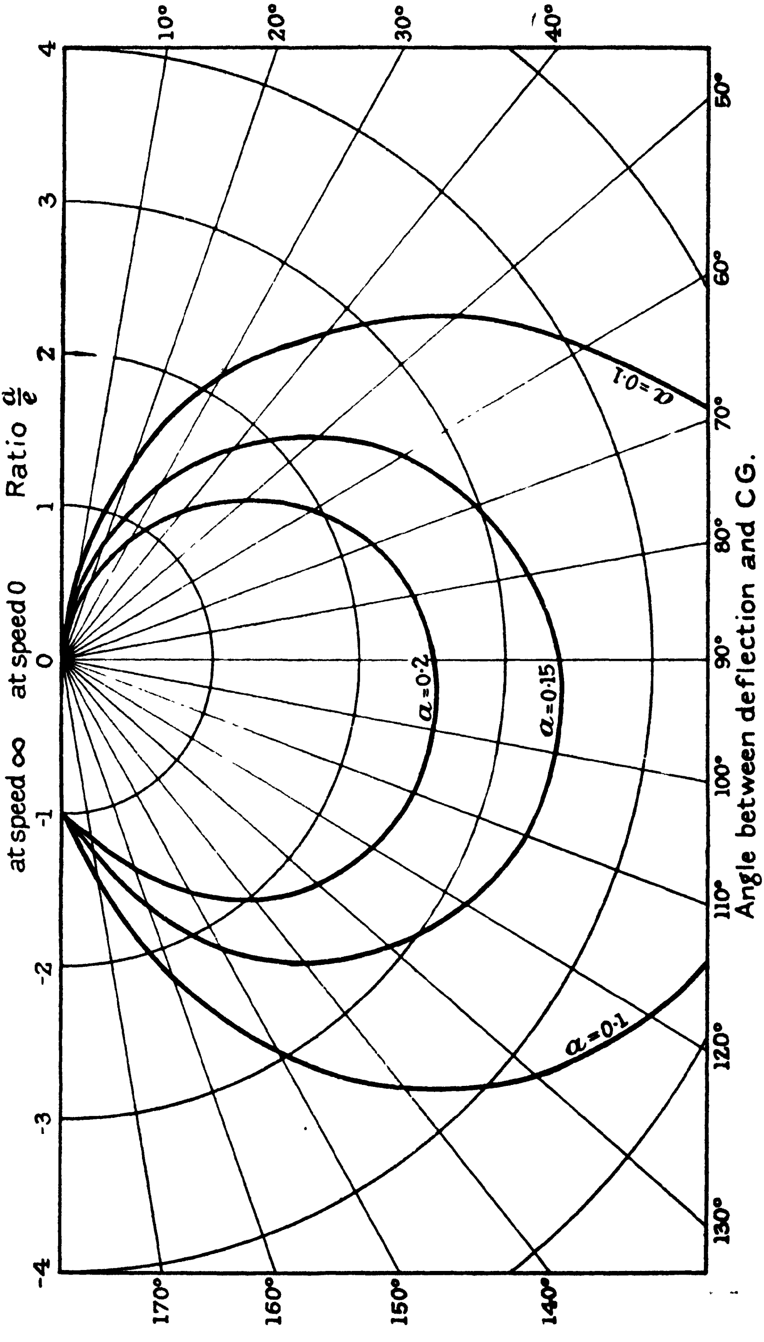

FIG. 670.—The ratio of the deflection d to the original eccentricity e for different values of k and a.

not overlapping, but a mirror of the first about a definite axis which indicates the line of the out-of-balance mass-centre.

A further difficulty lies in the fact that the want of balance may vary at different speeds, due to slight shifting of the rotor coils and elastic movements under great centrifugal force. While theoretically it would be best to balance at the critical speed, if it can be done, yet practice varies in this respect. In any case, however, in order to enable the rotor winding to take up its final position so far as possible, it should be run up to speed or to an overspeed, when hot, before balancing.

With a rotor constructed of thin laminations threaded on the shaft, the effective moment of inertia is practically only that of the shaft; the critical speed then loses its definiteness, since the laminations absorb part of the vibration energy.

The second critical speed in the case of turbo-rotors is considerably higher than twice the fundamental critical speed, and the ratio of the former to the latter may be about $2^{3/2} = 2{\cdot}83$. If, as would be the case under the action of gravity, the deflection is proportional to $\sqrt{L^3}$, and the shaft deflects in two halves with a node in the middle, so that $L' = L/2$, this result is reached, and in practice it is found that the second critical speed is about 2·6 to 2·8 times the first. It is not often, therefore, that it needs consideration, since if the first critical speed is 70 per cent. or less of the running speed, the second critical speed would only be reached through an accident.

During rapid acceleration, as when passing quickly through the critical speed, the conditions are modified by the appearance of mechanical hysteresis, and become highly complex, so that above it is presupposed that the speed is held constant until the shaft has had time to reach its steady condition.

§ 49. The predetermination of the first critical speed.—Although of value from an introductory point of view, the preceding results of § 47 have presupposed and are only strictly applicable to a particle of mass M rotated round a centre point with concentrated centrifugal force and under restraint from a concentrated controlling force, c. Under these conditions

$$\omega_{crit} = \sqrt{\frac{c}{M}} = \sqrt{\frac{c \times 32{\cdot}2}{W}}$$

$$N_{crit} = \frac{30}{\pi}\sqrt{\frac{32{\cdot}2\,c}{W}}$$

where W is the weight in lb. and c is the controlling force in lb. per one foot of deflection. But actually the unbalanced centrifugal force is distributed, it may be quite irregularly, along the shaft, and so also is the mass. Even when c is interpreted as the controlling force in lb. per one foot of deflection at its maximum point due to the stiffness of the shaft, and is identified with F/y, where y is the maximum deflection found to be due to some distribution of unbalanced centrifugal force, the proportion of the total centrifugal force which is to be taken as F concentrated upon the mass of the rotor remains doubtful.

There is, in fact, a fundamental difference between the cases of a particle and of an actual shaft, inasmuch as the displacement at any section of the latter is not necessarily due to nor proportional to the unbalanced force on that section. The above equations are not, therefore, valid for the predetermination of the critical speed of the actual rotor.

Two speeds must be carefully distinguished. About the centre line of the deflected shaft, which at some moment has the deflection $y = RC$ from the centre-line between the bearings in some particular direction, the iron and copper mass is rotating with the constant angular velocity ω, which is the normal running speed as ordinarily understood. But at the same time the deflected centre-line of the shaft may itself also be revolving round the centre-line between the bearings in an ellipse or in a circle at some constant angular velocity ω', which may or may not be the same as ω. It is the latter motion that constitutes whirling,[1] and there is no inherent connexion between ω and ω'. The value of ω' and its relation to ω will be considered later.

At the outset during the process of design the sole datum available for calculation is the rotor in a state of perfect balance to which the structure, when built, only approximates. It must be premised, therefore, that by the critical speed ω_k is here to be understood the speed which renders possible the stable deflection of a perfectly balanced shaft if damping resistance and internal friction or viscosity be supposed removed, the inertia forces being just balanced by the elastic controlling forces ; this unique speed is a mathematical function of the properties of the shaft, which must be calculated for it as if perfectly balanced,[2] since the extent and location of the out-of-balance are entirely unknown. Actually the deflection in this case is indeterminate, and it is the addition of unbalanced loads and of damping and friction which finally determine the speed at which maximum deflection of the real rotor occurs and the magnitude of the actual maximum deflection.

A perfectly straight and perfectly balanced shaft, free from internal hysteresis losses, *i.e.* perfectly elastic, will fail to start whirling at any speed, and, if forcibly deflected, it will quickly return to its initial straight condition. But in the absence of damping and internal hysteresis there is one particular shape of deflection curve and one particular speed which are related together in a special and peculiar manner; for, if the shaft were forcibly deflected to this particular shape when running at its critical speed without damping, it would continue to run, retaining this shape. The conditions presupposed are ideal, as shown by the requirement that no damping or hysteresis must be present, but the bearing of this ideal case and its practical importance will be more apparent later. The particular shape in question must be such that, whatever the absolute value of the deflection y_x at any point x, the forces from the unbalanced centrifugal loading at every section are exactly balanced by the controlling forces due to the stiffness of the shaft; and it can be found by solution of the differential equation

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[1] The possibility of the two motions and their independence seem indisputable from the experiments described by Dr. B. L. Newkirk in *Gen. El. Review*, Vol. 27, 1924, p. 169, to which the reader is especially referred.

[2] For the views here put forward as to critical speed, the writer is greatly indebted to Mr. S. Neville, whose valuable help is hereby acknowledged. At the same time, the writer has freely availed himself of the arguments in regard to internal friction given by Dr. B. L. Newkirk, *loc. cit.*, p. 172. See also A. L. Kimball, *Gen. El. Rev.*, Vol. 27, p. 244.

$$\frac{d^2}{dx^2}\left(EI_x \frac{d^2 y_x}{dx^2}\right) = m_x (\omega')^2 y_x \quad . \quad . \quad . \quad (297)$$

If w_x is the weight in lb. per inch run at any section, $m_x = w_x/g$ is the mass per inch run in engineers' units, and $m_x(\omega')^2 y_x$ is the centrifugal load per inch length of shaft at any section x, due to the speed ω' at which the deflected shaft is revolving round the centre line of the bearings, y_x being the distance between shaft axis and bearing centre-line. EI must be retained within the bracket, since the moment of inertia I is a function of x, as also is w_x.

Turning to the mathematical expression, the terminal conditions are

$$\left.\begin{aligned} y &= 0 \\ \text{and } \frac{d^2 y}{dx^2} &= 0 \end{aligned}\right\} \text{at} \left\{\begin{aligned} &x = 0 \\ &x = \text{the length between bearings} \end{aligned}\right.$$

Either, then, all the constants of integration are zero, and there is no deflection; *i.e.* the shaft remains straight, and $y_x = 0$, so that ω' becomes meaningless, since the shaft axis is not rotating round the centre-line between the bearings.

Or, the shaft is deflected and all the constants of integration can be eliminated; one particular value for ω' is found for which the constants are indeterminate, *i e.* y_x may have any value.[1]

By this second alternative, the value for ω' is precisely defined, and will now be symbolized as ω_k; when the degrees of out-of-balance and of damping are both small, it is very closely the speed at which the actual maximum deflection will occur, but so far as the above mathematical equation alone is concerned, it is for the perfectly elastic and balanced shaft the only possible value of ω'; if with the same shape of deflection curve the ideal shaft were forcibly compelled to revolve round the centre-line between the bearings at some other speed, and the external constraining force were suddenly removed, its speed would instantly rise or drop to ω_k (but at an altered radius to retain the same total energy), just as a pendulum, forcibly vibrated, when the external force is withdrawn, immediately resumes its own natural frequency of vibration.[1] The speed of rotation ω round the shaft centre-line is immaterial to the equation, and, in fact, it might be zero.

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[1] As stated by Mr. H. A. Webb, *Engineering*, Vol. 104, p. 455, § 2 (2nd Nov., 1917).

The solution of the particular shape of deflection curve and thence of the particular speed ω_k can be obtained by a process of trial and error now to be described. In the first instance some curve of y_x and absolute magnitude of the maximum y must be assumed after the shape of Fig. 671 (*cp.* Fig. 674).

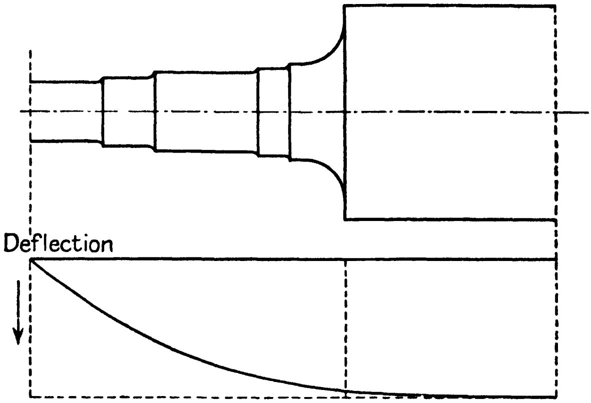

FIG. 671.—Deflection curve of turbo-alternator rotor.

Upon a horizontal base-line representing the shaft axis (*cp.* Fig. 675*a*), erect ordinates of $m_x y_x$ to form a "load line" for the centrifugal force per inch run for a speed of 1 radian per second. Integration of the area under the "load line" gives the total centrifugal force f for the speed of 1 radian per sec., which force acts as a resultant through the centroid of the area. Let the distance of a vertical passing through the centroid from the centre of the left-hand bearing be a, and from the centre of the right-hand bearing be b; then th reaction of the left-hand bearing is $R = f\frac{b}{a+b}$. At any point x the shearing force is

$$S_x = R - \int_0^x m_x y_x \, dx$$

By plotting the values of S_x as ordinates to points along the horizontal base-line, a "shearing-force curve" is obtained and integration of the area enclosed by this with the base-line for any distance x gives the bending moment at x in lb.-ins., *i.e.*

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[1] It will be understood that under the idealized conditions assumed, the rotor in the above case, when once started rotating round the centre-line of the bearings, requires no force to maintain its rotation, the accelerating force towards the centre being supplied by the stiffness of the shaft (*cp.* the solar system).

$$B.M. = \int_0^x S_x\, dx$$

which can in turn be plotted relatively to the base-line as the "bending moment curve." Integration of the area of the latter curve from the centroid towards either bearing will then give a curve proportional to the slope dy/dz at any point z reckoned from the vertical through the centroid, and integration of the slope curve from either end will give y_x (*cp.* Fig. 675*b*). As in the ordinary treatment of the deflection of beams in text-books of mechanics, the slope of the beam is dy_x/dx,

the bending moment is $EI_x \dfrac{d^2 y_x}{dx^2}$

the shearing force is $\dfrac{d}{dx}\left(EI_x \dfrac{d^2 y_x}{dx^2}\right)$

and finally

$$\frac{d^2}{dx^2}\left(EI_x \frac{d^2 y_x}{dx^2}\right) = m_x y_x$$

Successive integrations of the last differential equation with proper constants of integration added, once, twice, thrice, and four times would thus yield the shearing force, bending moment, slope, and deflection for 1 radian per sec. But the *shape* of the deflection curve thus deduced for $\omega = 1$ can now be compared with the assumed curve. If they agree in the sense that multiplication of the deduced curve throughout by a constant k makes it coincide with the curve originally assumed, then a special speed $\omega_k = \sqrt{k}$ and a special shape of curve have been found , that it is an unique speed is shown by the consideration that if the assumed deflection curve, while remaining of the same shape, has its ordinates *e.g.* halved, all the areas of the curves to be integrated are halved, and multiplication of the deduced curve by the same value ω_k^2 will again cause agreement. If there is discrepancy between the shapes of the deduced and assumed curve, the above processes must be repeated with adjustment of the original assumption until agreement is reached. A shape is thus found for which, as the mathematical expression shows, the absolute value of the deflection is indeterminate, the centrifugal forces at the special speed ω_k and the controlling forces from the stiffness of the shaft being exactly balanced, whatever the magnitude of the deflection.

The idealized shaft has now to be brought into closer accordance with the practical rotor by the successive addition of (1) mechanical damping, (2) internal friction, and (3) a cause to start the whirling.

(1) Applying mechanical damping by air friction to the whirling shaft as originally assumed (whether itself rotating or not) will have the effect of making the damped critical speed slightly less than ω_k. But work is now done, so that the whirl, when artificially established, will quickly die out in the above hypothetical case of the ideal shaft suddenly set free.

(2) Following the analysis of Dr. B L. Newkirk's article, it is evident that if an element of the material of the deflected shaft *a* (Fig. 672*a*) is alternately on the concave and on the convex side of the shaft, it is subjected to alternate compression and stretching. This is the case when the shaft is not itself rotating ($\omega = 0$) or ω is less than ω_k, and when ω is greater than ω_k.

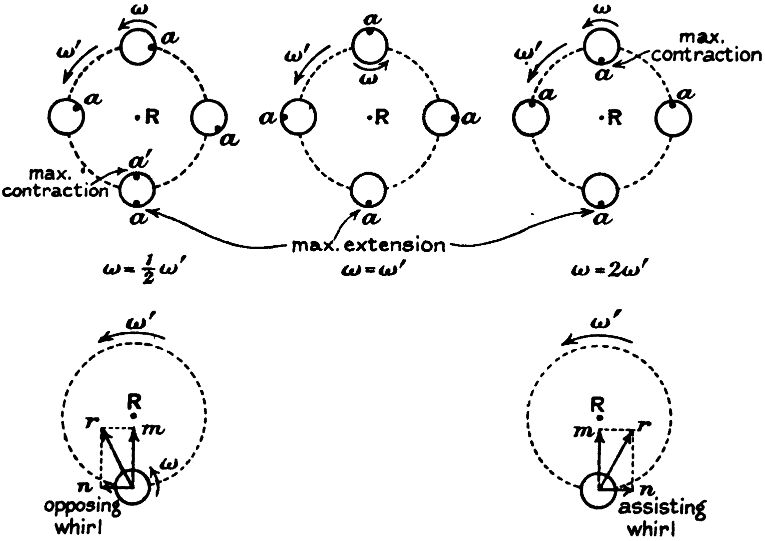

FIG 672*a* —Speed of rotation half, equal to, and double the speed of whirl.

But if $\omega = \omega_k$, a shaft element which is at any moment in a particular position on, say, the concave side remains at all other moments in the same position on the concave side (intermediate case in Fig 672*a*). There is then no internal friction, and we return to the first ideal case, in which, without damping, the whirling persists for ever.

But for all other speeds of rotation, a shaft element situated initially on the neutral plane, if becoming shorter as rotation proceeds, exerts more than its true elastic longitudinal thrust by reason of the additional resisting force due to internal friction. After reaching its maximum contraction, it exerts less than its true elastic longitudinal thrust as it recovers its neutral length owing to the internal friction opposing the change in length. Contrariwise when, as rotation proceeds, it begins to be stretched, it exerts more than its true elastic longitudinal pull through the additional resistance from friction, and, lastly, after reaching its maximum extension, it exerts less than its true elastic pull. The consequence is that the shaft rotating with $\omega \neq \omega_k$ (or non-rotating, but whirling), when regarded as a beam, is stiffer along one plane *oq* (Fig. 672*b*) inclined at 45° to the deflection plane in which extension and contraction reach their maxima than along a plane *op* also inclined at 45° to the deflection plane, but at right angles to the first. The resultant of the two component restoring forces *or* due to the stiffnesses is then not in line with or entirely opposed to the deflection ; when transferred to Fig. 672*a*, it is seen that it has itself a component *on* at right angles to the deflection, *i.e.* tangential to the circle of the whirl.

The difference between $\omega < \omega_k$ and $\omega > \omega_k$ (apart from the difference in frequency of alternate extension and contraction) is that when the shaft is rotating at a lesser angular velocity than the speed of whirl, the same shaft elements pass through extension and contraction in an order the reverse of that which holds when $\omega > \omega_k$. The consequence is that the resultant restoring force appears ahead of instead of behind the deflection; the component not balanced by the deflecting force, but tangential to the whirl, thus opposes the whirl when $\omega < \omega_k$, and assists the air-friction in damping it out. But when $\omega > \omega_k$, it assists the whirl; at some speed, therefore, it may counterbalance the air friction and cause the whirl to build up.

(3) The initial cause starting the whirl may be either (*a*) an external shock (subsequently reinforced when $\omega > \omega_k$ by the cramping effect of a hub fitting closely on the shaft, as in the cases experimentally investigated and described by Dr. Newkirk), or (*b*) an unbalanced load due to the mass-centre of the rotor being eccentric to the shaft centre. For our present purpose, case (*b*) is alone in question; and, in this case, since the unbalanced load rotated by the driving power at the speed ω produces a centrifugal force causing the shaft to whirl at that speed, it follows that the speed of whirl must be the same as the speed of rotation, *i.e.* $\omega' = \omega$.

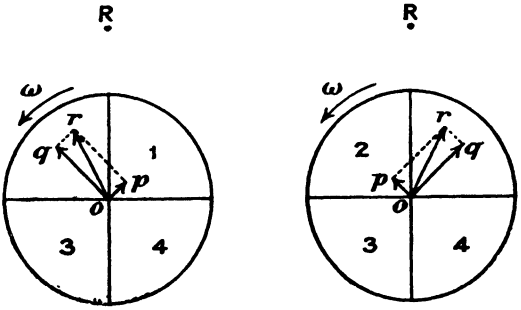

FIG 672*b*.—Resultant force arising from unequal stiffness in planes *op*, *oq*.

For every section of the unbalanced shaft when rotating with damping, a diagram, similar to Fig 668, giving the relative positions of *R*, *C*, and *G* for the eccentric mass per inch run, exists; it is not, however, available for calculation owing to the inter-connexion of the successive sections, and *G* might appear at very different distances from *R*, or even on either side of *C* at different parts of the shaft situated close beside each other. Strictly speaking, to overcome this difficulty in the treatment, it would be necessary to start with all the out-of-balance forces, assumed known and resolved along two planes, an *X* plane and a *Y* plane, at right angles to each other. At any section along the shaft the total disturbing force per inch run, due to the speed and want of balance, is $m_x\omega^2$ (*RG*), where *RG* is the distance between the centre-line of the bearings and the mass-centre. This total force has to balance not only the elastic restoring or controlling force due to the stiffness of the shaft, but also the damping resistance. The only component, therefore, that can cause bending of the shaft is that which is in phase with the final deflection. But a complexity in the problem must here be pointed out, viz., that just as the deflection at one section may be caused by unbalanced forces at another section, so also the damping resistance on one section may be overcome by a forward lead of unbalanced forces on other sections, the shaft between assuming in some measure a sinuous form. The same holds equally for the component systems at right angles to each other. Considering separately, therefore, the unbalanced forces, say, in the *X* plane, lagging behind them by some angle η which is not necessarily uniform, but varies along the shaft, lies the deflection, and the controlling force, due thereto owing to the stiffness of the shaft, equals the component of the unbalanced *X* force along the line of the deflection less the damping component of the unbalanced *Y* force. Completing the analysis for both systems, and combining the deflections after trial and error, some shape of deflection curve fulfilling the required conditions would be determined, but whether the speed was actually the critical speed would still be indeterminate, since there would be no uniform angle of lag, ϕ, along the whole length of shaft, which could be identified with 90° for some speed and shape. Indeed, the critical speed itself might be difficult of establishment under this procedure, owing to the shape of the deflection curve altering at different speeds according to the position of the unbalanced loads ; thus, if these are on opposite sides of the shaft, the first critical speed might almost be masked on a curve of deflections plotted against speeds, as this curve rose towards the second critical speed.

But for a simplified treatment of approximate accuracy, it must suffice to assume that the final deflection for the first critical speed

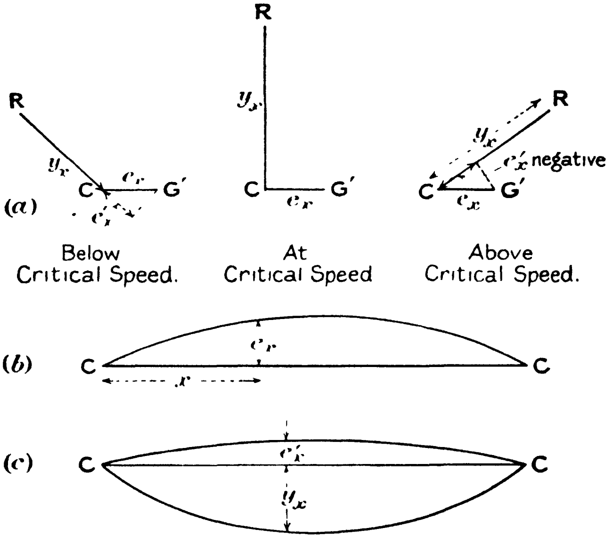

FIG 673 —The curves of deflection and inferred bending force.

lies in some one plane which also contains the running axis *RR* through the centre of the bearings, and that its continuous curve is of the simple bowed type shown in Fig. 673 (*c*). For each section there may then be inferred after the analogy of Fig. 668 a corresponding position of a virtual mass-centre *G′*, which takes into account the effect of the continuity of one section with another. Whether the curve containing the several positions of *G′* lies in one plane at some fixed angle $CRG' = \theta$ ahead of the deflection plane, or whether with real values given to the eccentricity e_x, θ varies along the shaft's length, in either case the component of the disturbing force that actually causes bending and lies uniformly in the plane of the deflection is $m_x \omega^2 (RG') \cos \theta$. Now according to whether the speed is below or well above the critical speed, $(RG') \cos \theta$ exceeds or is less than *RC*, and the difference e_x' is the projection of e_x on RC, *i.e.* $e_x \cos \phi$, where ϕ is the angle between CG' and RC produced (Fig. 673 (*a*)). The deflecting force is, therefore, $m_x \omega^2 (y_x \pm e_x')$, as shown plotted in Fig. 673 (*c*) along the centre-line of the bearings for a speed below the critical value when e_x' is positive, *i.e.* additional to y_x, the deflection. It will now be noticed that as the speed is increased up to the critical speed, the deflection increases and e_x at any section bears a decreasing ratio to y_x. More than this, the projection of e_x upon RC, *i.e.* e_x', is a smaller and smaller part of e_x, until finally it vanishes when RC is at right angles to e_x. As soon as this critical speed is passed, e_x' becomes negative, and although the deflection at first continues to increase with increasing speed, with a small amount of damping it quickly reaches its maximum and begins to decrease. In the differential equation for the deflected axis of the shaft below and above the critical speed (equation 297) the right-hand side would be

$$= m_x \omega^2 (y_x + e_x') \text{ for speeds below the critical speed}$$

and

$$= m_x \omega^2 (y_x - e_x') \text{ for speeds above it.}$$

When passing through the critical speed, e_x' vanishes, and we return to precisely the same equation (297) as for the balanced shaft without damping, viz.,

$$\frac{d^2}{dx^2}\left(EI_x \frac{d^2 y_x}{dx^2}\right) = m_x \omega^2 y_x$$

which must for the reasons given above yield the unique value $\omega = \omega_k$, and for which the solution has been found. There is justification, therefore, for the view that our ignorance as to the exact location and degrees of eccentricity and the impossibility of specifying precisely the relation of the disturbing force at each section to the deflection plane need form no bar to the discovery of the critical speed which will with small damping give little short of the maximum deflection.

It remains to add some further notes on the method of applying the solution proposed above.

§ 50. The procedure for determining the critical speed and example.—When the number n of portions of the shaft having practically the same loading under the assumed deflection is not large, the reaction R of the left-hand bearing can be found by the usual method of a polar diagram for n parallel forces. But preferably integration of the whole area in steps by planimeter enables not only the value of the total unbalanced centrifugal force f to be found, but its position corresponding to one-half of the area to be located. Of course, with a rotor symmetrical on either side of a vertical through the centre of its core, only the half load-curve is required, and the area enclosed by this gives the reaction *R* in lb. immediately.

The practical shaft is of varying diameter, and the rotor core with its air-holes, air-ducts, etc., is not solid. Hence in the first place wherever a cross-section shows air-holes, slots, etc., the diameter *D* of an equivalent solid shaft for that portion must be determined by finding the smallest moment of inertia (about a diameter) of the actual cross-section from a drawing ; for the value of the deflection depends on the moment of inertia, *I*. But this is not all that is necessary.

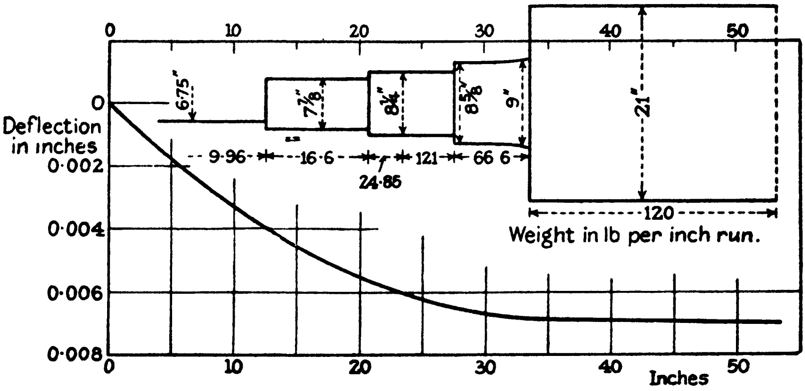

FIG. 674.—Weights per inch run and assumed deflection curve.

If the whole structure were simply a solid steel shaft of uniform diameter D' inches

$$(BM)_z = -EI\frac{d^2y}{dz^2}$$

where y is the deflection, z is distance reckoned from the vertical through the centroid towards the left bearing, $E = 29{,}000{,}000$ lb. per sq. in., and $I = \pi(D')^4/64$ in (inches)^4. Integration of the area of the bending moment curve from the centroid towards the left bearing would then give a further curve of $EI\,dy/dz$, where dy/dz is the slope of the beam at any point z from the centre, and integration of the "slope curve" (Fig. 675*b* from the left-hand side) would give $EI\,y$, whence y would be found. But the actual value of y being dependent upon the moment of inertia of any section, where the actual structure is stiffer and of diameter D larger than a shaft of some assumed uniform diameter D', the actual bending moment must be reduced in the proportion $(D')^4/D^4$, and where it is weaker and of smaller diameter, the actual bending moment must be analogously increased in the proportion $(D')^4/D^4$. This done, the assumed shaft of uniform diameter D' has been placed

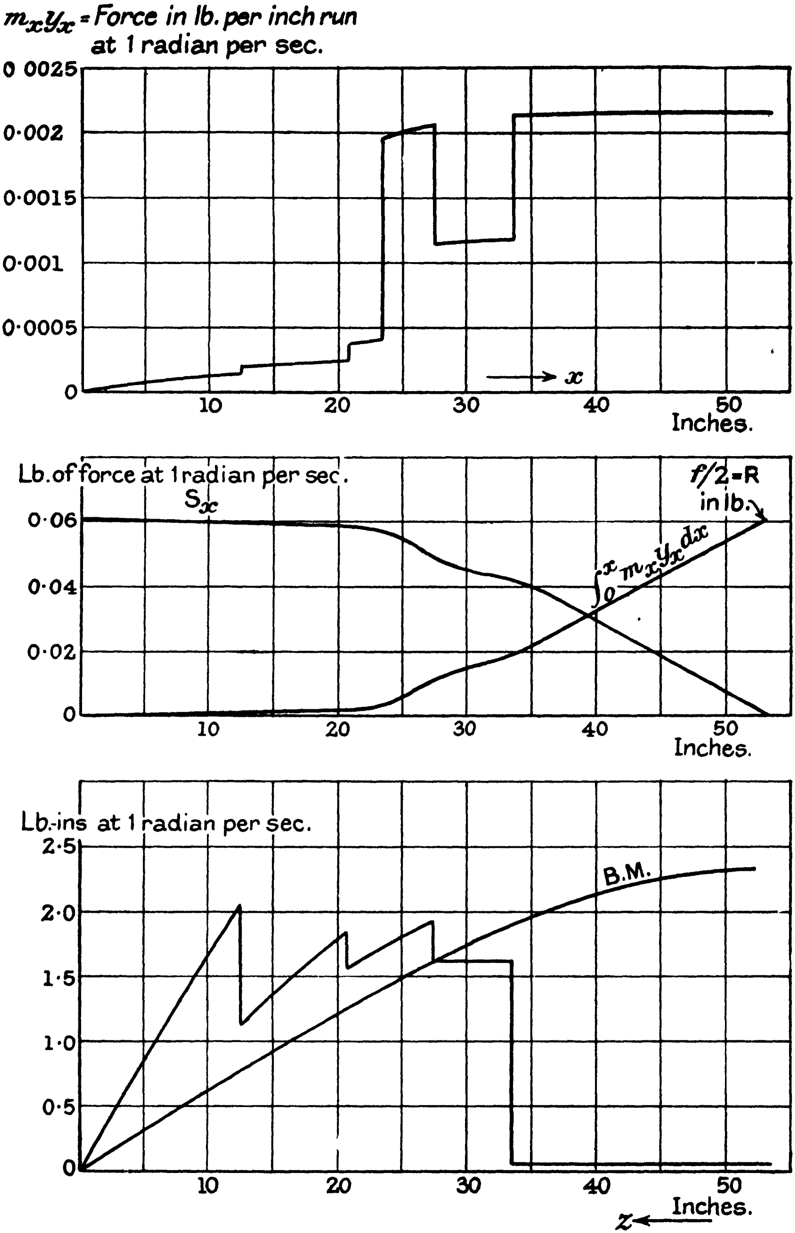

FIG. 675*a*.—Load per inch run, its integration, and shearing-force curve, all for 1 radian per second; bending-moment curve for shaft of uniform diameter, and the stepped inferred bending-moment curve for the different diameters.

under the same bending conditions as the actual shaft of varying diameter. For D' may be chosen the diameter near the body of the rotor core, and the values of the original bending-moment curve must be multiplied for each section by $(D')^4/D^4$. The " inferred bending-moment curve " having thus been obtained as an intermediate stage, the first integration of its area from the centre will yield a slope curve, and a further integration of this from the bearing end when divided by $\pi (D')^4 E/64$ will yield equally the curve of deduced deflection of the actual or of the assumed shaft.

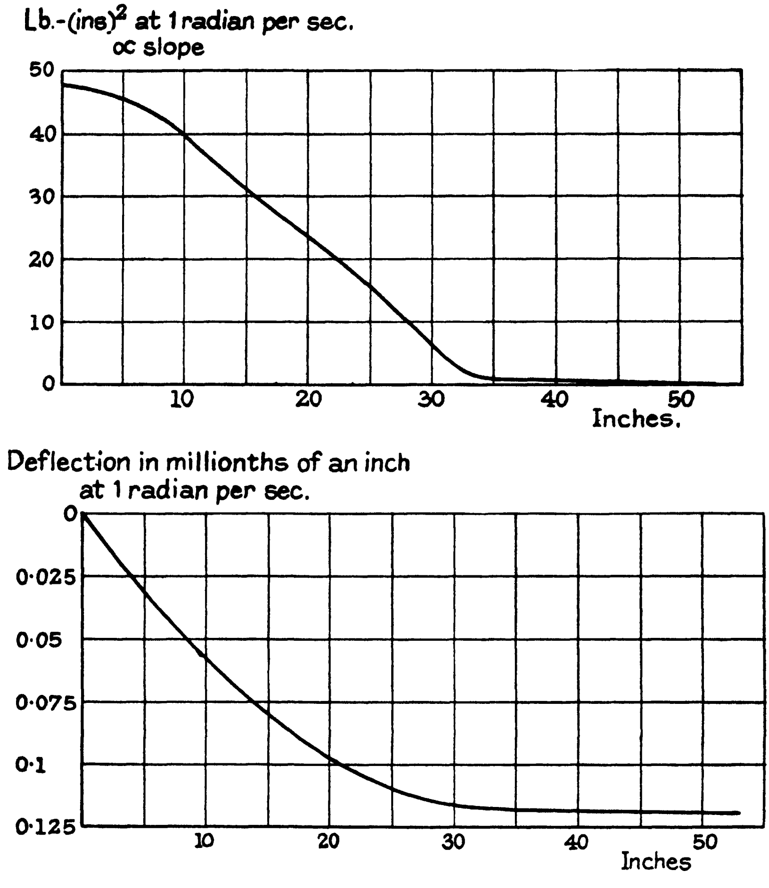

FIG. 675*b*.—The slope curve and the deflection curve, all for 1 radian per second.

If now the shape of the deduced deflection curve departs considerably from the assumed shape, the process must be repeated. The assumed and deduced shapes very quickly converge, so that with some experience the calculation, though tedious, is not unduly lengthy. When agreement is reached, so that the deflection curve for 1 radian per second multiplied by $k = \omega^2$ coincides with the assumed deflection curve, the critical speed ω_k is found. A shape has, in fact, been found for which, as already stated, the actual deflection is indeterminate, the centrifugal forces and controlling forces from the stiffness of the shaft being exactly balanced when the shaft is perfectly balanced to start with.

In the same way by assuming a double curvature or other types of deformation of the shaft, a second or higher critical speeds could be determined. If the mass centre-line passes on either side of the central axis of the shaft, such higher critical speeds might become of importance.

An example of the determination of the first critical speed of a rotor is given in Figs. 674–5. In the first of these figures is given the weight in lb. per inch run for each longitudinal portion of half of the rotor which is of uniform construction, the two halves of the rotor being similar, together with the assumed deflection in curve in inches.

The diameter of the solid shaft equivalent to the rotor core is taken as 21 ins., and this with the other real diameters is marked on the sketch of the shaft.

The products of weight × deflection after division by $g = 386{\cdot}4$ in inch units are plotted in the upper part of Fig 675*a*; integration of the area gives the total force from half the rotor at 1 radian per sec. as 0·0605 lb., or the reaction on the left-hand bearing, whence the shearing-force curve S_x is obtained. This integrated gives a total of 2·3132 lb.-ins., and the smooth B.M. curve of Fig. 675*a*; taking $D' = 8{\cdot}625$ ins., multiplication of the ordinates of the smooth B.M. curve by $(8{\cdot}625/D)^4$ yields the stepped inferred B.M. curve at 1 radian per sec. Integration of this gives 47·88 lb.-(ins.)^2, and a curve proportional to the slope (Fig. 675*b*). Integration of the slope curve and division by

$$\frac{\pi \times (8{\cdot}625)^4 \times 29{,}000{,}000}{64}$$

gives a deflection curve for 1 radian per sec., with a maximum of $0{\cdot}1187 \times 10^{-6}$in. Multiplication of the ordinates by $58{,}000 = 241^2$ reproduces so closely the deflection curve originally assumed in Fig. 674, that one is justified in accepting 241 radians per sec. = 2300 revs. per min. as the critical speed to a close degree of approximation.

§ **51. Operating details.**—In the operation of large generators, *unequal* expansion of the component parts of the rotor is especially to be guarded against, the ideal condition being the maintenance of the winding at a constant temperature of, say, 90° or 100° C. If, therefore, when a turbo-generator is shut down, its rotor becomes very cold, it should be run at a low speed with excitation sufficient to bring the temperature up to about 75° C., in about one half-hour before its speed is raised to the normal.[1]

To quench the field of the turbo-alternator in the shortest possible time in case of emergency when a breakdown occurs in the stator insulation, the breaking of the exciting circuit without any discharge resistance is the most effective means, but greatly increased stresses are thereby thrown on the insulation of the rotor winding and exciter (perhaps 1000 volts), and large currents flow through keys, end-rings, and shaft forming a damping winding.[2]

Reversal of the polarity of the exciter after a short-circuit may be prevented by a few turns of compound winding on the exciter field-magnet. An auxiliary winding separately excited from a station battery is also used for the same purpose. The exciter on the rotor shaft must not become unstable or show any tendency for its voltage to creep when its field-magnet is only lightly excited at very light loads and when synchronizing; saturation gaps are sometimes employed to secure stability under these conditions.[3]

Under short-circuit conditions at full speed dangerous local heating due to eddy-currents may occur, so that to dry an alternator on site, it is best to run it below half speed with current passing through the short-circuited stator winding.

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[1] W. J Foster, E. H. Freiburghouse and M A Savage, *Journ. Amer I E.E.*, Vol 43, p 930.

[2] For means for protecting generators, see J. A. Kuyser, *Journ. I.E.E.*, Vol. 60, p 761 with discussion, and for means for protecting the network and apparatus of distributing systems, see J Biermanns, *E T Z.*, Vol. 40, p. 593 ff.

[3] See especially J Rosen, *Journ I E.E.*, Vol 61, p 452.

CHAPTER XXXI

THE WORKING OF ALTERNATORS IN PARALLEL

§ 1. Parallel working of alternators.—The series operation of, say, two alternators, both as generators, is not possible unless they are connected together by a rigid mechanical coupling. If only connected together electrically, they exert no mutual control on one another ; if for any reason one machine falls slightly behind another in phase, the lagging machine has the greater part of the load thrown upon it, which causes it to lag still more, until finally the two settle down into exact opposition of phase, their E.M.F.'s neutralize each other, and no current flows.

Alternators can, however, be run as generators in parallel, the mutual control which they exert electrically upon one another being in most cases sufficient to keep them " in step," as it is termed. If two alternators be running at the same frequency, and further with equal E.M.F.'s rising and falling together, that is to say, if they *synchronize* in phase, their armature circuits may be coupled together, and they will continue to run in parallel, dividing the load between them in proportion to the power supplied to each. If a generator is to be thrown into parallel with another, or is to be joined to a network on which other generators are already working, then at any instant there should be no difference of potential between any pair of corresponding terminals which are to be united together, so that no cross current will flow between the machines due to such difference of potential. Thus there are required for parallel operation (1) *equality of frequency*, (2) *synchronism of phase*, and of less importance, (3) *equality of voltage*. The general principle of the connexions for working two low-tension 3-phase alternators in parallel is shown in Fig. 676. Machine A is at work, its triple-pole switch *S* being closed on the three leads, *a*, *b*, *c* of the distributing network. The current in one phase is read on the ammeter *I*, while the wattmeter *W* measures the power of one phase, its main terminals being in series with one main lead and its shunt coil being connected to the ends of one arm of the Y winding, *i.e.* between *a'* and the centre of the Y winding. The shunt-wound exciter *E* supplies the exciting current to the field-magnet *M* with a regulating rheostat *R* in series. The machine B is now run up to its full speed, and its excitation regulated by means of the rheostat *R'* in series with its field-winding until the voltmeter *V'* gives the same reading as the voltmeter *V*. The virtual voltages of the two machines are now alike, and assuming their E.M.F. curves to be similar, so also are the maxima values of the E.M.F.'s. A further and finer regulation of the speed and phase must now be made by means of the phase lamps *PP* belonging to machine B. It will be seen that in virtue of their connexion they are in series between two arms *a′* and *c′* of the armature winding on the one side and two bus bars *a* and *c* of the network on the other side, so long as the main switch of B is open; they therefore indirectly measure by their light the difference of potential between these points, and when this is zero their light goes out, and the triple-pole switch of B may be closed. When the machines are completely out of step, in the arrangement shown, the voltage on each pair of lamps may rise to the sum of the terminal volts of two machines.

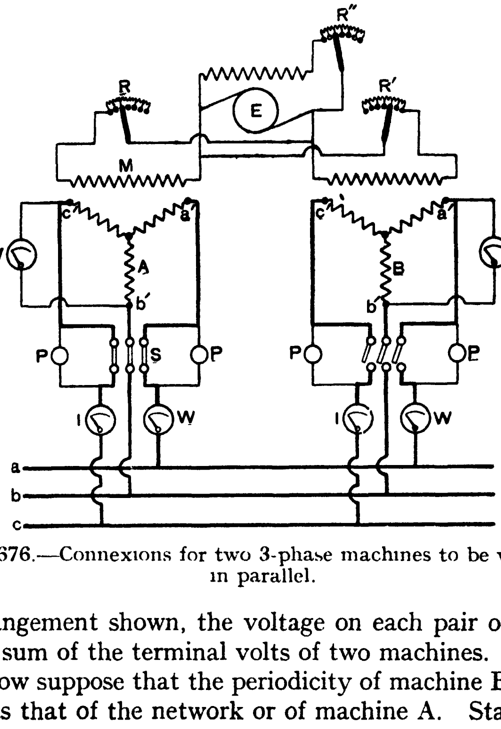

FIG 676.—Connexions for two 3-phase machines to be worked in parallel.

Let us now suppose that the periodicity of machine B is not quite the same as that of the network or of machine A. Starting from a point of time when the instantaneous voltage of both machines is zero, let the full-line curve E_1 (Fig. 677) represent the instantaneous voltage between the bars *a* and *c*, while the dotted curve E_2 represents the instantaneous voltage between the terminals *a′c′* of machine B. The algebraic difference of the two shown below by the heavy line e_c gives the difference of potential on the lamps, proportional to which is the current through them. From the nature of the resultant curve it will be seen that the lamps are traversed by an alternating current of which the crest value, and therefore also the virtual value, waxes and wanes; in other words, the lamps show alternate periods of brightness and darkness. The number of complete waves of the alternating current through the lamps itself coincides with the number of periods of the more rapid of the two periodic voltages, and one " beat " or pulsation of the light from complete darkness to maximum brightness and back to darkness occurs in the time in which the numbers of complete periods passed through by the voltage E_1 and the voltage E_2 differ by 1 (in our figure 8 and 7 respectively) ; *e.g.* if the frequency of the machine A be $f = 50$, and that of machine B is only 49, at the end of one second the difference of their numbers of periods will be 1, and the lamps will have passed through one pulsation. One beat per second

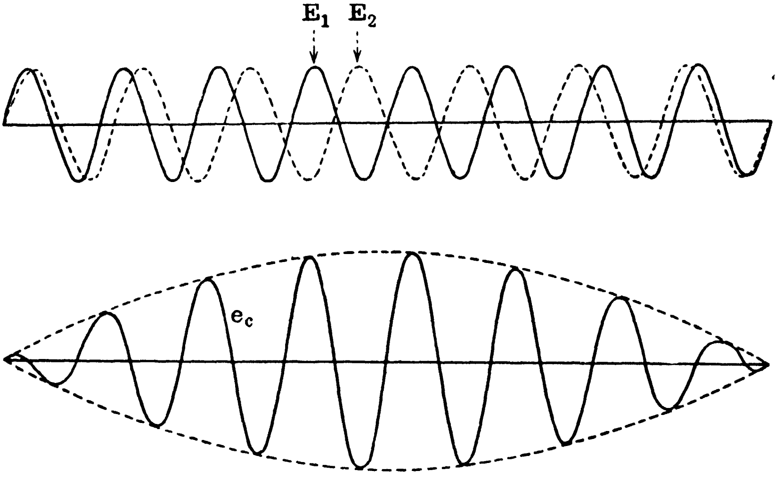

FIG. 677.—Pulsating voltage on phase-lamp.

of the light will, therefore, indicate a difference in the frequencies of 1/50 = 2 per cent. But by regulation of the driving power each pulsation of the light may easily be extended to, say, 5 seconds, when the difference of the frequencies will be only 2/5 per cent., or to 10 seconds with large machines. Complete synchronism may not be exactly obtained, yet if the triple-pole switch is now closed *in the middle* of an interval of darkness, but little cross current will flow, and the two machines will fall into step.

The two instruments of Fig. 676 enable a machine to be conveniently taken out of parallel as follows. The steam stop-valve or inlet gate of the turbine is closed until the wattmeter reads zero. If the ammeter still shows some cross-current passing, the field rheostat of the machine is altered until it is reduced to zero, when the main switch can be opened without disturbance to the machines still remaining at work and without spark.

§ 2. Phase indicator.—The arrangement of phase lamps shown in Fig. 676 is the simplest and earliest form of *synchronizer* or *phase* *indicator*, to ensure that the difference of phase between the two machines may not be too great at the moment when they are thrown into parallel. It is, however, only suitable for low-tension machines, and many other devices are employed based on the same or similar principles. Thus the method may be immediately adapted to high-tension machines, if, instead of the phase lamps being directly connected between the corresponding terminals, the primaries of two small transformers be inserted, to the secondaries of which the lamps or a voltmeter are applied as before. Or the secondaries of two small synchronizing transformers are connected in series with one or more lamps in circuit. A still more common variation is to employ a single transformer with two primaries wound on it; one of these p_1 is connected to the bus bars, and the other p_2 to the terminals of the alternator which is to be thrown into parallel, as in Fig. 678 which shows the arrangement for single-phase machines. A common secondary *s* has its terminals connected to an incandescent lamp, and the advantage is gained that the phase is indicated without any direct connexion of the machine to the network; when the phases are in synchronism, both poles of the incoming alternator B are for the first time connected to the bus bars by the closing of the double-pole switch. The primaries may be arranged to oppose one another when the machines are in step, so that the lamp is dark at the proper moment for closing the switch (synchronizing " dark "), or they may be arranged so that when the E.M.F.'s synchronize the lamp burns brightly (synchronizing " bright "). The latter is perhaps the best, since the moment of maximum brightness is better distinguished than the middle of intervals of total darkness, and, when synchronizing " dark,"

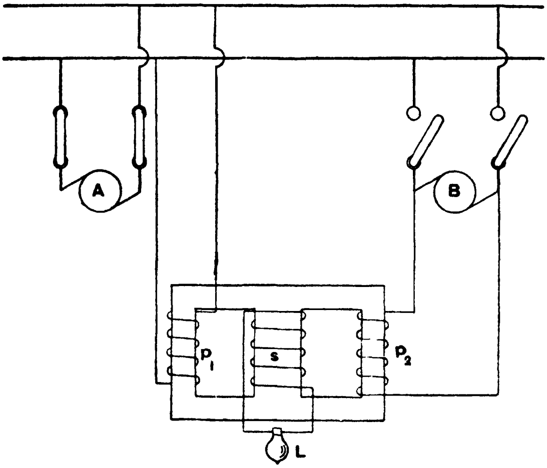

FIG. 678.—Diagram of phase indicator for parallel working.

rupture of the filament of the lamp might happen to coincide with the coming of the machines into step.[1] In either case the lamps should not be fully incandesced, but should be run below their true voltage in order that the eye may not become fatigued by their brilliancy. Small-candle-power high-voltage lamps are best, since their thin filaments heat and cool quickly with less time-lag behind the variations of current. A preferable arrangement is obtained with a hot-wire parallelling voltmeter in place of the lamps, the machines being switched into parallel when the voltmeter gives its maximum reading. The determination of this permits of greater accuracy, and the instrument has little or no time-lag.

But none of the above methods indicate whether the want of synchronism is due to the incoming machine running too fast or too slow, and the moment of exact synchronism is not definitely indicated, so that to effect a good parallel may take a considerable time. Hence various forms of synchronizer have been devised which possess the additional advantage of signalling whether the incoming prime mover requires to be slowed down or speeded up. One of the earliest of these was the Siemens and Halske 3-phase synchronizer, in which three lamps arranged in a triangle brighten in a clockwise or counter-clockwise order of succession according as the speed of the incoming machine is too high or low. They have, however, now given place to " synchroscopes " with a pointer which moves in one or the other direction according as the incoming machine is too fast or too slow. These are usually single-phase, and either depend on the action of a small induction motor with an iron armature magnetized by one machine and moving in a field supplied by splitting a phase of the other machine, or are of the dynamometer type like a single-phase power-factor meter with a fixed coil connected across one machine and moving coils at right angles connected across the other machine.[2] Automatic synchronizers[3] are also used in large stations for machines of great power, with electrically or electro-pneumatically controlled switches.

While in normal working it is sufficient to synchronize one phase of a polyphase machine, when a new machine is first set to work it is necessary to verify the connexions of each phase, or at least of two out of the three phases.

§ 3. Alternator vector diagrams.—The case of two synchronous alternating-current machines running in conjunction, whether both as generators on an external network or one as generator and the other as motor, or one as generator and the other as motor with the former also feeding an external network, may be represented diagrammatically in two ways, either (1) by a clock diagram of vectors whose directions and signs have reference to each of the two branch circuits *ABEFGA* and *CDEFGC*, which the two machines respectively form with the external network, or (2) by a diagram of vectors whose directions and signs have reference to the series circuit *ABEDCGA* existing between the two machines (Fig. 679).

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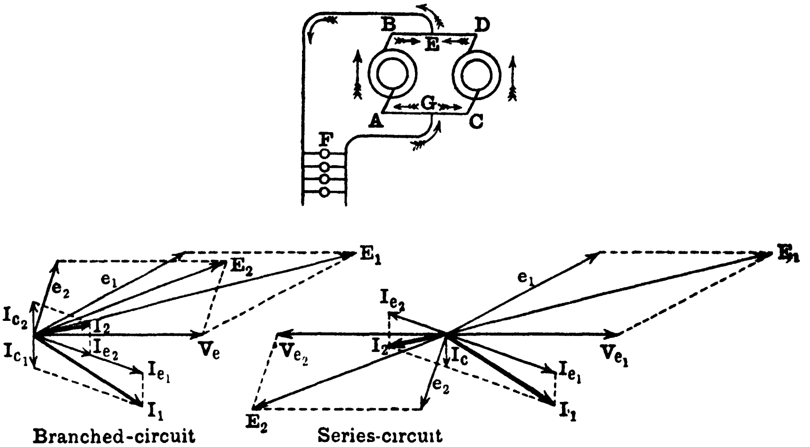
[1] But *cp.* F. A. Robbins, *Elect. World,* Vol. 70, p. 813; and E. Styff, *E.T.Z.*, Vol. 38, p. 461.

[2] See D. J. Bolton, *Electrical Measuring Instruments*, p. 261.

[3] *E.g.* the Brown, Boveri synchronizer used at Ritom (*cp. Brown, Boveri Rev.*, Vol. 10, p. 211), and Siemens-Schuckert apparatus (*E.T.Z.*, Vol. 42, p. 1098).



Branched-circuit diagram **Series-circuit diagram**

FIG. 679.—Two methods of representing alternators in parallel.

On the first method of representation the vectors of the terminal voltages which are necessarily equal in the two machines must also be shown as coincident; so also must the vectors of the currents I_{e1} and I_{e2}, supplied by each to the network when both are acting as generators, coincide in direction, although not necessarily equal in magnitude. But these currents may be only part of the total currents passing through the machines; there may be also a permanent "corrective" component[1] I_c flowing round the circuit *ABEDCGA* between the two machines in series, if their respective excitations and steam supplies are not so balanced that their terminal voltages are naturally equal. Or, if the phases of the induced armature E.M.F.'s are not at any moment correctly related to the excitations and steam supplies, there may be a temporary or fluctuating "synchronizing" component I_s tending to supply the necessary correction of phase; or, finally, one machine may be driving the other as a motor, so that the only current through the latter is a series current. The vectors of any such " series " current or component, whether corrective or synchronizing, must in the two machines necessarily have opposite directions in relation to the two branch circuits which are the basis of the present method of representation, and must be equal when only two machines are assumed, since the one supplies it to the other.

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[1] Any "cross current" round the circuit *ABEDCGA*, if it is permanent and due to stable conditions, is here regarded as "corrective." A similar current, if varying in magnitude from cycle to cycle, as arising from unstable conditions, is regarded as "synchronizing."

On the second method, the terminal voltage of one machine at *EG*, or the bus-bar voltage, appears as diametrically opposite to the terminal voltage of the other machine. The shares of the total external current which are supplied by each to the external network now appear as opposite in direction, while the vector of any corrective or series current has the same direction for both.

Method (1), which may be shortly described as the " branched-circuit " diagram, is the more appropriate to the case of two alternators working in parallel, and is thus to be adopted for our present purpose. Method (2), or the " series-circuit " diagram, is the more appropriate to the case of a synchronous motor driven by a source of alternating voltage, since, if the external network is suppressed, the diagram is reduced to the form most suitable to a transmission of power between a single generator and motor in series with a pair of intervening transmitting lines.

By the branched-circuit method the E.M.F. actually induced in the armature of each machine or E_t is to be regarded as the vector sum of the terminal voltage V_e and the voltage consumed by the impedance of the machine $e = z_a I$, but since the angle between the two components may be more than 90° when they are transferred to the common centre, either component may be larger than the induced E.M.F. which they combine to equal. In the series method of representation, especially when used for the synchronous motor case, the bus-bar voltage may be treated as combining with the induced E.M.F. of the motor to give a resultant E.M F. which is the voltage consumed by the impedance of the armature of the motor. The working of the synchronous motor and of the alternator in parallel is, in fact, intrinsically the same. *E.g.* an over-excited synchronous motor run at the far end of a transmission line, and absorbing a leading current so as to raise the power-factor, may from many points of view also be considered as a generator run in parallel with the station generators and supplying the necessary lagging current at the far end, so that the transmitting line is relieved from the necessity of transmitting anything but the working current.[1]

Thus the whole problem of the representation of the working of alternators in diagram form may be said to consist in finding the possible value which the third vector may take with given values of the other two vectors in order to form a closed triangle, and it is of the greatest importance to adopt a definite method of representation of the three vectors which may be applied to different problems without fear of confusion.

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[1] *Cp.* M. B. Field, " Idle Currents," *Journ. I.E.E.*, Vol. 37, p. 87.

§ 4. The power as dependent upon the phase angle between E_i and V_e.— The possible values of the impedance-voltage vector $e = z_a I$ are with constant values of E_i and V_e entirely dependent upon the phase angle γ between the induced and terminal voltages; further, for any one possible value of the current (except the minimum or maximum) there are always two possible phase relations between E_i and V_e, in one of which E_i leads before V_e and in the other lags behind it by the same angle. From the position of e is also fixed the phase of the current which lags behind e by the angle ϕ_a, so that what is obtained by an analysis of the possible values of e for different values of the angle γ is a knowledge of the different values which the *power* of the machine can assume, and whether this power is positive, *i e* the machine is acting as a generator, or is negative, when the machine is acting as a motor.

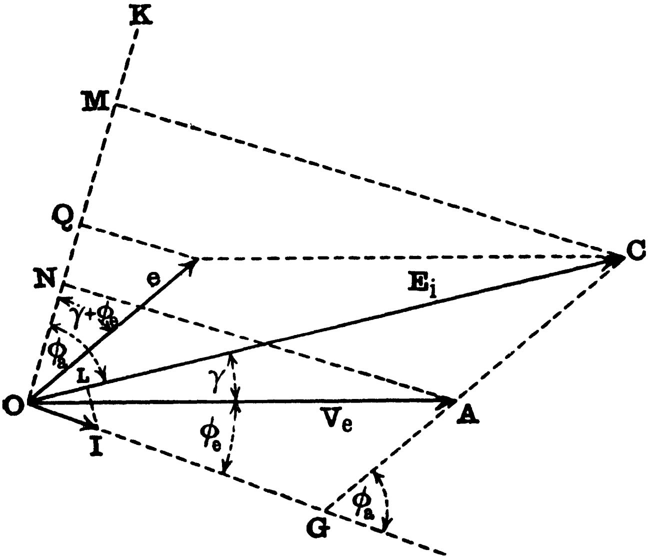

FIG. 680.—Total power of alternator as dependent upon variable angle γ.

The total rate at which energy is developed per phase by the alternator is $E_i I \cos(\gamma + \phi_e)$, and is, therefore, given by the product of the induced E.M.F. OC with the projection upon it of the current or OL (Fig 680) At the angle ϕ_a ahead of the current vector OI, let a line be drawn on which falls the E M F. consumed by the impedance z_a of the armature, *i e* AC, which is also shown as e when transferred to the centre. Draw OK at the angle ϕ_a ahead of the induced E M F.; this line is then also as much ahead of or behind e as E_i is ahead of or behind I; *i e* the angle between OK and e is $\gamma + \phi_e$. The projection of e upon OK is, therefore, $OQ = e \cos(\gamma + \phi_e) = z_a I \cos(\gamma + \phi_e)$. The component OQ divided by z_a thus gives the working component of the current or OL. This current can then be expressed in terms of the assumed constant quantities E_i and V_e and of the variable γ if OQ can be so expressed, and this is effected when it is seen that OQ is the difference of the projections of E_i and V_e upon OK. The angle between E_i and OK is ϕ_a, and between V_e and OK is $\gamma + \phi_a$; therefore, $OQ = OM - ON$

$= E_i \cos\phi_a - V_e \cos(\gamma + \phi_a)$. The total rate at which energy is developed by the machine electrically as a generator or mechanically as a motor is thus

$$P = \frac{E_i}{z_a}\{E_i \,.\, \cos\phi_a - V_e \cos(\phi_a + \gamma)\}$$

In precisely the same way is found the expression for the rate at which energy is delivered to the network, or is developed from it in the armature of the machine. Draw OK' ahead of the vector e by the angle ϕ_e, so that it is as much ahead of or behind e as V_e is ahead of or behind I (Fig. 681).

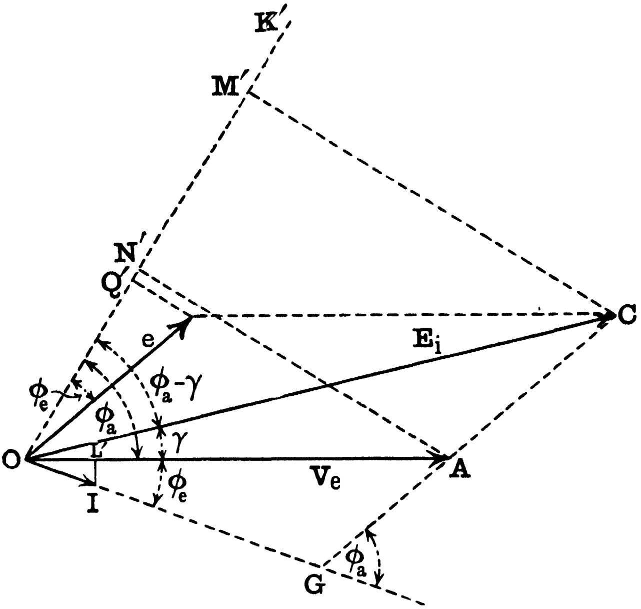

FIG. 681.—Useful power of alternator in terms of variable angle γ.

The rate at which energy is received by the network from the machine, or is received by the machine from the network, is $V_e I \cos\phi_e$, and is given by the product of the terminal voltage with the projection upon it of the current vector or OL'. The projection of e upon OK' is $OQ' = e \cos\phi_e = z_a I \cos\phi_e$; so that when this is divided by z_a it gives the current component OL' which is in phase with the terminal voltage. But OQ' can again be expressed in terms of E_i and V_e and of the variable γ when it is seen that it is the difference between the projections on OK' of E_i and V_e, i.e. $OQ' = OM' - ON' = E_i \cos(\phi_a - \gamma) - V_e \cos\phi_a$.

Therefore, the rate at which energy is delivered to or received from the network is

$$P' = \frac{V_e}{z_a}\{E_i \cos(\phi_a - \gamma) - V_e \,.\, \cos\phi_a\}$$

According as the energy is delivered from the alternator to the network, or *vice versa*, P' is positive or negative.

The algebraic difference between the above expressions must necessarily be the loss over the resistance of the armature, or $P - P' = I^2 r'_a$

$$= \frac{(E_i^2 + V_e^2)\cos\phi_a - 2\,E_i V_e \,.\, \cos\phi_a \,.\, \cos\gamma}{z_a}$$

The current is always a minimum when E_i and V_e coincide, or $\gamma = 0$, and is always a maximum when E_i and V_e are diametrically opposite, and $\gamma = 180°$.

By giving all values to γ in the above expressions, and plotting the results either by rectangular co-ordinates or by polar co-ordinates, a complete analysis is obtained of the possible combinations of the three E.M.F.'s in a given alternator with constant induced and terminal voltages, apart, of course, from the question whether all these combinations are equally practicable.

Unless $E_i = V_e$, and the machine is either a generator on open circuit or a synchronous machine which is being driven at synchronous speed by a continuous-current motor which supplies the losses from friction, etc., so that the synchronous machine is itself entirely neutral, there must always be some current flowing. The result of progressively increasing the angle γ though a complete cycle is then in general divisible into four stages, according as—

(1) P is positive, P' negative, the former increasing and the latter decreasing.

(2) Both P and P' are positive.

(3) P is again positive and P' negative, but the former is finally decreasing and the latter is increasing

(4) Both P and P' are negative.

Thus, if $E_i \lessgtr V_e$ there is some value of γ, say, γ_1, for which $V_e \,.\, \cos(\phi_a + \gamma_1) = E_i \cos\phi_a$, this value being an angle (negative) of lag of E_i behind V_e when $E_i > V_e$, and an angle (positive) of lead of E_i before V_e when $E_i < V_e$. For this value of γ, $P = 0$, and P' is negative; γ_1 will, therefore, form our starting-point in the cycle. There must also be some other value γ_2 similar to γ_1, but for which $E_i \,.\, \cos(\phi_a - \gamma_2) = V_e \cos\phi_a$, and P' in its turn becomes zero. Between these two limits we have the first stage (1); as γ is made larger than γ_1, i.e. as the angle of lag is made smaller or the angle of lead is made larger, P becomes positive and increasing, while P' is negative and decreasing. Thus, during this comparatively short stage both the mechanical energy absorbed by the armature of the alternator from its prime mover and converted into electrical energy, and also the electrical energy delivered to it directly from the bus bars or network, combine to heat the armature.

(2) From this point between $\gamma = \gamma_2$ and $\gamma = 2\phi_a - \gamma_2$ the machine is acting as a generator in the normal manner, supplying not only the whole of its own copper loss, but also a surplus of useful electrical energy to the network. E_i then precedes V_e, and the useful electrical output reaches its maximum when $\gamma = \phi_a$, and thence declines to zero when $\gamma = 2\phi_a - \gamma_2$ and $E_i \,.\, \cos(\phi_a - \gamma)$ again becomes $= V_e \,.\, \cos\phi_a$

(3) The third stage between $\gamma = 2\phi_a - \gamma_2$ and $\gamma = 360° - (2\phi_a + \gamma_2)$ resembles the first, in so far that both the absorbed mechanical energy of the generator and the energy received electrically from the network combine to heat the armature of the generator. But the former declines, while the latter increases, so that finally, when $\gamma = 360° - (2\phi_a + \gamma_2)$, P is again zero.

(4) In the final stage $V_e \,.\, \cos(\phi_a + \gamma)$ exceeds $E_i \cos\phi_a$, so that P is negative, and so also is P'. The network is, therefore, not only supplying the copper loss of the alternator, but is also driving it as a motor.

The total rate of development of energy by the alternator is a maximum when $\gamma = 180° - \phi_a$, and this value is positive, so that the machine is then acting as a generator.

If $E_i = V_e$, it is evident that both γ_1 and $\gamma_2 = 0$, and the first of the above four stages entirely disappears, so that in this case we have

(1) $\gamma = 0°$ to $\gamma = 2\phi_a$; generator supplying useful electrical output and its own ohmic loss.

(2) $\gamma = 2\phi_a$ to $\gamma = 360° - 2\phi_a$; both the absorbed mechanical energy and the electrical energy received from the network are expended in heating the alternator's armature.

(3) $\gamma = 360° - 2\phi_a$ to $360°$; *i.e.* for negative angles of lag from $0°$ to $2\phi_a$ the network not only supplies the ohmic loss, but also power to drive it as a motor.

Finally, if the armature reactance is high as compared with its resistance, so that, as is usually the case, ϕ_a approaches 90°, again both γ_1 and γ_2 become zero; and further, the intermediate stage also disappears, so that we simply have

(1) $\gamma = 0°$ to $180°$; generator. (2) $\gamma = 180°$ to $360°$; motor.

§ 5. Vector diagrams for steady conditions of working.—Before dealing with the problems of switching into parallel, the conditions when two alternators are in steady work in parallel will first be considered. These conditions, being permanent, can strictly be represented by simple phase diagrams of virtual E.M.F.'s or clock diagrams of instantaneous E.M.F.'s as given by vectors of maxima values rotating at an absolutely constant velocity.

The phase and value of each actual armature current in a number of alternators in parallel are always fixed by the vector $\overline{E_i \,.\, V_e}$, but no distinction is thereby drawn between that portion of the total armature current, I, which passes into or out of the external network and the portion which passes only as a cross current through the armatures of the other generators connected to the network. Now the vector of an actual alternating current or E.M.F. may be resolved into a pair of components in any number of ways, and the lines upon which it is so resolved should be chosen so as best to bring out the information which is required. In the present connexion, when a steady state has been reached, it is best to resolve each armature current along the external-current line and along a line at right angles to the terminal voltage, *i.e.* into I_e and I_c. The former component alone enters into the external output; the latter component may be strictly defined as a "corrective" current required to equalize some divergence of excitation or phase under permanent conditions of steady working. Such a method of resolution of the current is chosen because it gives at once the proportion of the total current of a machine which is actually supplied by it to the network or received by it therefrom. Thus when the complete diagram is drawn for two or more alternators working in parallel, the vector of the external current must be shown separately, and inclined to the terminal voltage at the angle of lag ϕ_e. This angle for a given network under fixed conditions is a constant dependent upon the relative values of its resistance and its reactance, *i.e.* upon its power-factor.

Since, in Fig. 508, $BG = IX_e$ where X_e is the reactance of the external circuit, and $OG = IR_e + Ir_a'$, the angle ϕ_i between the vectors of E_i and of the current is such that its tangent is $\dfrac{X_e + x_a}{R_e + r_a'}$. Taking first the case of two precisely similar alternators working in parallel under ideal conditions of perfect equality in all respects of excitation and steam supply, a similar diagram (Fig. 682) holds for both. But now each may be regarded as working separately on half the external network of which the resistance will be $2R_e$ and its reactance $2X_e$; the angle *COG* between E_t and I is, therefore, such that its tangent is $\frac{2X_e + x_a}{2R_e + r_a}$, and this angle is independent of what may afterwards be found to be the value of the E.M.F.'s, since the three sides of the triangle all vary together. The angle *COG* will later be seen to lend important assistance in the process of constructing diagrams to represent the effect of other altered conditions. Only so long as the similar alternators are working perfectly in parallel under precisely similar conditions, is it the ϕ_t of each; otherwise it is, as it were, the ideal ϕ_t for complete similarity.

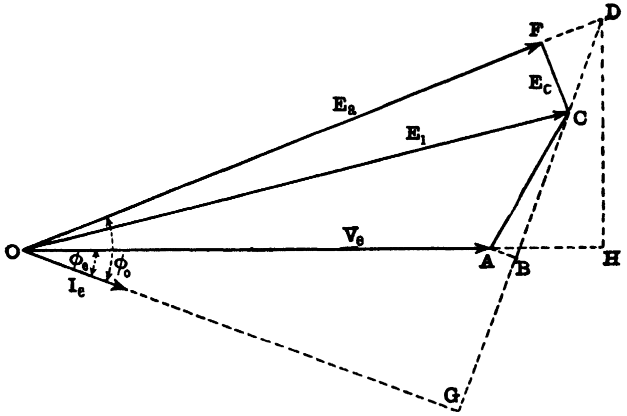

FIG. 682.—Single alternator, or two alternators in perfect parallel.

§ 6. Effect of unequal excitation.—But now let the conditions be altered, and in the first place, with the steam supply and frequency remaining equal, let the excitation of one be raised and of the other be lowered, and let a steady state be assumed to have been reached. The complete diagram of the two machines is shown in Fig. 683 in terms of the full quantities of Fig. 508. The vectors of E_{t1} and E_{t2} and also of E_{a1} and E_{a2}, are now no longer coincident, and from the divergence of the latter it follows that, in what may be defined as their true synchronous position for equality of the terminal voltages of the two machines, the pole-centres of machine 1 have fallen back relatively to the vectors of the terminal voltage or external current, while the pole-centres of machine 2 have conversely advanced. At the same time, the vectors of the total current in each machine have diverged. The current I_1 of the machine of larger E.M.F. falls behind the external current line, and its negative angle of lag behind the interpolar line of symmetry, or ϕ_{01}, is the larger. Conversely, the total current I_2 of the machine of smaller E.M.F. draws nearer to E_{a2}, or to the interpolar line of symmetry,

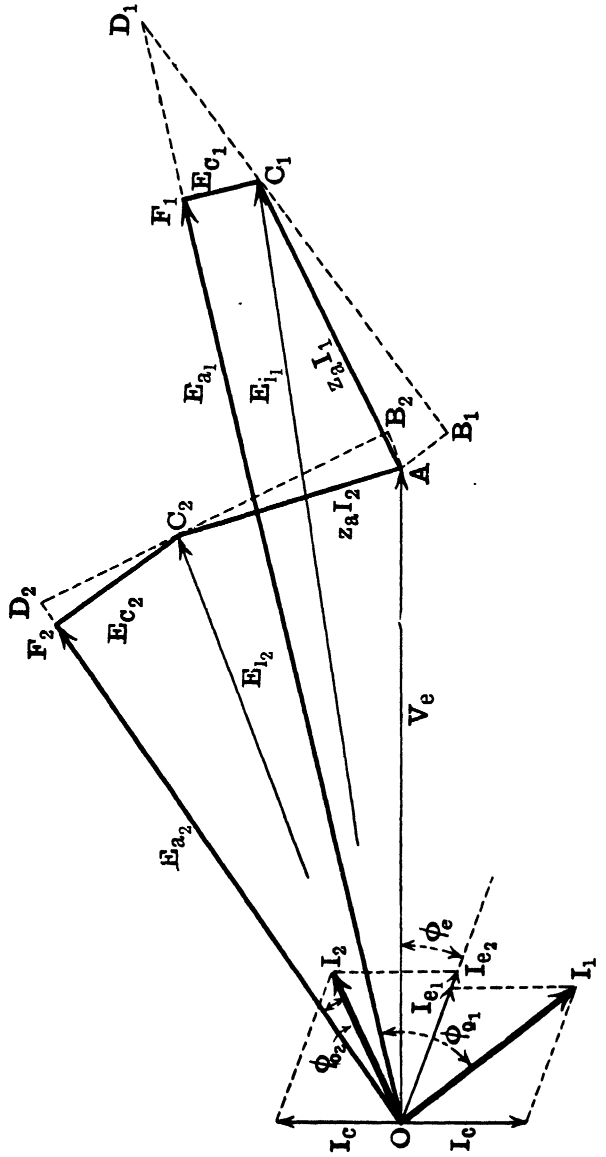

FIG. 683 —Two similar alternators, with equal steam supply and unequal excitation.

and its angle of lag ϕ_{o2} is reduced ; it may then precede E_{i2} and by its cross-magnetizing effect it increases E_{i2} as compared with E_{a2}. Resolving the currents as shown in the diagram, the component I_c lags in respect to E_{a1}, and, therefore, helps to demagnetize the machine of higher excitation ; on the other hand, since it leads in respect to E_{a2}, it assists its excitation, so that in the end E_{t1} and E_{t2} are not so far different as their unequal excitation would by itself cause, and their terminal voltages are actually identical. Further, since the component I_c is displaced widely from the positions of either E_{t1} or E_{t2}, it is very largely wattless and has no great effect upon the internal losses in the two machines or upon their output. The increase in the angle of lag ϕ_{o1} causes the total current I_1 to be more powerfully demagnetizing than it would be if the machines were equally excited, while the converse change in ϕ_{o2} lessens the demagnetization of the machine of smaller E.M.F., or positively produces forward ampere-turns upon its magnetic circuit. Thus the effect of an unequal excitation in altering the phases of the E.M.F.'s and currents is almost purely magnetic.

All the vectors of Fig. 683 have the same signification as in Fig. 508 ; *e.g.* the lines AC_1 and AC_2 are the E.M.F.'s consumed by the armature impedances, or z_aI_1 and z_aI_2, and the armature currents I_1 and I_2 lag $\phi_a°$ behind these E.M.F.'s. The products of E_{t1} and E_{t2} with the projections upon them of I_1 and I_2 respectively, are equal, since equality of steam supply has been presupposed. The ampere-turns corresponding to E_{a1} and E_{a2}, after taking into account the direct magnetizing turns proportional to $I_1 \sin \phi_{o1}$ and $I_2 \sin \phi_{o2}$, are the two unequal excitations. The machine of greater excitation has the larger armature current, but supplies the smaller share of the total external current.

While the finished diagram for two unequally excited alternators in parallel has been at once presented in Fig 683, the general method of obtaining this from given numerical values may be described It is best done by considering the E M F 's consumed by the armature impedances not in relation to the total armature currents, but in relation to their two components, *i.e.* the external and the corrective, separately and in succession

Let the joint impedance of the half system corresponding to one alternator be

$$2Z_t = \sqrt{(r_a' + 2R_e)^2 + (x_a + 2X_e)^2}.$$

Then $$\cos COG = \frac{r_a' + 2R_e}{2Z_t} = \frac{\text{joint resistance}}{\text{joint impedance}}$$

is the power-factor of the joint system of armature and half the external network, and from its value a line OCK can be drawn at the correct angle to the vector of the external network current before the position of C is determined. If now a line be drawn through the extremity of the vector of the terminal voltage and inclined at the angle ϕ_a to the external current vector OG, the point at which it intersects the line OCK gives the position of C (Fig 684) The length AC then represents the E M F. expended in overcoming the armature impedance in relation to the *mean* external current, or $z_a(I_{e1} + I_{e2})/2$. The mean external current I_e OC/ joint impedance as above defined.

Except in the case of unity power-factor, the corrective component I_c is from its definition inclined at a smaller angle to I_{e1} than to I_{e2} ; in consequence, if the external component of the two machines remained equal, the total armature current of machine 1 would be greater than that of machine 2, and the loss of watts over the resistance of the armature would be greater. But the steam supply and the total electrical energy developed in the two machines are *ex hypothesi* equal, so that the external output of machine 1

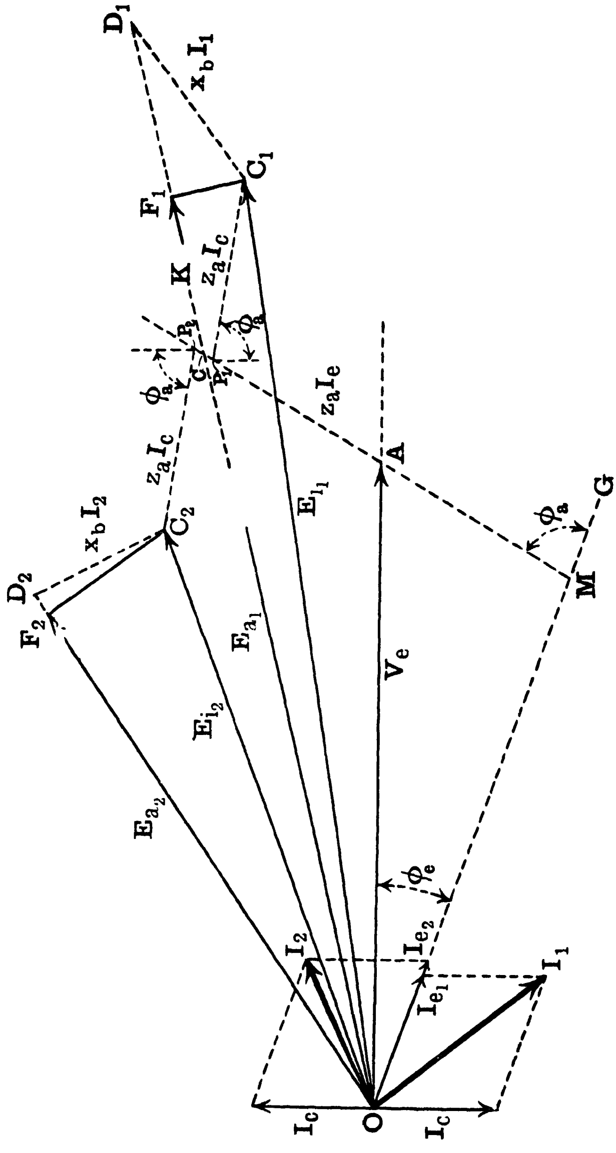

FIG 684 —Derivation of preceding diagram.

must be smaller than that of machine 2. Thus the presence of the corrective component destroys the perfect equality of the two external outputs, and the external component of machine 1 which has the strongest excitation must be made slightly smaller than that of machine 2, which has the weaker excitation. Since, however, the loss over the armature in either case is but small in comparison with the external output, no great difference is required in order again to reach equality of the total electrical energy developed. The diagram can, therefore, be drawn as follows—

After drawing *OCK* at the correct angle to the current line *OG* to represent the given values of the network resistance and reactance, draw *MAC* at the angle ϕ_a to *OG*, intersecting *OCK* at the point *C* which is a little less than the mean value of E_{01} and E_{02} Upon *AC* take two other points P_1 and P_2, the former as much below *C* as the latter is above it, and neither differing widely from *C*; then AP_1 and AP_2 represent respectively the E.M.F.'s consumed by the armature impedances in relation to the external components, or $z_a I_{e1}$ and $z_a I_{e2}$. From P_1 and P_2 draw two lines P_1C_1 and P_2C_2 of equal length, and each inclined to the vertical at an angle ϕ_a; then P_1C_1 and P_2C_2 represent the E.M F.'s consumed by the armature impedances in relation to the corrective component which has the same value I_c, but is in opposite directions in the two machines The two lines OC_1 and OC_2 then give the phases and magnitude of the impressed E M F.'s E_{t1} and E_{t2}, and the former must fall well within the no-load voltage E_{01}, while the latter is more nearly equal to the no-load voltage E_{02}. By dividing AP_1 and P_1C_1 by z_a, the values of the two component currents I_{e1} and I_c of the machine of larger excitation are found; when compounded, they give the phase and magnitude of the total current I_1 Similarly the quotients of AP_2 and P_2C_2 when divided by z_a give I_{e2} and I_c, and, when compounded, I_2 If the products of E_{t1} and E_{t2} with the projections upon them of I_1 and I_2 are not found to be equal when equality of steam supply has been presupposed, the lengths AP_1 and AP_2, P_1C_1 and P_2C_2 must be corrected until a possible diagram is obtained At right angles to the currents I_1 and I_2 are then drawn C_1D_1 and C_2D_2 representing I_1x_b and I_2x_b, and the diagram is completed in the same manner for each machine by letting fall the perpendiculars of the cross E M F 's, C_1F_1 and C_2F_2, which mark the extremities of the armature E M F.'s E_{a1} and E_{a2}. The excitations corresponding to E_{a1} and E_{a2}, when the direct magnetizing turns proportional to $I_1 \sin\phi_{01}$ and $I_2 \sin\phi_{02}$ are duly taken into account, must then tally with the two unequal excitations If the disagreement is found to be considerable, the position of the line *MAC* may require to be shifted inwards or outwards, thus altering the terminal voltage, and after one or two trials a correct solution of the problem is obtained That the diagram fulfils all the required conditions is evident, since it reproduces the same final result as Fig 683 for equal steam supply

If the magnetic and electric data of the machines are such that, as already explained in Chapter XXVII, § 42, *OD* may approximately be identified with E_0, the open-circuit voltage produced by the total excitation on the alternator (*i.e.* if the synchronous-impedance method may safely be applied for approximate calculation), the diagram is still further simplified. The line *MAC'* (Fig 685) must now be drawn inclined to the current vector at the angle ϕ_a' of which the tangent is x_a'/r_a', where x_a' is the synchronous reactance; similarly, the line *C'O* must be drawn at an angle *C'OG* to the current vector of which the tangent is $\dfrac{2X_e + x_a'}{2R_e + r_a'}$. The lines $P_1'D_1$, $P_2'D_2$ are drawn at the angle ϕ_a' to the vertical, and now represent the E.M F.'s consumed by the synchronous impedance z_a' in relation to the corrective components, while AP_1' and AP_2' represent the E.M F.'s consumed by z_a' in relation to the external currents. Such points must then be found that $P_1'D_1$ and $P_2'D_2$ are equal, and in the two cases respectively terminate upon the circles representing the values of the no-load voltages E_{01} and E_{02}, and give the required ratio of steam supply.

In Fig 684 the relative quantities used merely for the purpose of illustration are as follows: Half of the external network is assumed to have a resistance $2R_e = 4$, and a reactance $2X_e = 1{\cdot}456$ or $\cos\phi_e = 0{\cdot}94$ and $\phi_e = 20°$, while for the alternators $r_a' = 0{\cdot}25$, $x_a = 1{\cdot}4178$, so that $z_a = 1{\cdot}44$ and $\phi_a = 80°$. Thence $\cos COG = \dfrac{0{\cdot}25 + 4}{\sqrt{4{\cdot}25^2 + 2{\cdot}874^2}} = 0{\cdot}829$, and $COG = 34°$; the joint impedance $2Z_t$ is 5·13.

In Fig 685, on the synchronous-impedance method, when OD_1 and OD_2 are identified with the open-circuit voltages, $E_{o1} = 24$ and $E_{o2} = 11{\cdot}4$; $x_b = 0{\cdot}9$, whence $x_a' = x_a + 0{\cdot}9 = 2{\cdot}318$, and $z_a' = 2{\cdot}33$, while $\phi_a' = 83{\cdot}85°$. The joint impedance is then 5·68, and $\cos C'OG = \frac{4{\cdot}25}{5{\cdot}68} = 0{\cdot}748$, whence $C'OG = 41{\cdot}6°$.

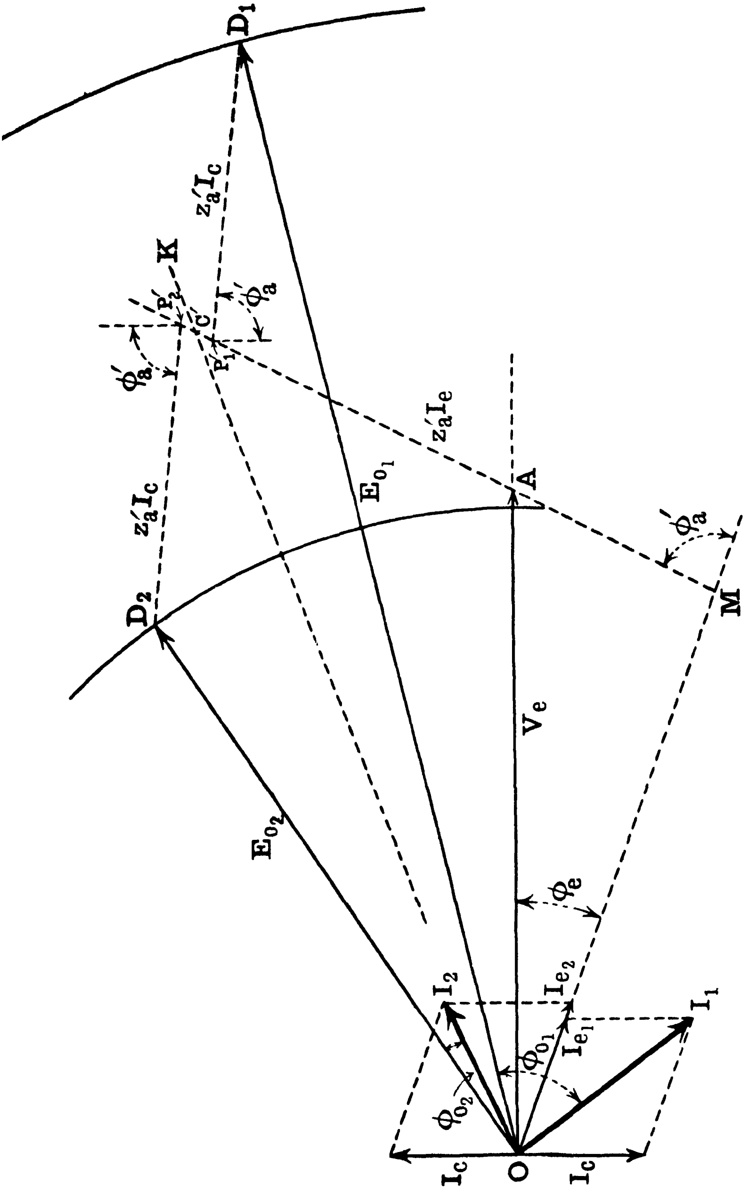

Fig 685 —Derivation of Fig. 683 on synchronous impedance basis.

In the case of unity power-factor, since the corrective component as before determined is in each case at right angles to the external current, the armature current I_2 is as much ahead of the terminal voltage as I_1 is behind, and both remain equal in value. Hence with equal steam supplies the external outputs remain equal throughout. Thus with unity power-factor in the external circuit, alteration of the field excitation of the two alternators will not affect the equal division of the external load, but with external power-factors less than unity, while their total loads remain equal in correspondence with their equal steam supplies, there will be some divergence from equality in their external outputs by reason of their internal losses becoming unequal.

In all cases the total heating loss over the armatures of the two unequally excited alternators exceeds that which would result with equal excitation for a given external load, and this excess may seriously detract from the rated capacity of the machine when the cross-current is large. It is not, however, equally divided between the two machines except in the case of unity external power factor, when I_c is at right angles not only to the external voltage, but also to the external currents, and the excess loss is then $2I_c^2 r_a$. In the case of a lagging external current, the additional loss in the generator of higher excitation exceeds the additional loss in the generator of lower excitation, so that as mentioned above the external load of the former becomes reduced

§ 7. Further examination of the preceding case.—The question then arises as to whether there is any limit to the inequality of excitation that is possible.

When in normal work and equally excited (Fig. 682), the horizontal distance *OH* along the line of terminal voltage to the perpendicular let fall from *D*, *i.e* the projection of E_0 on the line of terminal voltage, is

$$V_e + r_a' I_e \cos\phi_e + x_a' I_e \sin\phi_e$$

The height of the perpendicular *DH* is

$$x_a' I_e \cos\phi_e - r_a' I_e \sin\phi_c$$

The square root of the sum of their squares reduces to

$$E_0 = \sqrt{V_e^2 + 2V_e I_e r_a' \cos\phi_e + 2V_e I_e x_a' \sin\phi_e + (I_e z_a')^2}$$

When unequally excited, in the alternator of greater excitation, the horizontal distance *OH* becomes

$$V_e + r_a' I_{e1} \cos\phi_e + x_a'(I_{e1} \sin\phi_e + I_c)$$

and the perpendicular *DH* becomes

$$x_a' I_{e1} \cos\phi_e - r_a'(I_{e1} \sin\phi_e + I_c)$$

and

$$E_{01}^2 = \{V_e + r_a' I_{e1} \cos\phi_e + x_a'(I_{e1} \sin\phi_e + I_c)\}^2$$
$$+ \{x_a' I_{e1} \cos\phi_e - r_a'(I_{e1} \sin\phi_e + I_c)\}^2$$

Analogously in the alternator of weaker excitation,

$$E_{02}^2 = \{V_e + r_a' I_{e2} \cos\phi_e + x_a'(I_{e2} \sin\phi_e - I_c)\}^2$$
$$+ \{x_a' I_{e2} \cos\phi_e - r_a'(I_{e2} \sin\phi_e - I_c)\}^2$$

For each value of V_e on a given fixed external network, there is only one value possible for the total external current, viz , $\frac{V_e}{\sqrt{X_e^2 + R_e^2}} = \frac{V_e}{Z_e}$, the half of which is the mean external current I_e. With equal steam supply presumed,

$$I_{e2} > I_e > I_{e1}$$

i.e. the machine of weaker excitation supplies more than half the total external current (Fig. 684) The difference in the external outputs, as explained above, is due to the greater loss over the armature resistance of the machine of greater excitation owing to the lesser angle between I_c and I_e when the power-factor of the external circuit is less than unity. It is only when the armature resistance is so small as to be negligible that, with a lagging external current,

$$I_{e2} = I_e = I_{e1}$$

since then all the electrical power of the alternators must appear in the network and with equal steam supply they must give equal external loads.

But with some armature resistance, if the effect of reducing E_{02} or the excitation of machine 2 while maintaining constant that of machine 1 be traced on the assumption of constant armature impedance, it is of interest to note that there is a maximum possible inequality of excitation that can be equalized, and that this is reached when the divergence of E_{02} and E_{01} approaches a right angle, and the new terminal voltage is a little more than one-half of the original V_e when both machines were equally excited with the excitation maintained on machine 1. The calculation cannot indeed be pushed to an accurate conclusion under such extreme conditions owing to the assumed data not being, in fact, constant quantities. But the case is introduced as bearing on the question of what results when one alternator loses its excitation through some accident when running in parallel, and the answer will be found to turn entirely on the way in which the governors of both machines react.

The preceding analysis under steady conditions will have shown that directly the armature E.M F 's have diverged in phase, and still assuming that the speed is not increased and that the excitation of machine 1 is not raised to counterbalance the reduced excitation of the second machine, the terminal voltage at the bus bars is inevitably lowered, and in consequence the total external output into the network *must* have become reduced It follows, therefore, that unless the increased losses in the two armatures can have taken up the excess of the driving power which could not be the case in practice, the governor of one or both machines, while maintaining sensibly constant speed, must have reduced the total steam or water supply Above it has been implicitly assumed that the governors of *both* machines have moved equally inwards to reduce the supply to each equally

If the governor of machine 1 cuts off the driving power more rapidly than that of machine 2 which has become unexcited, we still have $I_{e2} > I_e > I_{e1}$; but if the reverse is the case, $I_{e1} > I_e > I_{e2}$, and the normally excited machine absorbs the greater power and develops the greater output Its E M F vector relatively advances, and that of machine 2 falls back, while their current vectors move contrariwise Machine 2, therefore, tends to approach the condition of a motor; but with salient-pole machines, owing to the powerful magnetizing action of the leading current I_2, it is possible for the second machine still to continue to act as a generator and to absorb mechanical power even when its field excitation is zero.[1]

On a network fed by such large alternators that the bus-bar voltage may be regarded as fixed, the effect of decreasing the field excitation and lowering the induced E M F of a single small alternator with a constant steam supply, is to increase the angle γ between the induced and terminal voltages until the point of the lowest induced E M F. at which the machine can take the power of the steam supply is passed, and it falls out of step. A V-curve of current analogous to that of a synchronous motor can be traced, but the best excitation in this supposed case is not that which gives the minimum loss in the single machine, but that which makes its vector diagram of E.M F.'s coincide with those of the other machines.

The greater the short-circuit current for a given excitation, or, in other words, the lower the armature reaction, the greater the cross-corrective current which must flow to counterbalance a given difference in the field strengths of two machines. Hence at least a moderate amount of armature reaction is on this score to be desired, so as to prevent the cross current from becoming unduly high.

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[1] *Cp.* H. Herman, *Electr World*, Vol. 51, p. 1110, where the action is traced to the variation in the inductance of the armature according to the varying position of its coils relatively to the poles within a cycle: C. P. Steinmetz, Chap. XVI of *Theory and Calculation of Electrical Apparatus.*

§ 8. Unequal steam supply with equal excitation.—In the case of equal excitation but unequal steam supply to two similar alternators the line *MAC* inclined at the angle ϕ_a to the external current vector must intersect *OC* at some point *C* falling within the circle corresponding to the open-circuit E.M.F. for the given excitation

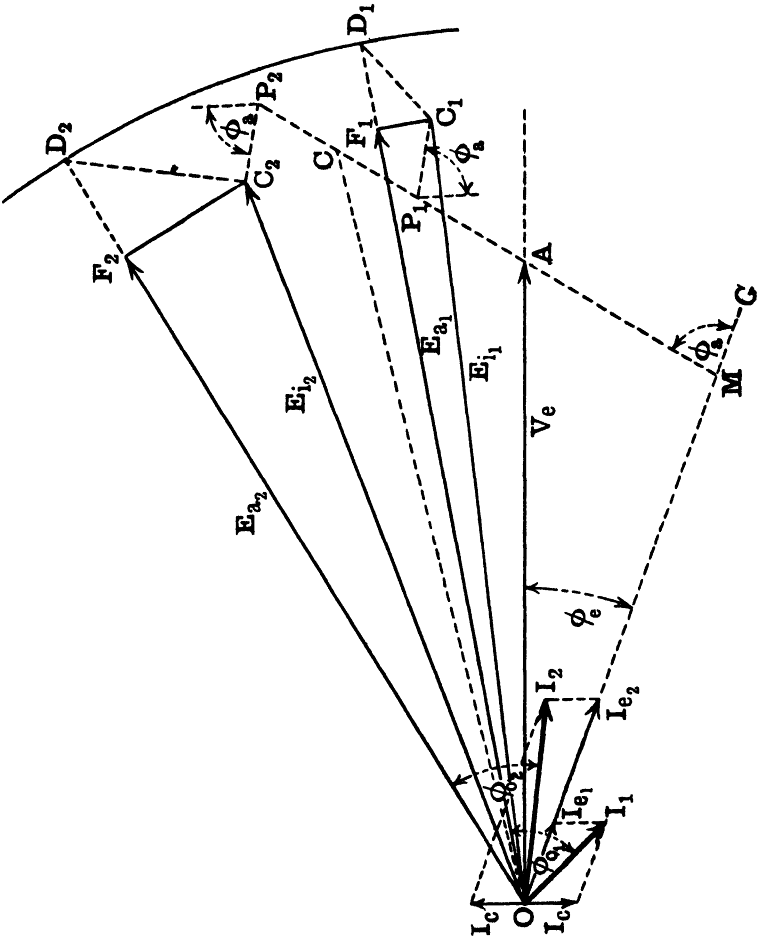

FIG. 686.—Two similar alternators equally excited, with unequal steam supply.

(Fig. 686). The product of the mean external current (as deduced from $AC = z_a \times$ mean I_e) when projected upon *OC* and multiplied by the value of *OC* will then give the mean horse-power absorbed in developing the mean external output. On either side of *C* must be taken the two points P_1 and P_2 such that AP_1 and AP_2 are nearly in proportion to the unequal steam supplies. By drawing P_1C_1 and P_2C_2 inclined at the angle ϕ_a to the vertical, and thence calculating the corrective current I_c and combining it with I_{e1} and I_{e2}, the diagram can be completed; and the values must be adjusted until the lines C_1D_1 and C_2D_2 representing I_1x_b and I_2x_b drawn at right angles to the currents I_1 and I_2, and proportional to them, terminate upon the circle of equal E.M.F.'s. The phases of the armature E.M.F.'s and positions of the centres of the poles are then

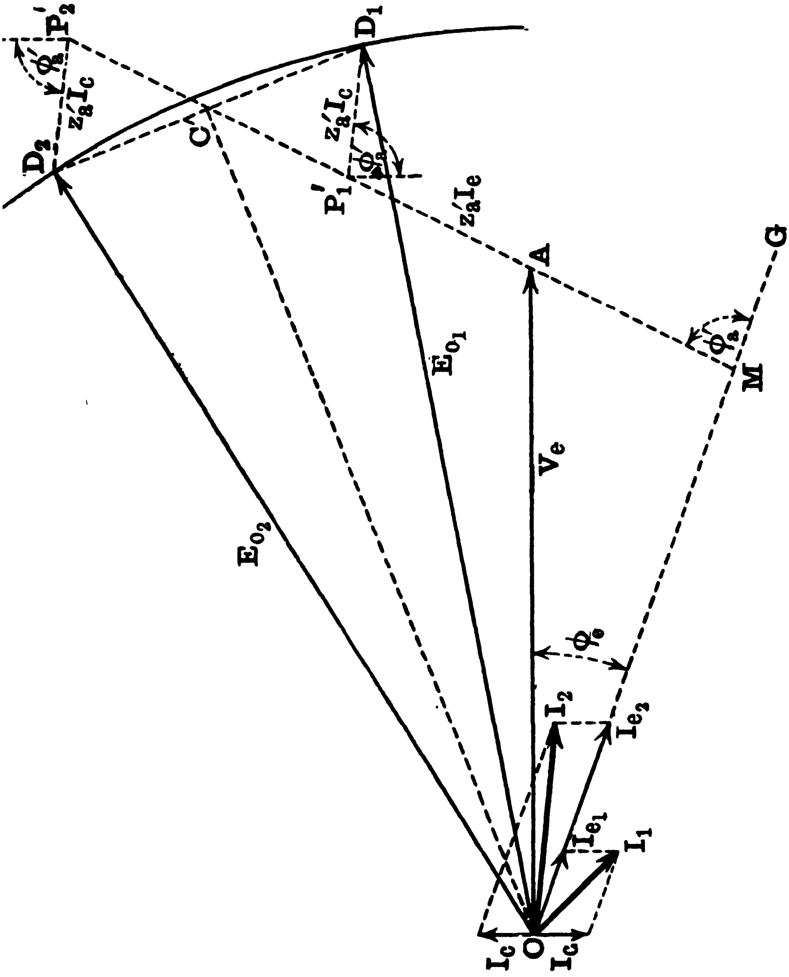

Fig. 687.—Preceding diagram on synchronous-impedance basis.

fixed by OD_1 and OD_2. It will be seen that, since the two machines cannot give equal powers, a corrective current is present which causes the greater current of machine 2 to lag behind E_{a2} by a lesser angle than the current of machine 1 lags behind E_{a1}; their respective demagnetizing turns are, therefore, more nearly equalized than would follow from the simple values of the currents, and in consequence their terminal voltages are equalized. The two induced E.M.F.'s

E_{i1} and E_{i2} diverge equally on either side of the mean induced E.M.F. OC.

When expressed in terms of the apparent synchronous impedance of the armature z_a' and of E_o, the diagram of Fig. 686 is redrawn as Fig. 687. The calculations of relative power thence deduced, although not strictly correct, are not far from the true values obtained when the induced E.M.F.'s are used instead of the open-circuit E.M.F.'s.

The effect of unequal steam supply with unequal excitation can also be traced by similar diagrams. The maximum value of the output and efficiency for a given total steam supply is reached when the phases of the two induced E.M.F.'s E_{i1} and E_{i2} coincide, and when the reactance of the armatures is high as compared with their resistances, which is the usual case of practice, *i.e.* when ϕ_a approaches 90°, the required ratio of the two steam supplies reduces to the same ratio as that of the induced E.M.F.'s E_{i1}/E_{i2}. This is again not far different from E_{a1}/E_{a2}, but the ratio of E_{o1}/E_{o2}, or of the two excitations, does not afford any direct clue, since the demagnetization affects the machine of larger excitation to a preponderating extent.

§ 9. Alternative resolution of the currents to meet a different condition.—The principle upon which the preceding diagrams have been drawn is thus simple. In Fig. 684 or Fig. 686 the vector $\overline{E_{i1}\, V_e}$ (not shown) or AC_1 has been treated as made up of the combination of AP_1, and P_1C_1, and the vector $\overline{E_{i2}V_e}$ or AC_2 as made up of AP_2 and P_2C_2, the first in each component pair being z_aI_{e1} or z_aI_{e2}, and the second z_aI_c. Similarly in Fig. 685 or Fig. 687, the vector $\overline{E_{o1}V_e}$ or AD_1 has been treated as made up of AP_1' and $P_1'D_1$ and $\overline{E_{o2}V_e}$ or AD_2 as made up of AP_2' and $P_2'D_2$. But these vectors might equally well be the result of the combinations respectively of AC with CC_1 and CC_2 or of AC' with CD_1 and CD_2. The vector $\overline{E_{i1}V_e}$ for example, is thus split up in Fig. 686 into AC proportional to the *mean* I_e of the external currents and CC_1 proportional to a different corrective current I_c'. This amounts to considering the vector $\overline{E_{i1}\, E_{i2}}$ which passes through C. This again is equivalent to assigning to each machine an *equal* share of the external current $I_e = \frac{I_{e1} + I_{e2}}{2}$ proportional to which is the mean length AC or AC', and combining this equal current with a corrective component I_c' which is inclined to the vertical, and which is, therefore, no longer purely at right angles to the terminal voltage V_e. The new corrective component which has been symbolised by I_c', and which is obtained by dividing the vector $\overline{E_{i1}\, E_{i2}}$ or $\overline{E_{o1}\, E_{o2}}$ by $2z_a$ or $2z_a'$, must now be plotted at an angle ϕ_a or ϕ_a'

to the line C_1C_2 or D_1D_2. In Fig. 688 this has been done for the true diagram; the vector $\overline{E_{i2}E_{i1}}$ or C_2CC_1 is there shown transferred to the centre, and lagging behind OC_2 or OC_1 is $I_c' = C_2OC_1/2z_a$. In Fig. 689 the same process has been applied to the approximate diagram on the synchronous-impedance assumption; the

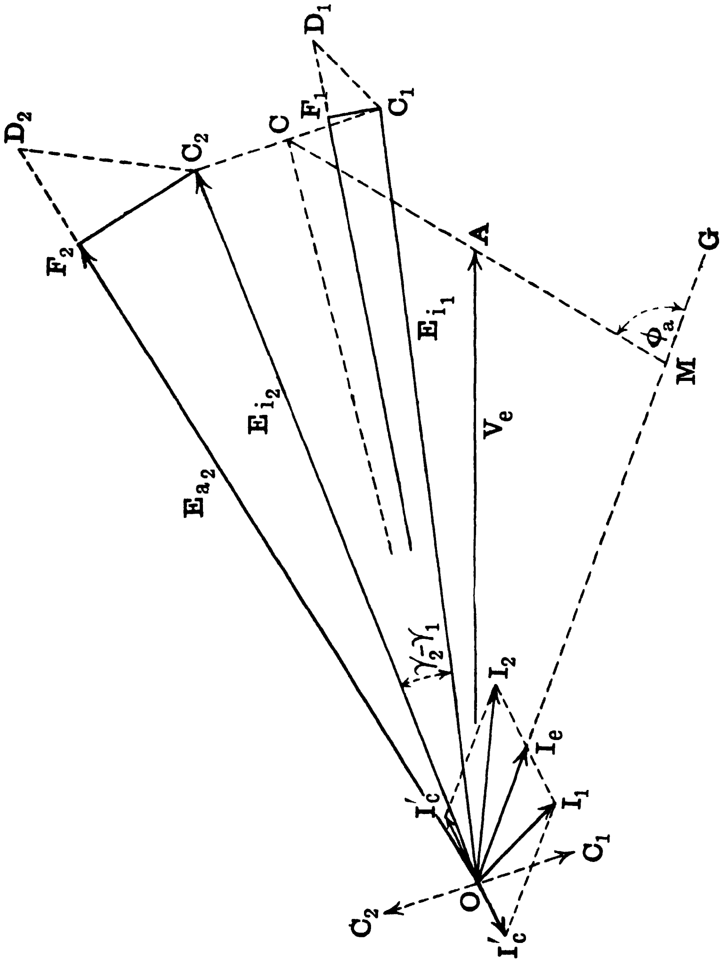

FIG. 688.—The diagram of Fig. 686 in terms of I_c'.

vector $\overline{E_{o1}\,E_{o2}}$ or $D_1C'D_2$ is transferred to the centre, and lagging behind it by the angle ϕ_a' is $I_c' = \dfrac{D_1C'D_2}{2z_a'}$. The resultant E.M.F.'s and currents are the same as before, but the diagrams meet a different condition. In Fig. 690 let the external circuit be imagined to be actually divided into two unequal portions, one attached to BA and the other to DC, demanding I_{e2} and I_{e1}

respectively; if say, AGC be cut at G and an ammeter of negligible impedance be inserted, it will then register I_c. Thus the corrective current at right angles to V_e, of which use was first made in order to show what proportion of the total external current was given by each machine to a common network, meets such a case. But now

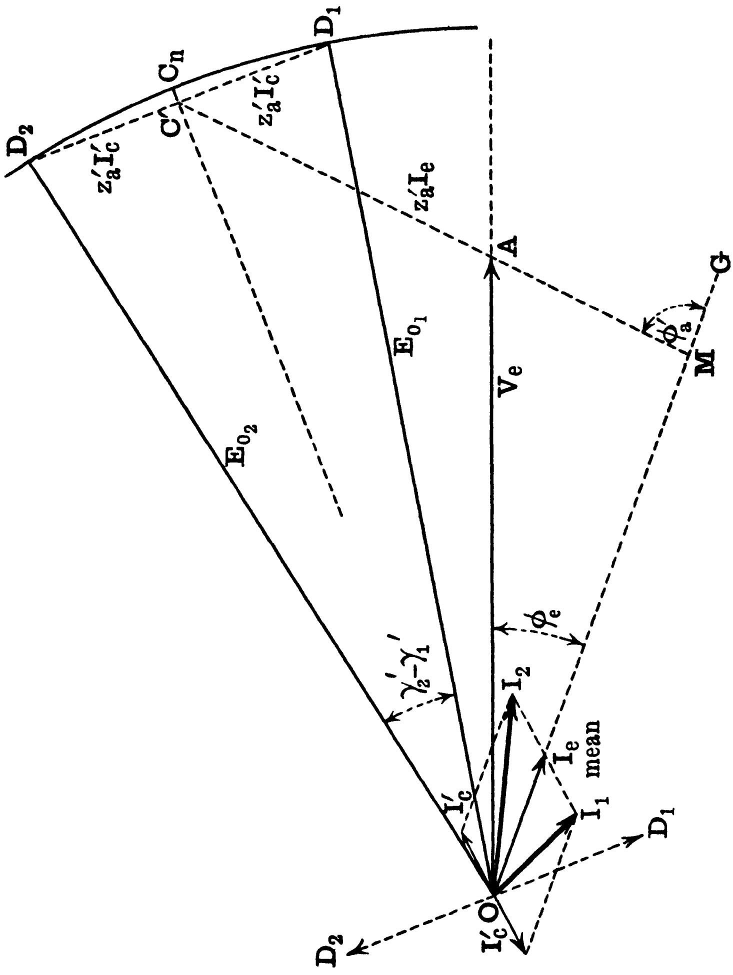

FIG. 689.—The diagram of Fig. 687 in terms of I_c'.

with the alternators unequally excited or unequally supplied with steam, let the two portions of the network be *equal*; then the ammeter will register I_c' or half the vector difference between I_1 and I_2, whose vector sum is $2I_e$ (mean), the total external current. Thus the new resolution of the currents shows the phase and magnitude of the interchange current I_c' flowing between the two machines, *i.e.*, through the series circuit of armatures and bus bars.[1] It is now seen that this latter component, being nearly in phase with the E.M.F. of the leading machine, and nearly opposed to the E.M.F. of the lagging machine, alters the output of each machine into accordance with its steam supply. When a real current is resolved into any two components, the rate of development of electrical energy by it in relation to any E.M.F. is always given by the algebraic sum of the products when each component is separately projected on to the E.M.F. and the projections are multiplied by the E.M.F. Hence, returning to the true diagram of Fig. 688, let $\gamma_2 - \gamma_1$ be the electrical angle between the equal E.M.F.'s E_{t2} and E_{t1}; the power absorbed in each machine is then the sum of the products with the induced E.M.F. of the projections upon it first of the mean external current I_e and then of the component I_c', *i.e.* in the machine of which the E.M.F. E_{t2} leads relatively to the mean vector OC

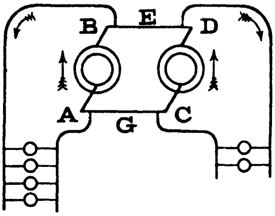

FIG. 690.—Two separate networks.

$$P_2 = N_{ph} E_{t2} \left\{ \text{mean } I_e \cos\left(COG + \frac{\gamma_2 - \gamma_1}{2}\right) + I_c' \cos\left(90^\circ - \phi_a - \frac{\gamma_2 - \gamma_1}{2}\right)\right\}$$

$$= N_{ph} E_{i2} \left\{ I_e \cos\left(COG + \frac{\gamma_2 - \gamma_1}{2}\right) + I_c' \sin\left(\frac{\gamma_2 - \gamma_1}{2} + \phi_a\right)\right\} \quad . \quad . \quad (298)$$

and in the machine of which the E.M.F. E_{t1} lags relatively to OC

$$P_1 = N_{ph} E_{t1} \left\{ \text{mean } I_e \cos\left(COG - \frac{\gamma_2 - \gamma_1}{2}\right) - I_c' \cos\left(90^\circ - \phi_a + \frac{\gamma_2 - \gamma_1}{2}\right)\right\}$$

$$= N_{ph} E_{t1} \left\{ I_e \cos\left(COG - \frac{\gamma_2 - \gamma_1}{2}\right) + I_c' \sin\left(\frac{\gamma_2 - \gamma_1}{2} - \phi_a\right)\right\} \quad . \quad . \quad (299)$$

The vectors E_{i1} and E_{i2} have diverged equally from OC, and the angle $COA = \frac{\gamma_2 + \gamma_1}{2}$. When E_{t1} and E_{t2} are (as assumed) equal, although for this the excitations will need to be slightly different, or with equal excitation if the ohmic resistance of the armatures is negligibly small or the difference in the steam supplies slight, the vector C_1CC_2 is at right angles[1] to OC, and $CC_1 = CC_2$.

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[1] Only permanent conditions are at present in view. It will later be seen in §§ 12, 25 that the same method of resolution is specially suited to the case of two similar machines whose speeds vary rhythmically and the conditions are not steady ;] in such a case, each machine does, in fact, give as its normal load half of the total external current.

I_c' is then $\frac{E_t}{z_a} \sin \frac{\gamma_2 - \gamma_1}{2}$, and we have

$$P_2 = N_{ph} E_t \left\{ I_e \cos \left(COG + \frac{\gamma_2 - \gamma_1}{2} \right) + \frac{E_t}{z_a} \sin \frac{\gamma_2 - \gamma_1}{2} \sin \left(\frac{\gamma_2 - \gamma_1}{2} + \phi_a \right) \right\}$$

$$P_1 = N_{ph} E_t \left\{ I_e \cos \left(COG - \frac{\gamma_2 - \gamma_1}{2} \right) + \frac{E_t}{z_a} \sin \frac{\gamma_2 - \gamma_1}{2} \sin \left(\frac{\gamma_2 - \gamma_1}{2} - \phi_a \right) \right\}$$

The permanent difference between the two powers is thus

$$P_2 - P_1 = N_{ph} E_t \left\{ \frac{E_t}{z_a} \sin \phi_a \sin (\gamma_2 - \gamma_1) - 2 I_e \sin COG \sin \frac{\gamma_2 - \gamma_1}{2} \right\}$$

or since $I_e = \frac{E_t}{2Z_t} \cos \frac{\gamma_2 - \gamma_1}{2}$ and $\sin COG = \frac{x_a + 2X_e}{2Z_t}$,

while $\sin \phi_a = \frac{x_a}{z_a}$

$$P_2 - P_1 = N_{ph} E_t^2 \left\{ \frac{x_a}{z_a^2} - \frac{x_a + 2X_e}{4Z_t^2} \right\} \sin (\gamma_2 - \gamma_1) \quad . \quad . \quad (300)$$

$$= N_{ph} E_t^2 \frac{x_a (2Z_t)^2 - (x_a + 2X_e) z_a^2}{z_a^2 \times 4Z_t^2} \sin (\gamma_2 - . \gamma_1) \quad (301)$$

It is, therefore, for equal induced E.M.F.'s and any given network equal to the sine of the angle of phase difference multiplied by a constant.

As regards external output only, the difference corresponds to that between the projections of the external currents I_2 and I_1 upon the horizontal V_e; since the angle between CC_2 transferred to the centre as OC_2 (Fig. 688) and the horizontal is $90° + \frac{\gamma_2 + \gamma_1}{2}$ and between I_c' and the horizontal is $90° + \frac{\gamma_2 + \gamma_1}{2} - \phi_a$, this is

$$I_c' \cos \left\{ 90° - \left(\phi_a - \frac{\gamma_2 + \gamma_1}{2} \right) \right\} \left[- I_c' \cos \left\{ 90° - \left(\phi_a - \frac{\gamma_2 + \gamma_1}{2} \right) \right\} \right]$$

$$= 2 I_c' \sin \left(\phi_a - \frac{\gamma_2 + \gamma_1}{2} \right)$$

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[1] In Fig. 689, on the synchronous-impedance hypothesis, the vector $D_1C'D_2$ must for equal excitations be at right angles to OC', and D_1C' or $D_2C' = E_0 \sin \frac{\gamma_2' - \gamma_1'}{2}$

and since $I_c' = \frac{E_i}{z_a} \cdot \sin \frac{\gamma_2 - \gamma_1}{2}$

the *difference of the external outputs* is

$$P_{e2} - P_{e1} = N_{ph} \frac{2E_i V_e}{z_a} \sin\left(\phi_a - \frac{\gamma_2 + \gamma_1}{2}\right) \sin \frac{\gamma_2 - \gamma_1}{2}$$

$$= N_{ph} \frac{2E_i V_e}{z_a^2} \left(x_a \cos \frac{\gamma_2 + \gamma_1}{2} - r_a \sin \frac{\gamma_2 + \gamma_1}{2}\right) \sin \frac{\gamma_2 - \gamma_1}{2}$$

Now,

$$V_e - I_c \times 2Z_c - E_i \cos \frac{\gamma_2 - \gamma_1}{2} \times \frac{2Z_e}{2Z_t}$$

Therefore,

$$P_{c2} - P_{e1} = N_{ph} \frac{E_i^2}{z_a^2} \sin (\gamma_2 - \gamma_1) \frac{Z_e}{Z_t} \left(x_a \cos \frac{\gamma_2 + \gamma_1}{2} - r_a \sin \frac{\gamma_2 + \gamma_1}{2}\right)$$

$$\cos \frac{\gamma_2 + \gamma_1}{2} = \frac{V_e + z_a I_e \cos (\phi_a - \phi_e)}{E_i \cos \frac{\gamma_2 - \gamma_1}{2}} = \frac{V_e \left(1 + \frac{r_a R_e + x_a X_e}{2Z_e^2}\right)}{V_e \frac{2Z_t}{2Z_e}}$$

$$= \frac{2Z_e^2 + r_a R_e + x_a X_e}{Z_e \times 2Z_t}$$

and

$$\sin \frac{\gamma_2 + \gamma_1}{2} = \frac{z_a I_e \sin (\phi_a - \phi_e)}{E_i \cos \frac{\gamma_2 - \gamma_1}{2}} = \frac{x_a R_e - r_a X_e}{Z_e \times 2Z_t}$$

Finally,

$$P_{e2} - P_{e1} = N_{ph} \frac{E_i^2 (2x_a Z_e^2 + X_e z_a^2)}{z_a^2 \times 2Z_t^2} \sin (\gamma_2 - \gamma_1) \text{ or analogously to (301)}$$

$$= N_{ph} \frac{E_i^2 \{x_a (2Z_e)^2 + 2X_e z_a^2\}}{z_a^2 \times 4Z_t^2} \sin (\gamma_2 - \gamma_1)$$

On the synchronous-impedance method, we only have to write *OD* for E_i and x_a', z_a', and ϕ_a' for x_a, z_a, and ϕ_a, and employ the new angle $\gamma_2' - \gamma_1'$ between E_{a2} and E_{a1}, while if the conditions are such that *OD* may be identified with E_o,

$$P_2 - P_1 = N_{ph} E_o^2 \left\{ \frac{x_a'}{(z_a')^2} - \frac{x_a' + 2X_e}{(r_a' + 2R_e)^2 + (x_a' + 2X_e)^2} \right\} \sin (\gamma_2' - \gamma_1')$$

This condition has been assumed in Fig. 689 where $\frac{\gamma_2' - \gamma_1'}{2} = 10{\cdot}8°$, E_o for both machines $= 15{\cdot}85$, and the mean external current with the same data as in the preceding diagrams is

$$I_e' = \frac{15{\cdot}85 \cos 10{\cdot}8°}{5{\cdot}68} = 2{\cdot}74$$

$$I_c' = \frac{15{\cdot}85}{2{\cdot}33} \sin 10{\cdot}8° = 1{\cdot}275$$

The two powers P_2 and P_1 are in the ratio 2·94 : 1·13, or by the above equation

$$P_2 - P_1 = N_{ph} E_o^2 \left\{ \frac{2{\cdot}318}{2{\cdot}33^2} - \frac{3{\cdot}774}{5{\cdot}68^2} \right\} \sin 21{\cdot}6°$$

The equations show that reactance must be present in the stator for the action to take place. For a given stator resistance the first term x_a/z_a^2 between the braces $\{ \}$ is a maximum when $x_a = r_a$; the second term, however, for a given network and value of r_a gradually falls as x_a is increased. Consequently the maximum of the whole term within the braces occurs when x_a slightly exceeds r_a. So small a value of x_a will not, however, obtain in practice, so that the condition is only of theoretic interest.

The above expressions cover the case of the leading machine driving the lagging machine as a motor, so that it develops electrical power; P_1 is then negative and $-P_1$ is additive to P_2. But, of course, so long as both machines are to act as generators, the maximum difference possible in their *steam* supplies is limited by the case of P_1 becoming zero, *i.e.* by such divergence of phase that the current of the lagging machine is exactly at right angles to its induced E.M.F., without any projection upon it, and this occurs before $\gamma_2 - \gamma_1$ reaches 90°. On open external circuit any divergence of phase means that the lagging machine is at least partially driven as a motor.

§ 10. Interconnected stations.—The bus bars by which interchange power flows from the leading to the lagging part of the diagram have above been assumed to have no impedance, so that the terminal voltages of the two machines are the same. The same class of diagram with I_c' but in a modified form is also specially adapted to the case when the bus-bars have impedance and the network is divided between two machines or between two stations of the same frequency interlinked by a connector of resistance r_i and reactance x_i per phase.[1] For interchange of power, there must then be some difference of phase between the terminal voltages per phase V_{e2} and V_{e1} of the leading and lagging machines. The diagrams of the two machines or stations are, therefore, separated by an intermediate portion OB_2AB_1O (Fig. 691) covering the interconnector, the central line of which, OA, is the voltage at the centre of the connector. Lagging behind B_1B_2 by the angle $\phi_i = \tan^{-1} x_i/r_i$ falls $I_c' = \dfrac{\overline{V_{e2}V_{e1}}}{z_i}$. Since this now has an objective existence, and enters directly as a component into the total external current I_{e1} of one network, it is simpler to compound it with I_1 to give I_{e1}, rather than to produce it backwards and regard I_1 as the resultant of I_c' produced backwards, and I_{e1}. But an essential point to be noted is that it alters the internal power-factor of each machine or station, so that neither can have the angle of lag that would normally correspond to the power factors of the networks. If the difference between the values of the terminal voltages projected on to OA is allowed to correspond exactly with the ohmic loss over the connector, I_c' falls on OA, but when the lagging machine 1 is excited more strongly than the leading machine

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[1] In the present elementary discussion, no transformation of the voltage is contemplated. In practice the diagram for the high-pressure transmitting cable, including the resistance and reactance of the transformers referred to the high-pressure side, would be considered. See especially L. Romero and J. B. Palmer, *Journ. I.E.E.*, Vol. 60, p. 287, with discussion, and E. C. Stone, *Trans. Amer. I.E.E.*, Vol. 38, Pt. II, p. 1651.

2, the terminal voltages can be rendered alike, and I_c' precedes OA.

Assuming the two portions of the network or total load to have the same power-factor, the external currents I_{e1} and I_{e2} fall behind V_{e1} and V_{e2} respectively by the angle ϕ_e. Of the total power per phase of the leading machine 2 (given by the product of E_{i2} with the projection upon it of I_2), the power delivered to the interconnector $= V_{e2}$ multiplied by the projection upon it of I_c'; the interchange power at centre of connector $= OA$, the mean voltage, multiplied by I_c' projected on it; the power received at the terminals of the other part of the network $= V_{e1}$ multiplied by I_c' projected on it. The power delivered by machine 1 to its own network is the product of V_{e1} with the projection upon it of I_{e1}. Power is not delivered from machine 2 to machine 1, but the latter is saved from some portion of the total load on its network.

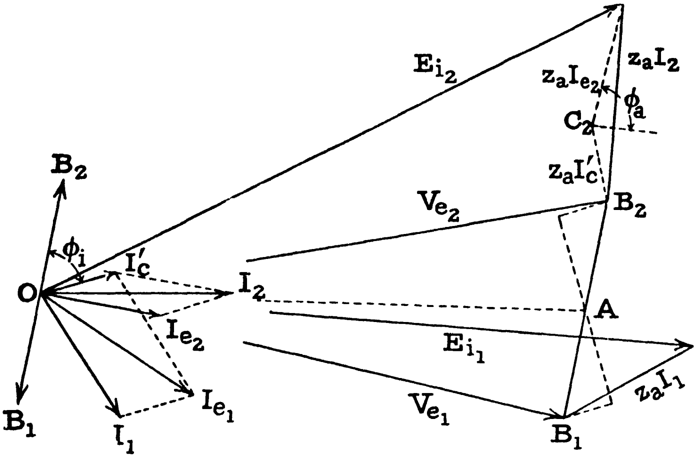

FIG. 691.—Two machines or stations interlinked by a connector.

Between the two portions of the bus bars or networks there may often be special impedance coils ("reactors"), and, when the voltages of two stations are not exactly equal, special voltage regulators to enable some adjustment to be made.[1]

In the special case of two stations or machines excited to give the same terminal voltage but with a difference of phase (Fig. 692), owing to the network load on each not being proportioned to their respective capacities, let α be the angle of displacement between the two voltages, each of value V_e, the vector $\overline{V_{e2}\,V_{e1}}$ is then $2V_e \sin\frac{\alpha}{2}$, and $I_c' = \frac{2V_e}{z_t}\sin\frac{\alpha}{2}$.

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[1] See K. M. Faye-Hansen and J. S. Peck, "Current-limiting Reactances on Large Power Systems," *Journ. I.E.E*, Vol. 52, p. 511; J S. Peck, "The Parallel Operation of Electric Power Stations," *Journ. I.E.E.*, Vol. 55, p. 61; E. B. Wedmore, "The Control of Large Amounts of Power," *Journ. I.E E.*, Vol. 56, pp. 274 and 276; E. P. Hollis, "Reactance and Reactance Coils in Power Circuits," *Journ I E.E.*, Vol. 52, p. 254; R F. Schuchardt, "The Use of Reactors on Large Central Station Systems," *Trans. Amer. I.E.E.*, Vol. 39, Part II, p. 1195; C. P. Steinmetz, "Power Control and Stability of Electric Generating Stations," *Trans. Amer. I.E.E.*, Vol. 39, Part II, p. 1215; and A. H. Gibson with collaborators, *Hydro-electric Engineering*, Vol. 2, pp. 140-145.

The powers per phase delivered from the leading machine to terminal 2 of the connector, at its centre, and received at terminal 1 are then respectively

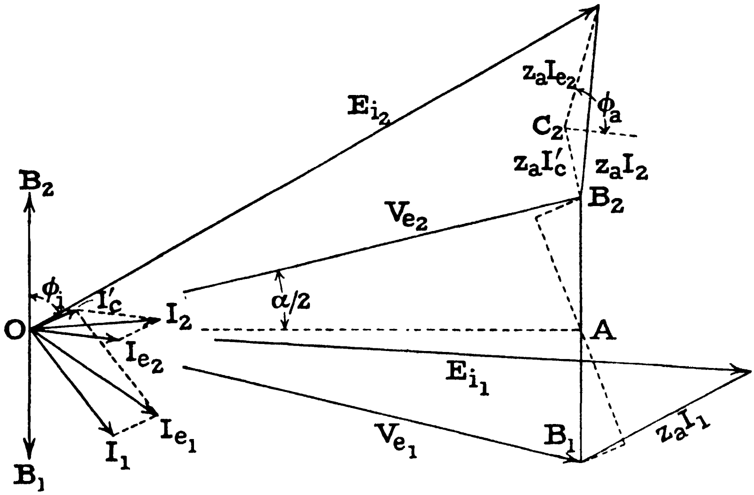

FIG. 692.—Two machines or stations with equal terminal voltage interlinked

$$V_e I_c' \cos\left\{(90^\circ - \phi_t) - \alpha/2\right\} = \left(\frac{V_e}{z_t}\right)^2 (x_t \sin \alpha + 2r_t \sin^2 \alpha/2)$$

$$V_e I_c' \cos \alpha/2 \times \cos (90^\circ - \phi_t) = \left(\frac{V_e}{z_t}\right)^2 x_t \sin \alpha$$

$$V_e I_c' \cos\left\{(90^\circ - \phi_t) + \alpha/2)\right\} = \left(\frac{V_e}{z_t}\right)^2 (x_t \sin \alpha - 2r_t \sin^2 \alpha/2)$$

the difference between the first and the third being, of course, $= (I_c')^2 r_t$. The second expression shows that reactance must be present in the connector for interchange of power, and that for a given impedance the interchange is directly proportional to the reactance ; the connector must necessarily have some resistance, but for a given angle α, the smaller the resistance, the greater the power interchange. So long, however, as r_t is small as compared with x_t, it does not have any great effect ; in practice it is found advisable for x_t to be not less than twice the resistance.

To render the case clear by a diagram, the relative values assumed as data for the purpose of illustration in Fig. 692 are per phase as follows: The equal steam supply or capacity of stations 1 and 2 corresponds to 22 watts, and their equal terminal voltage is 10·39. But the impedances of the networks at the two ends with the same power-factor ($\phi_e = 20°$) are $Z_{e1} = 3{\cdot}6$ and $Z_{e2} = 8{\cdot}61$. The network loads in true power are, therefore, $10{\cdot}39 \times 2{\cdot}88 \times \cos 20° = 28{\cdot}08$ watts and $10{\cdot}39 \times 1{\cdot}205 \times \cos 20° = 11{\cdot}72$ watts.

The remaining resistances and reactances are $r_i = 2$, $x_i = 5$, $z_i = \sqrt{29} = 5{\cdot}385$, $\phi_i = \tan^{-1} 2{\cdot}5 = 68{\cdot}2$, r_a of each machine $= 0{\cdot}25$, $x_a = 1{\cdot}418$, $z_a = 1{\cdot}44$, $\phi_a = 80°$.

By trial, giving different values to a, it is found that the conditions are satisfied when the angle between the terminal voltages is 27°. I_c' then $= \dfrac{2 \times 10{\cdot}39}{5{\cdot}385} \sin 13{\cdot}5° = 0{\cdot}9$.

The angle between I_c' and I_{e2} is $(90° - \phi_i) + \phi_e - 13{\cdot}5° = 28{\cdot}3°$, and compounding the two, the resultant $I_2 = 2{\cdot}04$, displaced from I_{e2} by the angle 12·05°. The total power of machine 2 is thus expended as follows—

In heating the stator, $2{\cdot}04^2 \times 0{\cdot}25$ = 1·04 watts

Delivered to network 2, $10{\cdot}39 \times 1{\cdot}205 \times \cos 20°$ = 11·72 „

„ connector at end 2, $10{\cdot}39 \times 0{\cdot}9 \times \cos(90° - \phi_i - 13{\cdot}5°)$ = 9·24 „

22·00 „

Power passing the centre of the connector $= 9{\cdot}24 - 0{\cdot}9^2 \times 1 = 8{\cdot}43$ „

„ delivered from machine 2 to network $1 = 8{\cdot}43 - 0{\cdot}9^2 \times 1 = 7{\cdot}62$ „

The angle between I_c' and the resultant I_{e1} is $(90° - \phi_i) + 13{\cdot}5° + \phi_e = 55{\cdot}3°$; the other component I_1 is thence found as 2·48, displaced behind I_{e1} by the angle 17·33°. The total power of machine 1 is then expended as follows—

In heating the stator, $2{\cdot}48^2 \times 0{\cdot}25$ = 1·54 watts

In network 1, $10{\cdot}39 \times 2{\cdot}48 \times \cos(\phi_e + 17{\cdot}33°)$ = 20·46 „

22·00 „

The required values for E_{i2} and E_{i1} are found trigonometrically from the tangents obtained when their several components projected on to the vertical and horizontal axes are respectively summed up. Thus for E_{i2}

Vertical component $= V_e \sin a/2 + z_a I_2 \sin\{\phi_a - (\phi_e - a/2 - 12{\cdot}05°)\}$

Horizontal „ $= V_e \cos a/2 + z_a I_2 \cos\{\phi_a - (\phi_e - a/2 - 12{\cdot}05°)\}$

and for E_{i1}

Vertical component $= V_e \sin a/2 - z_a I_1 \sin\{\phi_a - (\phi_e + a/2 + 17{\cdot}33°)\}$

Horizontal „ $= V_e \cos a/2 + z_a I_1 \cos\{\phi_a - (\phi_e + a/2 + 17{\cdot}33°)\}$

Thence $E_{i2} = 11{\cdot}61$ at an angle 27·38° above the horizontal

and $E_{i1} = 13{\cdot}22$ „ 2·95° below „

The respective powers can then be checked as

$P_2 = 11{\cdot}61 \times 2{\cdot}04 \times \cos\{(27{\cdot}38° + (\phi_e - a/2 - 12{\cdot}05°)\}$ = 22 watts

$P_1 = 13{\cdot}22 \times 2{\cdot}48 \times \cos\{(17{\cdot}33° + \phi_e + a/2) - 2{\cdot}95°\}$ = 22 „

If the resistances of the two machines were entirely negligible, the power crossing the centre of the connector would be exactly half the difference between the total network loads. But since the stator windings have some resistance, actually to secure exact equality of the loads, the power crossing the centre must be a little more than one-half of the difference, owing to the difference in the heating losses in the two machines.

§ 11. Governor regulation in parallel working.—Enough has been said to show that two similar alternators can be satisfactorily run in parallel when unequally excited or with unequal driving power, but that even under the most favourable conditions there is a certain loss of efficiency in the system due to the additional watts expended over the armature resistances by reason of the corrective current-component, which equalises their terminal voltages to a common value. Beyond the general conclusion that equal excitation has not the paramount importance that equality of frequency and synchronism of phase have, the above diagrams have chiefly a theoretical interest as assisting to a clear understanding of the phenomena. The relative dimensions of the vectors have been chosen not as representing practical cases, but merely to show how the action of machines in parallel depends upon the steam, water, or gas supplies to their respective prime movers, and so finally upon the governor which controls the supply of the working fluid. When once thrown into parallel, equal division of the load between two alternators cannot be secured as in continuous-current dynamos simply by altering the excitation of one. Any such change only alters the phase angle of the current relatively to the E.M.F., or, in other words, causes a cross-current magnetizing one machine and demagnetizing the other.

It is, however, desirable that all generators which are worked in parallel should have similar load characteristics, in which case when in parallel their separate field rheostats might be mechanically coupled together, or the exciting voltage of all may be simultaneously altered for any change of load by altering the rheostat in the shunt circuit of the exciter, without causing any cross current or loss of efficiency therefrom.

For a number of machines in parallel to share any permanent increase or decrease of the station load between them in strict proportion to their rated capacities, it is not merely sufficient that the total range of speed fixed by the governors should be alike; the changes of speed for increasing load should throughout the range be alike, and further the inherent regulation of each machine should have the same percentage value and their magnetization curves should bend over similarly. Such strict conditions cannot in practice be attained, but it will be evident that for ease of working a coarse regulation, both electrical and mechanical in the governor, is favourable to the sharing of the load in a definite manner between a number of units; that the slope of the speed curve from, say, three-quarters to full-load at the normal power factor should be the same is the requirement of chief importance.

Generators in parallel must run at exactly the same speed or at speeds strictly in inverse ratio to the number of poles; if, therefore, their speed regulation curves are different, as shown in Fig. 693, the one *A* having a regulation of 3½ per cent., and the other *B* of 2½ per cent., then if the governors are set to agree at full-load, and *the total load on the pair is halved, A will take* $\frac{5}{8}$ths and *B* $\frac{3}{8}$ths of the reduced load with a common rise of a little over 2 per cent. of the full-load speed ; should correct division be required, it can be secured without difficulty by hand adjustment of the governor or governors. If, on the other hand, the governors are set to agree at no-load, machine *B* will throughout take the greater share. If the two machines are of unequal capacity, the larger will preferably have the closer regulation. With a number of alternators in parallel, it is the machine of which the prime mover has the most sensitive governor that controls the speed of the rest, and thereby may affect the division of the load in due proportion between them.[1]

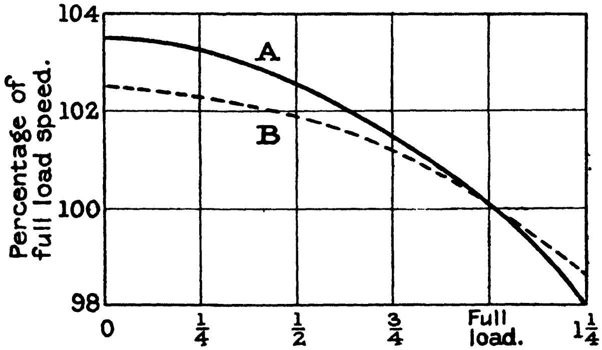

FIG. 693 —Difference in governor regulation.

§ 12. Synchronizing watts with two similar alternators and equal steam supply.—Equation (300) has a direct bearing upon the stability of running of alternators, which has now to be traced. Suppose that the steam supplies of the two similar alternators are equal, that they are equally excited and that they are running at the same mean speed, so that the true synchronous position of the rotor in each machine corresponds to coincidence of their E_i's, but suppose that for some reason at present unspecified, the rotor positions have diverged, so that the electrical angle between E_{i2} and E_{i1} is $\gamma_2 - \gamma_1 = 2\delta_e$. A line OC_n bisecting $2\delta_e$ gives the normal induced E.M.F. (*cp.* Fig. 689, which gives the present relation, but in terms of E_o and z_a') ; the normal mean current I_e is A_nC_n/z_a, and the mean output is $N_{ph}I_eE_i \cos COG$. But when the phases diverge, the point C_n travels inwards towards the centre to the position C (Fig. 688) ; the new mean current I_e' is, therefore, $I_e \cos \delta_e$, and the terminal voltage is also slightly reduced. At the same time the current component $C_1CC_2/2z_a$ arises, which will be symbolized as I_s, since, as is now to be shown, it is a synchronizing component in the total currents I_1 and I_2.

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[1] *Cp.* Prof. Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 215-217.

From equations (298) and (299) the changes of power when E_{i1}, E_{i2}, and E_{in} can be identified are then

in the leading machine 2

$$\begin{aligned}\Delta P_2 &= N_{ph} E_i \{I_e' \cos (COG + \delta_e) + I_s \sin (\phi_a + \delta_e) - I_e \cos COG\} \\ &= N_{ph} E_i [I_s \sin (\phi_a + \delta_e) - I_e \{\cos COG\,(1 - \cos^2 \delta_e) \\ &\qquad + \sin COG \sin \delta_e \cos \delta_e\}] \\ &= N_{ph} E_i [I_s \sin (\phi_a + \delta_e) - I_e \sin (COG + \delta_e) \sin \delta_e]\end{aligned}$$

and in the lagging machine 1

$$\begin{aligned}\Delta P_1 &= N_{ph} E_i \{I_e' \cos (COG - \delta_e) - I_s \sin (\phi_a - \delta_e) - I_e \cos COG\} \\ &= - N_{ph} E_i [I_s \sin (\phi_a - \delta_e) + I_e \{\cos COG\,(1 - \cos^2 \delta_e) \\ &\qquad - \sin COG \sin \delta_e \cos \delta_e\}] \\ &= - N_{ph} E_i [I_s \sin (\phi_a - \delta_e) - I_e \sin (COG - \delta_e) \sin \delta_e]\end{aligned}$$

Further, $I_s = \frac{E_i}{z_a} \sin \delta_e$, so that in each machine separately

$$\Delta P = \pm N_{ph} E_i \left\{\frac{E_i}{z_a} \sin \delta_e \sin (\phi_a \pm \delta_e) - I_e \sin (COG \pm \delta_e) \sin \delta_e\right\} \quad . \; . \; . \quad (302)$$

the upper signs when they differ referring to the leading machine, and the lower to the lagging machine. The change of power in the two machines is not precisely alike, being less in the leading than in the lagging machine. Treating ΔP_1 and ΔP_2 as not in themselves having any sign and adding them together

$$\Delta P_1 + \Delta P_2 = N_{ph} E_i \left\{\frac{E_i}{z_a} \sin \phi_a \sin 2\delta_e - I_e \sin COG \sin 2\delta_e\right\}$$

and analogously to (300), this becomes

$$\Delta P_1 + \Delta P_2 = N_{ph} E_i^2 \left\{\frac{x_a}{z_a^2} - \frac{x_a + 2X_e}{(r_a' + 2R_e)^2 + (x_a + 2X_e)^2}\right\} \sin 2\delta_e \quad . \; . \; . \quad (303)$$

similar to the difference $P_2 - P_1$ between the powers of two machines with unequal steam supply in steady work.

But the steam supplies have remained unchanged, and the driving torques are constant. The additional power to correspond to ΔP_2 watts will, therefore, when the initial disturbing cause is removed, in reality be obtained from the kinetic energy of the fly-wheel and rotor of machine 2, or it may be said that ΔP_2 divided by the actual angular velocity is an additional resisting torque, tending to bring the rotor back to its true synchronous position. On the other hand, the lessened electrical power of machine 1, divided by the actual angular velocity, gives a margin in its driving torque, and this surplus over the resisting torque tends to accelerate its rotor forwards and reduce its lag. In either machine, therefore, the tendency of the *synchronizing watts*, ΔP_2 or ΔP_1, which will now be symbolized as W_s is to bring the rotors back to their correct synchronous position. The sum of the retarding and accelerating changes of power, $\Delta P_2 + \Delta P_1$, is, therefore, often called the "synchronizing power," and in some cases as approximately the same, the difference of the external outputs, $\Delta P_{e2} + \Delta P_{e1}$. But more correctly the term, *synchronizing power*, should be limited to $d(\Delta P)/d\delta_e$, *i.e.* the rate of change of power for departures from the correct synchronous position.[1] This, of course, is proportional in the two machines to $\frac{d \sin 2\delta_e}{d\delta_e} = 2 \cos 2\delta_e$. When the machines are working in perfect parallel, and $\delta_e = 0°$, it is a maximum for the smallest divergence of phase; as the divergence is increased, it gradually falls off and becomes zero when $\delta_e = 45°$ and the two induced E.M.F.'s are in quadrature. As in § 9, there must be reactance present, and for any value of the resistance r_a' and angle of phase displacement the synchronizing power is a maximum when x_a is approximately $= r_a'$.

The opposite effects in the two machines are additive, since both combine to annul the divergence, and their sum corresponds to the total force holding the two machines in step.

There is thus a definite electric cause conducing to stability in parallel work and tending to prevent the machines from falling out of synchronism. It arises when there is present in the armature currents a component I_s lagging $\phi_a°$ behind the vector $\overline{E_{i2} E_{i1}}$, and from its phase having the direct effect of increasing the load on the machine of which the rotor is too far forward and of reducing the load on the machine of which the rotor has fallen back. Any such current or component of the total armature current which tends to alter the position of the rotor and its poles relatively to a steadily rotating current vector so as to bring the machine into such phase-relationships as satisfy the conditions of excitation or steam supply, may be defined as a *synchronizing* current; it does not necessarily tend to actual coincidence of the phase of either the induced or the armature E.M.F. of the alternator with that of other alternators connected to the same network when they are dissimilar or unequally supplied with steam, but it causes such a slowing down or speeding up of the rotor as would at that moment bring it more nearly into a position of stability for continuous running.

From the case chosen to illustrate a synchronizing current, it will have been seen that the same component current would have been "corrective" or I_c' if the steam supplies were actually unequal, and the rotors maintained their divergent positions, but becomes " synchronizing " or I_s when the rotors are not in their true running position. A synchronizing current is, therefore, from its nature either temporary or always varying in magnitude. To distinguish the two, the component I_c at right angles to V_e was first taken in § 6 as the permanent " corrective " component.

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[1] But *cp.* E. S. Henningsen, *Gen. Elect. Rev.*, Vol. 24, p. 951.

§ 13. The synchronizing watts of two similar or dissimilar alternators, with unequal steam supplies.—The preceding expressions may also be widened to cover the case of two alternators having similar electrical properties and excitation but unequal steam supplies. If the normal angle between E_{t2} and E_{t1} is $\gamma_2-\gamma_1$ and this is supposed to have for some reason altered to $\gamma_2-\gamma_1 \pm 2\delta_e$,

$$\Delta P_2 = N_{ph} E_i \Big\{ I_e' \cos\left(COG + \frac{\gamma_2-\gamma_1}{2} + \delta_e\right)$$

$$+ \frac{E_t}{z_a} \sin\left(\frac{\gamma_2-\gamma_1}{2} + \delta_e\right) \sin\left(\frac{\gamma_2-\gamma_1}{2} + \delta_e + \phi_a\right)$$

$$- I_e \cos\left(COG + \frac{\gamma_2-\gamma_1}{2}\right) - \frac{E_i}{z_a} \sin\frac{\gamma_2-\gamma_1}{2} \sin\left(\frac{\gamma_2-\gamma_1}{2} + \phi_a\right)\Big\}$$

$$\Delta P_1 = N_{ph} E_t \Big\{ I_e' \cos\left(COG - \frac{\gamma_2-\gamma_1}{2} - \delta_e\right)$$

$$+ \frac{E_t}{z_a} \sin\left(\frac{\gamma_2-\gamma_1}{2} + \delta_e\right) \sin\left(\frac{\gamma_2-\gamma_1}{2} + \delta_e - \phi_a\right)$$

$$- I_e \cos\left(COG - \frac{\gamma_2-\gamma_1}{2}\right) - \frac{E_i}{z_a} \sin\frac{\gamma_2-\gamma_1}{2} \sin\left(\frac{\gamma_2-\gamma_1}{2} - \phi_a\right)\Big\}$$

$$= - N_{ph} E_t \Big\{ - I_e' \cos\left(COG - \frac{\gamma_2-\gamma_1}{2} - \delta_e\right)$$

$$- \frac{E_t}{z_a} \sin\left(\frac{\gamma_2-\gamma_1}{2} + \delta_e\right) \sin\left(\frac{\gamma_2-\gamma_1}{2} + \delta_e - \phi_a\right)$$

$$+ I_e \cos\left(COG - \frac{\gamma_2-\gamma_1}{2}\right) + \frac{E_i}{z_a} \sin\frac{\gamma_2-\gamma_1}{2} \sin\left(\frac{\gamma_2-\gamma_1}{2} - \phi_a\right)\Big\}$$

Ignoring the difference in the signs outside the braces, which shows that the one is an increase and the other a decrease, and noting that

$$I_e' = I_e \frac{\cos\left(\frac{\gamma_2-\gamma_1}{2} + \delta_e\right)}{\cos\frac{\gamma_2-\gamma_1}{2}} = \frac{E_t}{2Z_t} \cos\left(\frac{\gamma_2-\gamma_1}{2} + \delta_e\right),$$

$$\text{and } I_e = \frac{E_t}{2Z_t} \cos\frac{\gamma_2-\gamma_1}{2}$$

the two added together give

$$\Delta P_2 + \Delta P_1 = N_{ph} E_t \left[\frac{E_t}{z_a} \left\{ \sin(\gamma_2 - \gamma_1 + 2\delta_e) - \sin(\gamma_2 - \gamma_1) \right\} \sin \phi_a \right.$$

$$\left. - \frac{E_t}{2Z_t} \left\{ \sin(\gamma_2 - \gamma_1 + 2\delta_e) - \sin(\gamma_2 - \gamma_1) \right\} \sin COG \right]$$

$$= N_{ph} E_t^2 \left(\frac{\sin \phi_a}{z_a} - \frac{\sin COG}{2Z_t} \right) \left\{ \cos(\gamma_2 - \gamma_1) \sin 2\delta_e - 2 \sin(\gamma_2 - \gamma_1) \sin^2 \delta_e \right\} \quad . \quad . \quad . \quad (304)$$

§ 14. The synchronizing watts and synchronizing torque of a single machine coupled to a large network.—In the case of a single alternator which is coupled to a network and in which the true synchronous position of the rotor under the given steam supply and excitation corresponds to some angle γ between E_t and V_e, again let it be supposed that for some reason at present unspecified the rotor has diverged to a new position such that the angle between E_i and V_e is $\gamma \pm \delta_e$. If the network and the size of the other machines supplying it are very large as compared with that of the alternator under consideration, any reaction of the latter upon the system will be negligible. The vector of the terminal voltage V_e may now be regarded as unalterable in speed and in magnitude; only one half of the diagram is before us, and the result is slightly different. The total normal watts in steady work should be from § 4

$$P_n = N_{ph} \frac{E_t}{z_a} \left\{ E_t \cos \phi_a - V_e \cos(\phi_a + \gamma) \right\}$$

while the actual watts are

$$P_1 = N_{ph} \frac{E_i}{z_a} \left\{ E_t \cos \phi_a - V_e \cos(\phi_a + \gamma \pm \delta_e) \right\}$$

The power to overcome the additional resisting torque when δ_e is positive, or the lessened power when the rotor lags is, therefore,

$$W_s = P_1 - P_n = \pm N_{ph} \frac{E_t V_e}{z_a} \left\{ \cos(\phi_a + \gamma) - \cos(\phi_a + \gamma \pm \delta_e) \right\}$$

When expanded, this becomes

$$W_s = N_{ph} \frac{E_t V_e}{z_a} \left\{ \sin(\phi_a + \gamma) \sin \delta_e \pm 2 \cos(\phi_a + \gamma) \sin^2 \frac{\delta_e}{2} \right\}$$

In all the above cases it is assumed that the maximum displacement falls within the second stage described in § 4, where the running as a generator is stable, *i.e.* that the varying values of the new γ fall within the limits of γ_2 (when $E_i \cos(\phi_a - \gamma_2) = V_e \cos \phi_a$), and $\pi - \phi_a$. Within these limits an increase of the angle γ by the amount δ_e calls for more power, and W_s is really synchronizing.

§ 15. The general action of the synchronizing current.—The action of the synchronizing torque due to the synchronizing current in all cases has the same general effect. Suppose that the initial cause of the divergence of the rotor from its stable position is removed or reversed; the rotor begins to move back to the synchronous position, but acquiring momentum under the action of the synchronizing torque it overshoots the correct position, only to be recalled by the synchronizing torque which itself is reversed, so that it again passes the correct position; oscillation about its mean or normal position, or "phase swinging," is thereby set up. Thus the synchronizing current, which always tends to bring the rotor back to its correct position for stable running, never in itself attains the desired end owing to the necessary presence of a certain amount of inertia in the rotor. Even if the disturbing cause is not itself periodically repeated, the oscillations would continue indefinitely, unless damped out; damping forces are usually present, and, as will be described later, gradually lessen the oscillations, until they cease after a short time.

§ 16. Proportionality of the synchronizing watts to the angle δ_e.—In the full expression (304) for $\Delta P_1 + \Delta P_2$ for two alternators having similar electrical properties but unequal steam supplies, if the total divergence $2\delta_e$ is small, and $\sin^2\delta_e$ can be neglected, the terms between the braces reduce to $\cos(\gamma_2 - \gamma_1)\sin 2\delta_e$. Lastly, if $2\delta_e$ radians may be written for $\sin 2\delta_e$, the whole expression reduces to

$$N_{ph} E_i^2 \left(\frac{\sin\phi_a}{z_a} - \frac{\sin COG}{2Z_t}\right) \cos(\gamma_2 - \gamma_1) \times 2\delta_e \quad . \quad (305)$$

In the case of § 12 for two similar alternators equally supplied with steam, $\gamma_2 - \gamma_1 = 0$. In either case the synchronizing watts actually increase less rapidly than the angle of phase displacement, although the divergence from proportionality is not very great for reasonable values of δ_e. But when the full expressions for each machine separately are examined, it is found that there is a greater error in assigning half of $\Delta P_1 + \Delta P_2$ to each machine, although this approximate assumption may be made for most purposes of calculation.

The synchronizing watts are thus affected by the external load, but on no-load or very light loads they reduce in each machine separately to the approximate value

$$\Delta P = W_s = N_{ph} E_i^2 \frac{\sin\phi_a}{z_a} \times \delta_e \quad . \quad . \quad . \quad (306)$$

In the last case of a single alternator connected to a network fed by other much more powerful generators and, therefore, with constant pressure vector (§ 14) upon the same assumption of such a small divergence that $\sin^2(\delta_e/2)$ may be neglected and $\sin\delta_e = \delta_e$

$$W_s = N_{ph}\left\{\frac{E_i V_e}{z_a}\sin(\phi_a+\gamma)\right\}\delta_e$$

which at switching into parallel when there is no surplus driving torque from the steam supply, and when $E_0 = E_i = V_e$ so that $\gamma = 0$, reduces to the same expression as above, viz.,

$$W_s = N_{ph}\left(\frac{E_i^2}{z_a}\sin\phi_a\right)\delta_e = \text{approx. } N_{ph}\left(\frac{E_0^2}{z_a'}\sin\phi_a'\right)\delta_e$$

In a recorded case,[1] to tally with the experimentally-observed frequencies of oscillation of 1·133 and 0·833 per second (the latter with added resistance and reactance), when two 2-phase alternators were paralleled slightly out of phase at 6600 volts, the synchronizing watts per radian should be 9280 and 5000 kilowatts respectively. The constants were

r_a = 0·16, and with added resistance 3·76 ohms
x_a = 1·6, ,, ,, reactance 7·2 ,,
x_b = 8
whence x_a' = 9·6 or 15·2
z_a' = 9·605 ,, 15·65 and $\sin\phi_a' = x_a'/z_a'$.

To illustrate, then, the approximate validity of the above simplified formulæ on switching into parallel on no-load,

$2 \times 6600^2 \times 9{\cdot}6/(9{\cdot}605)^2$ = 9050 kW per radian
$2 \times 6600^2 \times 15{\cdot}2/(15{\cdot}65)^2$ = 5400 ,, ,,

which would give frequencies of 1·115 and 0·865.

§ 17. Influence of the ratio of reactance to resistance.—Since $\sin\phi_a = \frac{x_a}{z_a}$, it is evident that reactance must be present to give a synchronizing effect to the change of power. As in §§ 9 and 10, for a given value of r_a', the maximum effect is produced when x_a is approximately = r_a', and on no-load, as *e.g.* on switching into parallel, when x_a is exactly = r_a'. The angle ϕ_a would then be 45°, and since $z_a = r_a'/\cos\phi_a$,

$$\frac{\sin\phi_a}{z_a} = \frac{\sin\phi_a \, . \, \cos\phi_a}{r_a'} \text{ is a maximum.}$$

Such a low value of the reactance would, however, subject the machine to great mechanical stresses due to the racking action of the synchronizing current, so that the higher ratios of the reactance to the resistance which alone occur in practice with toothed armatures are, in fact, better from the point of view of parallel working. The sudden acceleration or retardation of many tons of metal may evidently impose very severe stresses on the windings and mechanical construction.

§ 18. The synchronizing watts in terms of the short-circuit current.—For E_i/z_a may be substituted OD/z_a or approximately E_0/z_a', with some loss of accuracy in the powers thus obtained. But, as explained in Chap. XXVII, § 43, it is only under rare conditions[2] that E_0/z_a' may be identified with

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[1] A. R. Everest, "Some Factors in the Parallel Operation of Alternators," *Journ. I.E.E.*, Vol. 50, p. 526; and Mr. Shuttleworth in discussion thereon, p. 552.

[2] In the absence of these conditions, the substitution of I_{sc} for E_0/z_a' yields far too high values for the natural periodic time. *Cp.* A. R. Everest, *loc. cit.*

$E_0/z_{sc} = I_{sc}$, the current that would flow through the armature if short-circuited under the same excitation, in which case, on switching into parallel, W_s would become $N_{ph}E_0I_{sc} \sin\phi_a' \times \delta_e$.

§ 19. The synchronizing torque as proportional to the angular displacement.—If in a synchronous generator or motor the rotor owing, say, to some alteration of load is not in the true relative position for steady running when the angle between induced E.M.F. and terminal voltage has the value γ corresponding to the torque T arising from the load, but is so displaced that this angle actually is $\gamma \pm \delta_e$, the unbalanced torque available for accelerating or retarding the rotor is $\delta_e \dfrac{dT}{d\gamma}$. This is best followed in the case of a synchronous motor, in which the action is not complicated by any effect from a mechanical governor; under an increase of load, let its induced E.M.F. have fallen back behind V_s by the increased angle $\gamma + \delta_e$, so that its torque, which was T, is $T - \delta_e \dfrac{dT}{d\gamma}$, δ_e being taken as negative when in the direction of rotation of the motor. Then upon removal of the increase in the load, there is an unbalanced amount $\delta_e \dfrac{dT}{d\gamma}$ of torque to advance the rotor forwards.

When once it is granted that the synchronizing watts W_s may for practical purposes of calculation be treated as approximately proportional to the electrical angle of displacement, $dT/d\gamma$ becomes practically a constant. For the angular velocity may for small oscillations be taken as constant at its mean normal value of ω radians per second $= 2\pi N/60$, so that the *synchronizing torque* is $T_s = W_s/\omega$, corresponding units being employed as will be explained in § 22. Thence

$$T_s = \delta_e T_{eu} = \frac{\delta_e}{p} T_u = \theta \,.\, T_u$$

where T_{eu} = the synchronizing torque per radian of electrical displacement

T_u = the synchronizing torque per radian of mechanical displacement.

Since each pole-pair represents 360 electrical degrees, the mechanical angle θ corresponding to an electrical displacement δ_e will vary inversely as the number p of pole-pairs, or $\theta = \delta_e/p$, and T_u is p times T_{eu}.

The acceleration or retardation of the rotor by the synchronizing torque causes the rotor to catch up, or to fall back upon, the true position of steady running, but upon arriving there the rotor has more or less than its true steady velocity. In consequence, it does not then exert the steady-motion torque corresponding to the constant losses and to the steady electrical load, if any, but there is still a balance of torque accelerating or retarding it due to the speed being incorrect. If then all the varying conditions are mentally isolated from the steady conditions, the problem becomes amenable to the ordinary treatment of a mechanical system executing rotary oscillations, in which the controlling force is proportional to the angular displacement from the central position of rest or in our case of steady running.

§ 20. Rotary oscillation of a mechanical system.—Linear mechanical oscillations have been already described in Vol. I, Chapter VI, § 23, and now when we come to circular or rotary mechanical oscillations, the whole of the preceding results are immediately applicable when converted to meet the case of oscillation through the arc of a circle under the action of a true tangential force or twisting moment. In place of a body of mass M executing simple harmonic movements with varying velocity v under the action of an applied force $\mathbf{F}$, there must be substituted a body with moment of inertia Mk^2 (where k is the radius of gyration) executing oscillatory movements round its centre with varying angular velocity ω under a twisting moment or torque $\mathbf{T}$; an instance of this would be a heavy body hung from a wire, or a heavy ring fastened by a clock-spring to a central axis, acted on by a torque which twists it through an angle of θ radians.

If there is a controlling torque or in the electrical case a synchronizing torque proportional to the angular displacement from the central position of rest, also a couple or torque proportional to the angular velocity of oscillation $d\theta/dt$, *i.e.* $b\frac{d\theta}{dt}$, which may be damping or otherwise, and an external periodically varying torque of which the instantaneous value is $\mathbf{T}_0 \sin pt$, where $p = \frac{2\pi}{T_e}$, the general equation of the motion is

$$Mk^2 \frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + T_u \, . \, \theta = \mathbf{T}_0 \sin pt \quad . \quad . \quad (307)$$

exactly analogous to equation (20).

T_u is the proportionality factor between synchronizing torque and mechanical displacement. The steady driving and resisting torques cancel out, and the steady angular velocity similarly does not come into question, so that it is only the small changes in this velocity, *i.e.* $d\theta/dt$ giving the velocity of oscillation, with which we have to deal.

§ 21. The oscillations set up by the synchronizing torque of § 12.—When the driving torque is unvarying, or $\mathbf{T}_0 = 0$, the equation of motion reduces to that for the natural oscillation of the system, viz.

$$Mk^2\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + T_u\theta = 0 \quad . \quad . \quad . \quad . \quad (308)$$

The moment of inertia Mk^2 of the rotor multiplied by its angular acceleration $d^2\theta/dt^2$ is equal to the sum of the moments of the two remaining forces acting on the rotor about its axis of rotation. The nature of the oscillations set up as dependent on the value of b has been already explained in Chapter VI, § 23, but is here repeated. Usually $4T_u . Mk^2 > b^2$, and in this case the solution[1] of the differential equation is of the form

$$\theta = \varepsilon^{-\frac{b}{2Mk^2}.t}\,(C \sin p_1t + D \cos p_1t)$$

$$\text{where } p_1 = \frac{\sqrt{4T_u . Mk^2 - b^2}}{2Mk^2} \quad . \quad . \quad (308a)$$

It follows that if b is negative, however small its value, the amplitude of the oscillations continually increases until the machine falls out of step. If b is positive, the amplitude of the oscillations continually diminishes until they are finally damped out. If the damping is very small, we approach the ideal case of $b = 0$, *i.e.* $Mk^2 . \frac{d^2\theta}{dt^2} = - T_u . \theta$, or the angular acceleration is proportional simply to the synchronizing torque, and, as in the case of an undamped pendulum, the amplitude of the first oscillation is continually repeated unchanged ; there is nothing to increase it, and no damping to diminish it. In the assumed case that the exciting current is maintained absolutely constant, and that the field-magnets, armature core, and conductors are perfectly laminated, so that there is no damping from eddy-currents in any part of the machine, the value of b is approximately[2]

$$-\frac{r_a}{4\pi^2 f^2} . E_o^2 \frac{x_a^2 - r_a^2}{(x_a^2 + r_a^2)^2}.$$

Hence, if the resistance of the armature is negligible, the tendency is for the last case to be reproduced, *i.e.* oscillations of constant amplitude indefinitely repeated. If the reactance is less than the resistance, b is positive and the running is stable ; but since in practice x_a is invariably greater than r_a, the running is, so far as the present assumptions hold true, inherently unstable with any variation of load or of driving torque, although the negative value of b thus calculated is very small.

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[1] For the plotting of a curve of θ, see Prof. Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, p. 229.

[2] See Prof. B. Hopkinson, *Proc. Royal Soc.*, Vol. 72, p. 235 ; " The ' Hunting ' of Alternating-current Machines," and paper on " The Parallel Working of Alternators," British Assoc., 1903, reprinted in *Electr. Eng.*, Vol. 32, p. 467 ; also, Dr. A. Russell, *The Theory of Alternating Currents*, Vol. 2 (2nd ed.), pp. 210-223.

There is, however, in addition a true damping effect due to eddy-currents set up in the iron of solid pole-pieces, and also (except in special cases) to variation produced in the current flowing round the poles in the exciting coils. When phase-swinging or free oscillations are set up, the combined fundamental M.M.F. of the armature ampere-turns is no longer stationary in respect to the poles even in the polyphase machine; as it moves, both the direct magnetizing wave and the cross wave vary, so that there results a change both in the total flux of the field and also in the amount of its distortion. The general effect of the eddy-currents thereby set up is to diminish the amplitude of the changes of flux in the pole-pieces, and to cause them to lag a little behind the changes in the M.M.F. waves to which they are due. Whether this will render b positive and assist the damping will depend on their effect upon the torque at the moment when the rotor passes its position of steady motion with more than its steady-motion velocity. Prof. B. Hopkinson[1] has shown that the direct magnetization due to eddy-currents in short-circuited coils which merely surround the poles or variation in the exciting current may under certain conditions of load (if the watt current in phase with the armature E.M.F. is $<\dfrac{\sqrt{2}E_o\,.\,r_a}{x_a^2+r_a^2}\Big)$ increase the instability, but on the other hand the effect of eddy-currents crossing the face of solid poles always causes a true damping effect. Chiefly, therefore, from the additional damping given in general by eddy-currents in the iron and copper of both armature and field, it results that b is usually positive, even without any special damper winding, and the free oscillations are quickly damped out, so that alternators can be run with success in parallel.

With damping taken into account when $4\,T_u\,.\,Mk^2 > b^2$, it follows from (308*a*) that the full expression for the periodic time of a free oscillation with damping is

$$T_{pd}=\frac{2\pi}{p_1}=\frac{4\pi\,Mk^2}{\sqrt{4T_u\,.\,Mk^2-b^2}} \quad . \quad . \quad . \quad . \qquad (309)$$

§ 22. The natural periodic time of a free undamped oscillation.—But it is the periodic time of a free undamped oscillation T_p that possesses the more importance from its bearing upon the possibility of resonance between the frequencies of a free and a forced oscillation.

This may be obtained at once from equation (308*a*); p_1 takes the value $p_o=\sqrt{\dfrac{T_u}{Mk^2}}$, and $T_p=2\pi\sqrt{\dfrac{Mk^2}{T_u}}$. Or it may be derived by the same reasoning as in Chapter VI, § 23 (*a*) from the equation

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[1] *Electr. Eng.*, Vol. 32, p. 470.

$$\tfrac{1}{2} Mk^2 (\omega^2_{max} - \omega^2) = \int_0^\theta T_u \theta \, . \, d\theta$$

Thence

$$\omega = \sqrt{\omega^2_{max} - \frac{T_u \, . \, \theta^2}{Mk^2}}$$

and when $\omega = 0$, θ reaches its maximum value θ_{max}, or

$$\omega_{max} = \theta_{max} \sqrt{\frac{T_u}{Mk^2}}$$

When developed on the flat the linear displacement at any moment is equal to the arc of the angular displacement, or $s = \theta r$, and the radius of the circle round which a point moving with uniform velocity will give the equivalent harmonic motion is $s_{max} = \theta_{max} r$. The maximum velocity as the body passes the centre being $\omega_{max} r$, this is also equal to the uniform velocity $\frac{2\pi \, . \, \theta_{max} r}{T_p}$, whence $\omega_{max} = \frac{2\pi \, . \, \theta_{max}}{T_p}$

Equating the two expressions for ω_{max}, namely,

$$\theta_{max} \sqrt{\frac{T_u}{Mk^2}} = \frac{2\pi \, . \, \theta_{max}}{T_p}$$

$$T_p = 2\pi \sqrt{\frac{Mk^2}{T_u}} \quad . \quad . \quad . \quad (310)$$

or in words,

$$T_p = 2\pi \sqrt{\frac{\text{moment of inertia}}{\text{torque per mechanical radian}}}$$

$$= 2\pi \sqrt{\frac{\text{angular displacement}}{\text{angular acceleration}}}$$

since the actual torque T = moment of inertia × angular acceleration.

In the case of an alternator when the periodic time of a free oscillation is expressed in terms of $T_{eu} = T_s/\delta_e$, or the synchronizing torque which gives one radian of electrical displacement, rather than in terms of T_u, the synchronizing torque which gives one radian of mechanical displacement

$$T_p = 2\pi \sqrt{\frac{Mk^2}{p \, T_{eu}}} = 2\pi \sqrt{\frac{Mk^2}{p \, T_s/\delta_e}}$$

It only then remains to secure the expression of the synchronizing torque and moment of inertia in corresponding units.

On the C.G.S. system, if Mk^2 be given in kilogrammes of mass × square of radius of gyration k in metres, $Mk^2 \times 10^7$ will give the moment of inertia in the C.G.S. unit of grammes - (centimetres)^2, in agreement with the torque in dyne-centimetres. In British units, if the torque be in lb.-feet, the moment of inertia must be in engineers' mass-units × (feet)^2 $= \frac{W}{g} \times r^2 \times 2240$, where W is the weight in tons, and r is the radius of gyration in feet.

In both cases it is, however, more usual to express the torque through the synchronizing watts or kilowatts, coupled with division by ω or N, explicit or hidden. On the C.G.S. system, $W_s \times 10^7$ = ergs per second, and $\frac{W_s}{\omega} \times 10^7 = T_s$ in dyne-centimetres. Consequently with Mk^2 in kilogrammes - (metres)^2, the numerical factors cancel out, and

$$T_p = 2\pi \sqrt{\frac{Mk^2}{p\, W_s/\delta_e\, \omega}} \text{ seconds}$$

To avoid the need for the fraction W_s/δ_e, let

$W_{eu} = W_s/\delta_e$, the synchronizing watts per radian of electrical displacement,

$kW_{eu} = kW_s/\delta_e$, the synchronizing kilowatts per radian of electrical displacement, the magnitude of a radian being such that W_s per radian is usually many kilowatts. Then, since $\omega = 2\pi f/p$ or $2\pi N/60$, where f is the machine frequency, the preceding expression becomes

$$T_p = \frac{0{\cdot}497}{p}\sqrt{\frac{f \,.\, Mk^2}{(k\,W_{eu})}} = 0{\cdot}0083\, N \sqrt{\frac{Mk^2}{f\,(k\,W_{eu})}} \text{ seconds}$$

The reciprocal, or the frequency of the oscillations per second is

$$f_o = 2{\cdot}01\, p \sqrt{\frac{(kW_{eu})}{fMk^2}} = \frac{120{\cdot}5}{N}\sqrt{\frac{f\,(kW_{eu})}{Mk^2}}$$

If G = the weight in metric tons, *i.e.* in units of 1000 kilogrammes, $M = 1000\, G$, and if D = the diameter of gyration in metres, so that $k^2 = D^2/4$,

$$f_o = 0{\cdot}127\, p \sqrt{\frac{(kW_{eu})}{fGD^2}} = \frac{7{\cdot}62}{N}\sqrt{\frac{f\,(kW_{eu})}{GD^2}}$$

where GD^2 is in metric tons - (diameter in metres)^2, the unit by which the moments of inertia of fly-wheels are usually compared on the Continent.

In British units, $\frac{W_s \times 550}{746}$ = ft.-lb. per second, and T_s in lb.-feet $= \frac{kW_s \times 550 \times 1000}{\omega \times 746} = 737\ (kW_s)/\omega$. Hence when Wr^2 is in tons-ft.^{2},

$$T_p = 2\pi\sqrt{\frac{Wr^2\,\omega \times 2240}{gp\,(kW_{eu}) \times 737}},$$

and substituting the same expressions for ω,

$$T_p = \frac{4\cdot82}{p}\sqrt{\frac{f\,Wr^2}{(kW_{eu})}} = 0\cdot0805\,N\sqrt{\frac{Wr^2}{f\,(kW_{eu})}}\ \text{seconds}$$

$$f_o = 0\cdot207\,p\sqrt{\frac{(kW_{eu})}{f\,Wr^2}}\ \text{or}\ \frac{12\cdot42}{N}\sqrt{\frac{f\,(kW_{eu})}{Wr^2}} \quad . \quad . \quad (311)$$

Finally, if the energy stored in the rotating parts at the normal speed N be calculated, the reference to N can be eliminated;[1] for $\frac{1}{2}\,Mk^2\,\omega^2$ in foot-tons $= \frac{\text{lb.-ft.}^2}{2 \times 32\cdot2 \times 2240} \times \left(\frac{2\pi N}{60}\right)^2$, so that tons-ft.$^2 \times N^2$ = foot-tons $\times$ 5870, and

$$f_o = 0\cdot1625\sqrt{\frac{f(kW_{eu})}{\text{foot-tons}}} = 2\cdot86\sqrt{\frac{f\,(kW_{eu})}{\text{kilogrammetres}}} \quad . \quad (312)$$

(Foot-tons of energy × 2240 × 0·13825 = kilogrammetres)

The several expressions for T_p and f_o are given as it is variously expressed by different writers. It must be borne in mind that W or M in lb. or tons or kilogrammes must include all rotating parts (magnet, fly-wheel, crank discs, etc.).

The expressions for the periodic time of a free oscillation have been confirmed very closely by actual experiment;[2] it varies as the square root of the moment of inertia, and inversely as the square root of $E_0{}^2$, *i.e.* inversely as E_0, which is roughly proportional to the excitation.

The above expressions hold good either for two similar machines alone in parallel under a rhythmically varying driving torque, or for a single machine on a relatively very large network; in the former case the mean induced E.M.F. remains unaltered in phase, and in the latter case the terminal voltage remains stationary in phase, so that we have, as it were, one-half of the diagram of the former case.

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[1] *Cp.* A. R. Everest, *Journ. I.E.E.*, Vol. 50, p. 533.

[2] *Cp.* M. Paul Boucherot, "Coupling Fly-wheel Alternators in Parallel," *Trans. Intern. Electr. Congress St. Louis*, 1904, Vol. 1, p. 693.

§ 23 The natural periodic time of a free undamped oscillation for two dissimilar alternators.—Next let the two alternators be dissimilar in their number of poles, moments of inertia, or electrical constants—any or all combined. Assuming that the average driving and resisting torques and average speeds remain constant, so that the total input of mechanical power into the system and total output of electrical power are unaffected, the kinetic energy of the system remains constant; ΔP_1 and ΔP_2 must then be equal to ΔP or W_s. If θ_1 and θ_2 are the two mechanical displacements of the two rotors from their stable positions, corresponding to the abnormal electrical difference of phase β_e between the induced E.M.F.'s of the two machines, $\beta_e = p_1\theta_1 - p_2\theta_2$, one angle being positive and the other negative. The changes of torque must have opposite signs, and with negligible damping are in each of the two machines separately

$$\frac{\Delta P}{\omega_2} = M_2 k_2^2 \frac{d^2\theta_2}{dt^2} \text{ and } -\frac{\Delta P}{\omega_1} = M_1 k_1^2 \frac{d^2\theta_1}{dt^2}$$

ΔP being treated as not in itself having any sign.

Multiplying each expression by its respective value p_1 or p_2,

$$p_1 M_1 k_1^2 \frac{d^2\theta_1}{dt^2} + \frac{p_1 \Delta P}{\omega_1} = 0$$

$$p_2 M_2 k_2^2 \frac{d^2\theta_2}{dt^2} - \frac{p_2 \, . \, \Delta P}{\omega_2} = 0$$

Subtracting

$$\frac{d^2(p_1\theta_1 - p_2\theta_2)}{dt^2} + \left(\frac{p_1 \, . \, \Delta P}{\omega_1 M_1 k_1^2} + \frac{p_2 \, . \, \Delta P}{\omega_2 M_2 k_2^2}\right) = 0$$

$$\frac{d^2\beta_e}{dt^2} + \frac{\Delta P}{\beta_e}\left(\frac{p_1}{\omega_1 M_1 k_1^2} + \frac{p_2}{\omega_2 M_2 k_2^2}\right)\beta_e = 0$$

$$\frac{d^2\beta_e}{dt^2} + \mu\beta_e = 0$$

The solution being $\beta_e = A \sin(\sqrt{\mu} \, . \, t + \eta)$, the motion is simple harmonic, and the periodic time of the combination $T_p = 2\pi/\sqrt{\mu}$, or $f_o = \sqrt{\mu}/2\pi$, where

$$\mu = \frac{W_s}{\beta_e}\left(\frac{p_1}{\omega_1 M_1 k_1^2} + \frac{p_2}{\omega_2 M_2 k_2^2}\right) \quad . \quad . \quad (313)$$

Since $\frac{p}{\omega} = \frac{60^2 f}{2\pi N^2}$, with the moments of inertia in tons-ft.^{2}, this becomes

$$\mu = \frac{60^2 \times g}{2\pi \times 1{\cdot}35 \times 2240} \times \frac{f W_s}{\beta_e}\left(\frac{1}{N_1^2 W_1 r_1^2} + \frac{1}{N_2^2 W_2 r_2^2}\right)$$

Let T_1 and T_2 be the foot-tons of energy stored in the two machines respectively at their normal speeds. Then with kW_s kilowatts

$$\mu = \frac{60^2 \times g \times 10^3}{2\pi \times 1{\cdot}35 \times 2240 \times 5870} \times \frac{f(kW_s)}{\beta_e}\left(\frac{1}{T_1} + \frac{1}{T_2}\right)$$

$$= 1{\cdot}04\ \frac{f(kW_s)}{\beta_e}\left(\frac{T_1 + T_2}{T_1 T_2}\right)$$

$$\text{and } f_o = 0{\cdot}1625\sqrt{\frac{f(kW_s)}{\beta_e}\left(\frac{T_1 + T_2}{T_1 T_2}\right)}$$

This disposes of the mechanical differences between the two machines, but since their electrical constants may also differ, we have

$$\beta_e = \delta_{e1} + \delta_{e2}$$

Now in order to express the new combination in terms of the electrical properties of the separate machines, and retain their original values for kW_{eu} in the expression, each machine must be paired with another machine electrically similar; δ_{e1} would then be W_s/W_{eu1} and δ_{e2} would be W_s/W_{eu2}, that is, for the same W_s the proportion of β assignable to each machine will be inversely proportional to its synchronizing kilowatts per radian of electrical displacement. Or in terms of kilowatts

$$\delta_{e1} + \delta_{e2} = \frac{kW_s}{kW_{eu1}} + \frac{kW_s}{kW_{eu2}}$$

$$= kW_s \frac{kW_{eu1} + kW_{eu2}}{(kW_{eu1})\ (kW_{eu2})}$$

Substituting this value for β, the generalized expression[1] is

$$f_o = 0{\cdot}1625\sqrt{f \times \frac{(kW_{eu1})\ (kW_{eu2})\ (T_1 + T_2)}{T_1\, T_2\, (kW_{eu1} + kW_{eu2})}} \quad . \quad (314)$$

If one alternator, although similar in electrical design, has a very large moment of inertia, so that, *e.g.* $M_2 k_2^2$ renders the second term of eq. (313) negligible, the case is equivalent to a small alternator, coupled to very large alternators having a constant vector of terminal P.D. The displacement is then entirely in one machine, and β becomes equal to δ_e of one machine. The periodic time would then reduce to the same as for a single machine coupled to a large network.

The above expression in terms of one of the two machines may be derived as follows. The accelerating and retarding torques, being inversely as the mechanical speeds, vary directly as the numbers of poles, or

$$\frac{T_{s2}}{T_{s1}} = \frac{\Delta P/\omega_2}{\Delta P/\omega_1} = \frac{\omega_1}{\omega_2} = \frac{p_2}{p_1}$$

The variable velocities of oscillation ω_{v2} and ω_{v1} (not of constant rotation) from the two torques acting respectively on $M_2k_2^2$ and $M_1k_1^2$, and also the displacements, will have the constant ratio

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[1] *Cp.* A. R. Everest, *Journ. I.E.E.*, Vol. 50, p. 529.

$$\frac{\omega_{v2}}{\omega_{v1}} = \frac{p_2 M_1 k_1^2}{p_1 M_2 k_2^2} = \frac{\theta_2}{\theta_1}$$

and, reckoning both angles as positive,

$$\theta_1 + \theta_2 = \theta = \theta_1\left(1 + \frac{p_2 M_1 k_1^2}{p_1 M_2 k_2^2}\right)$$

But $\theta = \frac{\delta_{e1}}{p_1} + \frac{\delta_{e2}}{p_2}$; and if each machine was paired with another machine with similar electrical properties, this is

$$\begin{aligned}\theta &= W_s\left(\frac{1}{p_1 W_{eu1}} + \frac{1}{p_2 W_{eu2}}\right)\\ &= \frac{W_s}{p_1 W_{eu1}}\left(1 + \frac{p_1 W_{eu1}}{p_2 W_{eu2}}\right)\end{aligned}$$

Equating the two values of θ, we have

$$\frac{W_s}{p_1\theta_1} = \frac{W_s}{\delta_{e1}} = W_{eu1}\left\{\frac{1 + \dfrac{p_2 M_1 k_1^2}{p_1 M_2 k_2^2}}{1 + \dfrac{p_1 W_{eu1}}{p_2 W_{eu2}}}\right\}$$

This, then, is the new value of W_s/δ_e for the combination, to be inserted in (311), and it is seen to be different from W_{eu1} for a pair of machines similar to 1. When expressed[1] therefore in terms of tons-ft.^{2} and revolutions per min.,

$$f_o = \frac{12{\cdot}42}{N_1}\sqrt{\frac{f(kW_{eu1})\left(1 + \dfrac{p_2 W_1 r_1^2}{p_1 W_2 r_2^2}\right)}{W_1 r_1^2\left\{1 + \dfrac{p_1 (kW_{eu1})}{p_2 (kW_{eu2})}\right\}}} \quad . \quad . \quad . \quad (314a)$$

§ 24. The causes of the divergence δ_e.—The chief causes which may set up the divergence of the rotor from its final stable position which have above been unspecified are in general threefold—(1) variation in the driving torque of the prime mover causing rhythmical fluctuation of the speed, (2) sudden fluctuations of the load, and (3) the operations of switching into parallel. Each will shortly be considered in turn.

In the first case the variation of the driving torque is assumed to be a periodic function of the time, so that the conditions are continually repeated and result in a forced oscillation. The case may, therefore, be regarded as semi-permanent, and through it we pass to the truly transitory cases (2) and (3). But in all the cases the change of velocity of the rotor is slow as compared with the periodic time of the current, so that at any moment they may very closely be represented by a diagram of vectors rotating at a constant speed. The true facts cannot strictly be represented by any simple clock diagram of instantaneous or virtual values, but the assumption of a constant instantaneous velocity of the vectors remains near enough to the truth for purposes of investigation, provided that it is remembered that, strictly speaking, the vector diagram from period to period is continuously altering its shape under transitory synchronizing conditions.

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[1] *Cp.* R. E. Doherty, *Gen. Electr. Review*, Vol. 23, p. 125.

§ 25. Rhythmical fluctuation of speed.—With the wider use of steam and water turbines in central stations, the practical difficulties arising from want of uniformity in the turning moment of the prime movers have lost much of the seriousness that attached to them in the days when slow-speed steam-engines were more general. They still, however, arise in connexion with alternators driven by Diesel engines or other slow-speed internal-combustion engines. The importance of the degree of uniformity of the torque lies in the fact that there is always the possibility of the period of the free oscillation approaching too closely to the period of the " forced " oscillation of speed due to the unequal turning moment of the engine. In this case the rhythmic " hunting " or variation of speed which is set up is always objectionable as causing swinging of the ammeter needles and flickering of lights, and may even increase to such an extent as to throw the machines out of step.

If two similar alternators equally excited and running at the same mean speed with equal power supply vary rhythmically in driving torque and speed, the effect is at once seen from Fig. 689. Whatever the amount of the irregularity, it is always possible that the maximum velocity of one machine may coincide with the minimum velocity of the other machine, so that the displacement of each machine is a maximum but on opposite sides of the normal ; unless indeed, the mechanical phases of the prime movers were synchronized as well as their electrical phases. Such synchronization of the crank positions in the steam-engines has, however, only been resorted to and is only feasible in exceptional cases of very slow-speed machines. Apart, then, from such cases, when one machine runs ahead of its mean position by the electrical angle δ_e, the other machine may lag behind the mean position by the same angle, so that the total divergence of phase is $2\delta_e$, as was assumed in § 12. Fig. 689 then gives all the essential relations ; it is only necessary to write $\beta = 2\delta_e$ for $\gamma_2 - \gamma_1$. The line OC_n now bisects the angle β, and gives the normal E.M.F.

The nature of the mechanical oscillation is then as follows—When a periodic condition has been reached (*cp*. Fig. 694), a rotor passes its true normal position in one or other direction with either maximum or minimum angular velocity, and assuming the damping to be small, the driving torque then has nearly its mean normal value. Starting from the moment when the driving torque *is* normal and the angular velocity is nearly a minimum, the rotor, say of machine 1, passes its normal position and falls back. The displacement backwards or lag of the rotor causes the rate at which electrical energy is developed by machine 1 to fall below the normal. At the same time, the driving torque from the engine is now above the average, so that both the actual excess of the driving torque above the average and also any reduction in the opposing electrical torque are available in combination to accelerate the rotor up to its maximum velocity when once again in its normal position with nearly its normal driving torque. There follows a displacement forwards or lead, and correspondingly the development of electrical energy by machine 1 is above the normal. At the same time, the driving torque is below the average, so that both the deficit in the work from the prime mover and also the increase of electrical work are made up by a gradual reduction of the kinetic energy stored in the rotor and fly-wheel of the leading machine. This goes on until the displacement forwards has been annulled, and the rotor of machine 1 has again its normal position with minimum angular velocity. In the rotor of machine 2 the same processes have been occurring, but in reverse order, so that there results a cyclical displacement of the relative phases with a total amplitude $\beta = 2\delta_e$.

Two duplicate generators in parallel are above contemplated, but similar considerations apply equally to several alternators in parallel or even to a single alternator coupled to a network containing synchronous motors or a single synchronous motor, with which it is virtually working in parallel.

§ 26. Effect of the synchronizing torque in increasing the displacement.—The curve of the driving torque or tangential effort of the prime mover relatively to time may be regarded as made up of a variable portion, alternately positive or accelerating and negative or retarding, superposed upon a constant torque as a base line. A simple assumption will then be that the variable part of the driving torque may be replaced by a sine curve, with a periodic time T_e in seconds. The instantaneous value of the driving torque being $T_n + t_v$, where T_n is the mean value averaged over an entire cycle, the above simple assumption gives

$$t_v = \mathbf{T}_0 \sin\left(\frac{2\pi}{T_e} \times t\right) = \mathbf{T}_0 \sin 2\pi f_e t$$

where $\mathbf{T}_0$ is the maximum value of the variable t_v, and time within the cycle of duration $T_e = 1/f_e$ is reckoned from the moment when the torque has its normal value.

The angular acceleration $d\omega/dt$ is then also a sine curve, which with very little damping present is almost in phase with t_v. The acceleration curve being given or deduced, its time-integral is the instantaneous value of the varying angular velocity, ω_v, again a sinusoidal curve lagging 90° behind the acceleration, which when superposed on the mean constant velocity Ω gives the actual velocity $\omega = \Omega + \omega_v$. Finally, the time-integral of the varying velocity will give at any moment the angle of displacement θ (alternately lag and lead) which has been traversed by the alternator rotor on either side of the mean position corresponding to perfectly uniform rotation. The curve of θ, therefore, lags 90° behind ω_v, 180° behind the acceleration and nearly 180° behind t_v (Fig. 694).

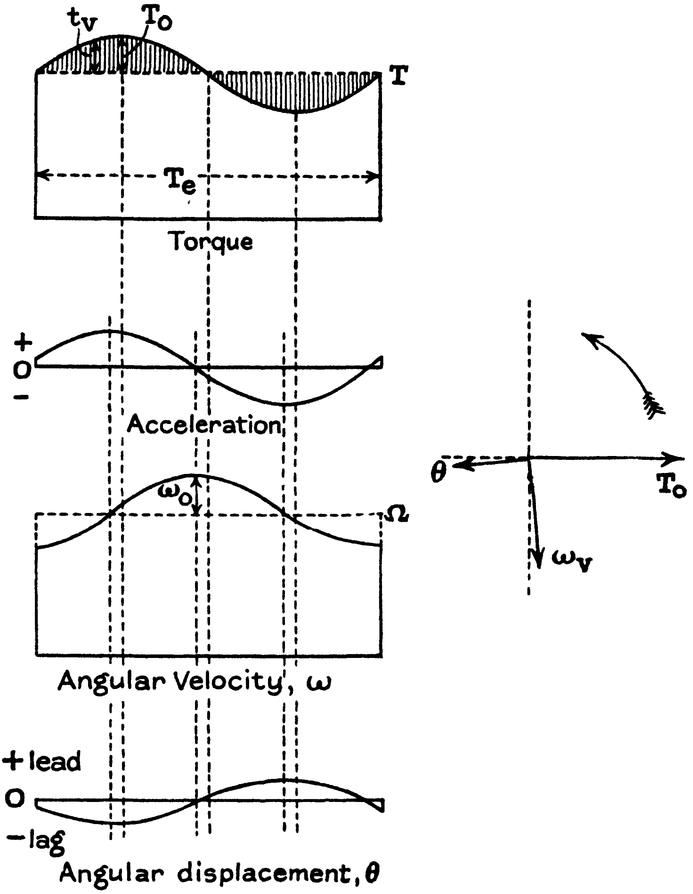

FIG. 694.—Torque, acceleration, velocity, and displacement curves for single machine with slight mechanical damping.

While the above is universally true, a marked distinction has now to be drawn between two cases, which affects the passage from t_v or its maximum value T_0 to $d\omega/dt$. If a single alternator is working on a load, *e.g.* of lamps, which furnishes a constant resisting torque, only the mechanical data of the prime mover and alternator come into the question, $d\omega/dt = t_v/Mk^2$, and the final displacement thence deduced is to be symbolized as θ_0.

But in the case of an alternator in parallel or coupled to a network containing synchronous motors as referred to at the end of the preceding section, the conditions are altered, since the alternator then reacts on its prime mover, and a further force arises owing to the resisting torque not being constant. It has already been shown (§ 25) that, as the armature E.M.F.'s of two similar generators equally excited diverge in phase from their normal position for synchronism, the electrical resisting torque of one rises and of the other falls, and the normal *minus* the actual resisting torque in either case gives the synchronizing torque acting upon the machine in question. The change of the resisting torque is in phase with the displacement, so that, when a machine leads, the electrical resisting torque is above the mean, or the variable part of the resisting torque is positive, but when a machine lags, the electrical resisting torque is below the mean, or the variable part is negative. A change of torque, alternately positive and negative, may, therefore, be regarded as superposed upon the normal constant torque corresponding to coincidence of phase in the two machines, and since its alternations coincide in phase with the displacement, it lags nearly 180° behind the variable part t_v of the driving torque (Fig. 694*a*), when the damping is small. The synchronizing torque is the same as the change of resisting torque but with reversed sign, so that it then almost coincides in phase with t_v. The *sum of the two* is, therefore, available for accelerating or retarding the rotor. Thus, although the synchronizing torque is always acting to reduce any difference of phase between two alternators running in parallel, the work which it does in accelerating or retarding the rotating mass has the effect already pointed out in § 25 of actually increasing the angular displacement which would be due solely to the unequal turning moment of the steam engine. Take the moment when the crank-pin of a single-cylinder steam engine is passing the dead centre; the rotor is then in advance of the position corresponding to uniform speed owing to the previous accelerating action of the excess driving torque, hence the synchronizing torque is holding back the rotor; at the same moment the driving torque is a minimum or, in other words, the variable part of the driving torque is negative and also retarding the rotor. Or, when the driving torque is nearly normal the speed is a maximum or minimum, but there is no displacement and no synchronizing torque to check it. Thus the synchronizing torque acts almost in harmony with the variable part of the driving torque, and the amplitude of the oscillations is

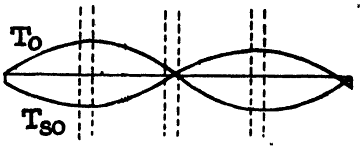

Fig. 694*a*.—Combination of variable driving torque and change of resisting torque.

increased.[1] The final maximum displacement θ_{mf} is thus some multiple of θ_0 or is $\sigma\theta_0$, but since it originates from and depends upon θ_0, the first step must be in the next section to determine the value of the initial displacement θ_0 which would arise if the alternator were working alone against a constant resisting torque.

§ 27. The angular displacement due to want of uniformity in the driving torque of an isolated alternator.—The complete process for determining the variation in speed and the angle of displacement in a single alternator working against a constant resisting torque as caused simply by the unequal driving torque of the steam engine will be as follows—From the indicator diagrams,[2] combined with a consideration of the inertia of the reciprocating parts, is obtained a curve of the driving torque (crank-pin effort), and thence a curve of the instantaneous value of the angular acceleration in relation to time. Two successive integrations by planimeter will then enable the velocity and the space traversed to be obtained, as stated at the beginning of the preceding section.[3]

The process described above is laborious, and also demands exact knowledge of the working of the engine,[4] so that an approximate simplification may meet all the needs of practice. It is evident that the variation of the speed and the angular displacement will depend upon the time during which the acceleration or retardation lasts, since their effects are cumulative; hence for a given permissible maximum angle of displacement the maximum permissible variation of speed may be higher in an engine with two cranks than in one with a single crank, since in the former case the excess speed only lasts half the time of the latter case. It will also depend upon the law which the fluctuations of the driving torque or of the acceleration follow. Assuming as in § 26 that the variable part of the driving torque is sinusoidal and that it depends on the successive impulses, its periodic time in seconds is

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[1] Boucherot, *Bull de la Soc Intern des Electr.*, 2nd series, Vol 1, pp. 529-553 (abstracted in *L'Éclairage Électrique*, Vol. 30, p 287); Longwell, "Paralleling of Alternators," *Electr World and Engineer*, Vol. 39, p 958; Oscanyan, *Electr. World and Engineer*, Vol 40, p 416.

[2] See especially Kruesi, "Speed Variations of Engines Direct-connected to Alternators," *Electr World*, Vol 37, p 591 ff; *Bull. Soc Intern des Electriciens*, Vol 1, 2nd series (1901), pp 536 and 555; P O Keilholtz, *Trans Amer. I E.E*, 25th October, 1901; and A R Horne, *Engineering*, Vol. 87, p. 719

[3] *Cp.* also M David, *Bull Soc Intern des Electriciens* (last cited), p. 503, where it is shown that the actual displacement as measured experimentally is almost always larger than that obtained by the calculation described above, and the reason for this discrepancy is found in the mean speed not being strictly constant For the methods of M. David and M. Cornu for measuring the degree of uniformity of speed or the angular variation in actual practice, see *Bull. Soc Intern. des Electriciens* abstracted in *Electr. Eng*, Vol. 28, p. 728, and a paper by Dr. Rudolf Franke, translated in *Electr. Review*, Vol. 50, p. 897. Tachograph records are shown in Prof. Miles Walker's *Specification and Design of Dynamo-electric Machinery*, p. 346, and *The Diagnosing of Troubles in Electrical Machines*, p. 226.

[4] For an example drawn from gas-engine practice, see especially Prof. Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 218 ff. *Cp.* A. W. Copley, *Elect. Journ.*, Vol. 17, p. 385

$$T_e = \frac{60}{N \times n_c}, \text{ and its frequency } f_e = \frac{N \times n_c}{60} \quad . \quad . \quad (315)$$

where n_c is the number of engine impulses per revolution, and $N =$ revs. per min. *E.g.* with a single-cylinder double-acting steam-engine there are two points of maximum speed and two minimum points in each revolution, so that the frequency of the forced oscillation is $f_e = N/30$ (Fig. 695); in a 2-crank engine with cranks at 90°, $f_e = N/15$ and in a 3-crank engine with cranks at 120°, $f_e = N/10$.

In the gas-engine with a single cylinder working on the 4-stroke Otto cycle with one impulse every two revolutions, the values of the turning moment do not repeat until after 2 revolutions. With two cylinders in tandem on the same crank and one impulse every revolution, the second impulse will not repeat exactly the same effect as the first, so that the same characteristic holds. When 2, 3 or more such tandem cylinders are arranged side by side, firing at equal intervals in each revolution and working on 2, 3 or more cranks, the values naturally approach much more nearly the constant mean, but the above characteristic, viz., that the values of the turning moment only repeat after 2 revolutions, never entirely disappears. Consequently there is a periodic rise and fall of the speed having the frequency of the cam-shaft, equal to $\frac{1}{2}\, N/60$. Upon this there are superimposed more rapid fluctuations, having the frequency $Nn_c/60$, where n_c is the number of impulses per revolution or number of cranks. It is also possible that the more rapid or principal fluctuations of the turning moment may have other frequencies, if for any reason the cylinders are not firing regularly.

In the present case with a constant resisting torque assumed, there is no synchronizing torque, and on the analogy of equations (20*a*) and (20*b*)[1]

$$\text{in radians, } \theta = \frac{\mathbf{T}_o}{\sqrt{4\pi^2 f_e^2 b^2 + (4\pi^2 f_e^2 Mk^2)^2}} \sin\,(2\pi f_e t - \eta)$$

$$\omega_v = \frac{\mathbf{T}_o}{\sqrt{b^2 + 4\pi^2 f_e^2\,(Mk^2)^2}} \sin\,(2\pi f_e t - \eta + 90°)$$

$$\text{where } \tan \eta = \frac{b}{-\,2\pi f_e Mk^2}$$

When b is positive but negligibly small, η is just short of 180°, and the variable part of the space traversed or the angular displacement from the position for truly uniform motion is closely

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[1] Vol. I, pp. 117, 118.

$$\theta = \frac{\mathbf{T}_o}{4\pi^2 f_e^2 Mk^2} \sin (2\pi f_e t - 180°),$$

with a maximum value

$$\theta_o = \frac{\mathbf{T}_o}{4\pi^2 f_e^2 Mk^2}$$

which may, since $2\pi f_e = n_c \Omega$, also be expressed as

$$\theta_o = \frac{\mathbf{T}_o}{n_c^2 \Omega^2 Mk^2}$$

Further, on the same supposition, *i.e.* without any damping

$$\omega_v = \frac{\mathbf{T}_o}{2\pi f_e Mk^2} \sin (2\pi f_e t - 90°)$$

with a maximum value

$$\omega_o = \frac{\mathbf{T}_o}{2\pi f_e Mk^2}\cdot$$

Thus when the torque has its mean value ($2\pi f_e t = 0°$ or $180°$), the angular velocity is at its minimum or maximum, and the angular displacement from the position of uniform rotation is zero. When the variable torque is positive or accelerating, and the angular velocity is rising, the angular displacement increases on the negative or lagging side and passes through its negative maximum when the torque is a positive maximum and the angular velocity is at its mean value. *Vice versa*, when the variable torque is negative or retarding, the angular displacement is positive or leads (Fig. 694).

If $\varepsilon = \frac{\omega_{max} - \omega_{min}}{\Omega} = \frac{(\Omega + \omega_o) - (\Omega - \omega_o)}{\Omega} = \frac{2\,\omega_o}{\Omega}$, we have $\omega_o = \frac{\varepsilon\Omega}{2}$. Now $\theta_o = \omega_o \frac{T_e}{2\pi}$; therefore $\theta_o = \frac{\varepsilon\,\Omega}{2}\frac{T_e}{2\pi}$.

But $\Omega\, T_e$ = the angle turned through in the time of one cycle $= \frac{2\pi}{n_c}$;

therefore $\theta_o = \frac{\varepsilon}{2n_c}$ radians $= \frac{\varepsilon}{2n_c} \times \frac{360°}{2\pi}$ mechanical degrees,

or $\delta_o = \frac{\varepsilon}{2n_c} \times \frac{360°}{2\pi} \times p$ electrical degrees . . (316)

where ε is the ratio of the speed variation to the average speed, and is known as the " cyclic speed-irregularity."

In reality the curve of the variable portion of the driving torque, with the effects of inertia taken into account, being of an irregular shape in the actual engine, can only be strictly expressed by a Fourier series of sine terms; yet one of these terms may very greatly predominate over the others. In the single-crank steam engine this is the term having the frequency

$$f_e = \frac{n_c \Omega}{2\pi} = \frac{2\Omega}{2\pi}, \text{ so that } t_v = \mathbf{T}_o \,.\, \sin 2\Omega t$$

Analysis of actual results[1] from a number of single-crank steam engines working against a constant resisting torque shows that except under extreme conditions of very light loads or exceptional inertia of the reciprocating parts, the displacement may safely be deduced from the coefficient of speed variation as by eq. (316) with $n_c = 2$. The remarkable result was also found that the same expression, namely,

$$\theta_o = \frac{\varepsilon}{4} \times \frac{360^\circ}{2\pi} \text{ degrees},$$

could at least, as an approximation, be extended to the case of engines with two or more cranks, regardless of the fact that n_c is then more than 2. Thus in conclusion, it may be said that with some caution the hypothesis of a sinusoidal variable torque as replacing the actual varying torque may generally be adopted in practice

§ 28. Calculation of necessary fly-wheel effect for given value of ε.—The fly-wheel effect required to keep the variation of speed within any assigned limits is calculated as follows Let a complete cycle of speed be performed in y revolutions ; usually y is one revolution or some fraction of a revolution, but in the case, *e g.* of a single-cylinder gas engine working on the Otto cycle, after one explosion two revolutions may be required to complete a cycle of operations before the second explosion occurs. The mean power is $HP \times \frac{33{,}000}{2240}$ ft -tons per minute, and the mean energy developed in the y revolutions is $E = y \times \frac{33{,}000}{2240} \times \frac{HP}{N}$ ft -tons The crank-effort curve, in which the tangential force in tons on the driving point or crank-pin of the engine is plotted as ordinates with the linear feet moved through by the crank-pin as abscissæ, is determined from the combined indicator diagrams of the several cylinders, the effect of inertia in the moving parts being also taken into account (Fig. 695). A mean effort-line being drawn corresponding to the mean energy E, the largest of the areas of excess lying above or of deficiency lying below this line represents the *fluctuation of engine energy* ΔE.

If W be the weight of the fly-wheel, and V be the mean velocity in ft. per sec. of its rim at the radius of gyration, the energy stored in the fly-wheel at its maximum velocity $= \frac{1}{2}\frac{W}{g} V^2_{max}$, and at its minimum velocity $= \frac{1}{2}\frac{W}{g} V^2_{min}$, so that the energy received and stored by it as its speed rises from V_{min} to V_{max}, or the *fluctuation of the fly-wheel energy*, is

$$\frac{1}{2}\frac{W}{g}(V^2_{max} - V^2_{min}) = \frac{1}{2}\frac{W}{g}(V_{max} + V_{min})(V_{max} - V_{min})$$

For small changes of speed, $V = \frac{V_{max} + V_{min}}{2}$, so that the above fluctuation in the fly-wheel energy is $\frac{W}{g} V \times V\varepsilon = \frac{W}{g} \,.\, V^2 \varepsilon$

$$= \frac{W}{g}\frac{4\pi^2 r^2 N^2}{60^2} \varepsilon \text{ ft.-tons, if } W \text{ is in tons} \quad . \quad . \quad . \quad (317)$$

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[1] Boucherot, *Bull. Soc. Intern. des Electriciens*, November, 1901, Vol 1, 2nd series, p. 537, and *Trans. Intern. Electr. Congress St. Louis*, 1904, Vol. 1, p. 695, which are here mainly followed. *Cp.* also Ceytre, *Rev. Electrique*, 1908, Vol. 9, p. 29.

Or in rotational measure, if $\frac{W}{g} \cdot r^2 =$ the moment of inertia,

$$\frac{1}{2}\frac{W}{g} r^2 (\omega^2_{max} - \omega^2_{min}) = \frac{1}{2}\frac{W}{g} r^2 \omega^2 \varepsilon$$

This quantity must then be equal to ΔE, or the average excess of the tangential pressure in tons on the driving-point or crank-pin of the engine multiplied by that portion of the y revolutions during which it lasts, expressed in feet travelled by the crank-pin. Or if it be a negative area which is the largest, and the fly-wheel gives out energy as it falls in speed from V_{max} to V_{min}, it is then the product of the deficit of the tangential pressure by the feet through which it lasts. The ratio of the fluctuation of energy ΔE to the mean energy E developed in the period of y revolutions (also expressed in ft.-tons) is the *coefficient of fluctuation of energy*, and forms a constant x

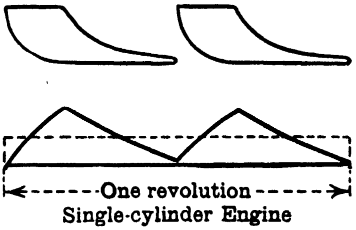

FIG. 695.—Crank-effort curve of single-cylinder double-acting steam engine.

for the engine in question, of which the value will depend largely upon the number of cylinders and their arrangement. Thus

$$\frac{\Delta E}{E} = x = \frac{\frac{W}{g}\frac{4\pi^2 r^2 N^2 \varepsilon}{60^2}}{y \times \frac{33{,}000\, HP}{2240\, N}}$$

whence

$$Wr^2 = \frac{4{\cdot}35xy \times HP \times 10^4}{\varepsilon N^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (318)$$

or combining together $4{\cdot}35xy$ into one constant c

$$Wr^2 = \frac{c \times HP \times 10^4}{\varepsilon N^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (319)$$

where W is in tons and r is the mean radius of the rim in ft.[1]

The determination of ΔE or of c is by no means easily made, but, as illustrating the subject, values found for c in particular cases[2] are here quoted, namely,

For a single-cylinder steam-engine	0·91
For a compound engine with cranks at right angles . .	0·4
And for a triple-expansion engine with cranks at 120° . .	0·23

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[1] Wr^2 in tons-ft.^{2} $= \frac{2{\cdot}204 \times 3{\cdot}28^2}{2240}$ kilogrammes weight-(radius in metres)^2

$= \frac{1}{94{\cdot}4}$ kilogrammes-(metres radius)^2

$\therefore$ (Wr^2 in tons-ft.^{2}) $\times$ 94·4 $=$ kilogrammes-(metres radius)^2

$= \frac{1000}{4}$ metric tons-(metres diam.)^2

$\therefore$ (Wr^2 in tons-ft.^{2}) $\times$ 0·3776 $= GD^2$ in metric tons-metres^2

[2] From Herr O. Lasche's paper on the "Construction and Manufacture of Alternators."

For gas-engines the values will, of course, be much larger.[1] From eq. (319) such values must be taken for W and r as will give a convenient size of fly-wheel; or, if the size of fly-wheel is given, the value[2] of ε is found.

§ 29. The final displacement with synchronizing torque taken into account.—The three lower diagrams of Fig. 694, which assume a constant resisting torque, will therefore need to be modified when the effect of the synchronizing torque (Fig. 694*a*) is added. As already mentioned, the curve of the variable driving torque can only be accurately represented in practice by a Fourier series of sine waves, although one of these may be predominant. But if any one of these components is isolated, the initial displacement due to it as arising from the engine alone can be deduced, and also the initial synchronizing torque which it calls into action; the combined action of the driving torque and of this initial synchronizing torque will then give an increased acceleration, and larger variable velocity and displacement diagrams; these again will give a larger synchronizing torque (although the increase except under special conditions will be less), so that the diagrams must be step by step rectified until all the conditions are satisfied. But since the component of the variable driving torque which is in question is sinusoidal, this gradual process of correction may be at once abbreviated by mathematical analysis. Let the variable component in question be expressed by $\mathbf{T}_o \sin 2\pi f_e t$ or $\mathbf{T}_o \sin (w_c \,.\, \Omega t)$, so that its periodic time is $T_e = \frac{1}{f_e} = \frac{2\pi}{w_c \,.\, \Omega}$, the symbol w_c being now not simply the number of engine impulses per revolution, but the number of complete waves of the component in question per revolution.

With damping and synchronizing torques present, we return to the full equation of motion (307), of which the solution on the analogy of (20*a*)[3] is in radians

$$\theta = \frac{\mathbf{T}_o}{\sqrt{4\pi^2 f_e^{\,2} b^2 + (T_u - 4\pi^2 f_e^{\,2}\, Mk^2)^2}} \sin (2\pi f_e t - \eta)$$

where $\tan \eta = \dfrac{b}{(T_u/2\pi f_e) - 2\pi f_e\, Mk^2}$

with a final maximum value for the mechanical angular displacement

$$\theta_{mf} = \frac{\mathbf{T}_o}{\sqrt{(2\pi f_e b)^2 + (T_u - 4\pi^2 f_e^{\,2}\, Mk^2)^2}} \quad . \quad . \quad . \quad (320)$$

The effect of the addition of synchronizing torque can now be judged by comparison of the value of θ_{mf} with that previously found for θ_o.

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[1] For gas-engines, *cp.* Unwin and Mellanby, *Elements of Machine Design*, Part II, p. 277.

[2] See M. David, *Bull. Soc. Intern. des Electriciens*, Vol. 1, 2nd series (1901), pp. 504, 505, where an example is worked out.

[3] Vol. I, p. 117.

The ratio of the final to the original displacement without any synchronizing torque is

$$\sigma = \frac{\theta_{mf}}{\theta_o} = \frac{\sqrt{b^2 + 4\pi^2 f_e^2 M^2 k^4}}{\sqrt{b^2 + \{(T_u/2\pi f_e) - 2\pi f_e Mk^2\}^2}} \quad . \quad . \quad (321)$$

At the resonant frequency, *i.e.* when the period of the force oscillation due to the varying component of the driving torque exactly coincides with the natural period of an undamped oscillation of the rotor,

$$T_e = T_p$$

$$\frac{1}{f_e} = 2\pi\sqrt{\frac{Mk^2}{T_u}}\text{, or } T_u/2\pi f_e = 2\pi f_e Mk^2 \quad . \quad . \quad (322)$$

If, therefore, the damping torque be small, as the resonant frequency is approached, the value of σ increases very rapidly, but it never becomes infinite so long as there is any damping, and it quickly diminishes again when T_e is less than T_p. The magnification of the displacement by the synchronizing torque is rendered clear by mentally dividing the process into stages. Let $\mathbf{T}_{so}$ be the amplitude of the original synchronizing torque called forth by the original θ_o due solely to the variation of the engine driving-torque, *i.e.* $\mathbf{T}_{so} = T_u \cdot \theta_o$, and let $q = \mathbf{T}_{so}/\mathbf{T}_o$, the ratio of the original synchronizing torque to the varying steam torque; then to the latter must be added an amount $q\mathbf{T}_o$ as increasing the displacement; this again will cause an additional increment to the synchronizing torque, and so on. The total effective variable torque or the sum of the original variable torque and the additional increments from the synchronizing action, so far as they act together in causing displacements, is not, however, the sum of the geometric series $\mathbf{T}_o + q\mathbf{T}_o + q^2 \cdot \mathbf{T}_o + \ldots\ldots$, since with each increment of displacement and speed the angle η between the vectors of applied torque and displacement alters; hence for each step in our mental process only the component of the varying engine torque at right angles to that component which balances the damping torque must be taken into account, *i.e.* only

$$\mathbf{T}_o \cos\eta = \mathbf{T}_o \frac{T_u - 4\pi^2 f_e^2 Mk^2}{\sqrt{(2\pi f_e b)^2 + (T_u - 4\pi^2 f_e^2 Mk)^2}}$$

is effective in causing displacement. Considering, therefore, the process as finished

$$\mathbf{T}_{mf} = \mathbf{T}_o \cos\eta\,(1 + q + q^2 + q^3 + \ldots)$$

and

$$\sigma = \frac{\mathbf{T}_{mf}}{\mathbf{T}_o} = \left(\frac{1 - q^n}{1 - q}\right)\cos\eta$$

The value of .

$$q = \frac{\mathbf{T}_{so}}{\mathbf{T}_o} = \frac{T_u \, . \, \theta_o}{4\pi^2 f_e^2 Mk^2 \, . \, \theta_o} = \frac{T_u}{4\pi^2 f_e^2 Mk^2}$$

is another expression for $\frac{T_e^2}{T_p^2}$ or the square of the ratio of the two periodic times, and when resonance occurs so that $q = 1$, $\eta = 90°$. Hence although $\frac{1 - q^n}{1 - q}$ becomes infinite, its product with $\cos \eta$ remains finite.

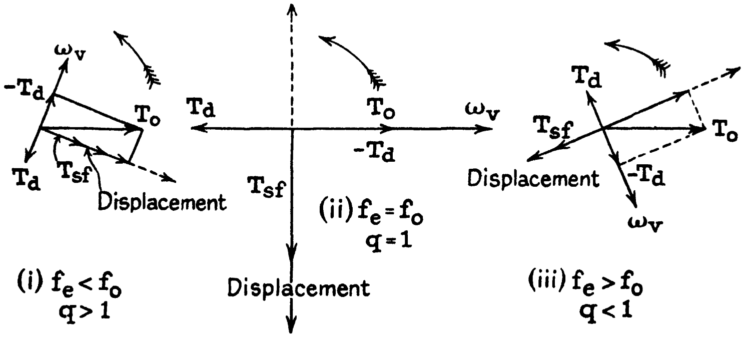

FIG. 696 —Varying torque, with damping.

So long as $f_e < f_o$, the speed curve precedes by 90° the component of the variable torque at right angles to the damping torque reversed, *i.e.* the component left after deducting vectorially the component that balances the damping torque; it lags behind it by 90° when $f_e > f_o$; the displacement curve always continues to lag behind ω_v by 90°. But the initial variable torque $\mathbf{T}_o$ owing to the damping component present in it is no longer at right angles to the speed and exactly opposite in phase to the displacement. As q is increased the varying speed draws nearer and nearer into phase with the vector $\mathbf{T}_o$, until, when $q = 1$, they coincide, and the whole of the initial variable torque now exactly balances the damping torque. The synchronizing torque then exactly supplies the accelerating torque. The speed is not then infinite, but just rises to such a value that this equality holds. When $q > 1$, the speed precedes the vector of the initial variable torque and the final displacement draws nearer in phase to $\mathbf{T}_o$. The relations are indicated in Fig. 696, which takes into account the effect of damping[1] (*cp.* Fig. 47).

If the damping, *i.e.* b, is negligibly small, $\eta = 0$, and $\sigma = \frac{1}{1 - q}$; $\mathbf{T}_{mf} = \mathbf{T}_o + \mathbf{T}_{sf}$, and the final synchronizing torque, $\mathbf{T}_{sf}$, to which the fluctuation of output is proportional, being $\sigma\mathbf{T}_{s0}$, is given directly as $\frac{q}{1-q}\mathbf{T}_o$.

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[1] *Cp.* also Dr. E. Rosenberg, *Journ. I.E.E.*, Vol. 42, pp. 533, 534; and Prof. Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 230–8.

Unless the damping is great, when $q = 1$ or near thereto, the oscillations once started by the small amount of speed variation which must persist even in an exceptionally good engine, will increase to such an extent from the pendulum action that the machine is eventually thrown out of step entirely. If $T_p > T_e$ or $f_e > f_o$, but is too nearly equal to it, the remedy must be to increase the fly-wheel effect; but in the reverse case of $T_p < T_e$ or $f_e < f_o$ a decrease in the fly-wheel will cause a greater divergence between the two quantities, and in certain exceptional cases in practice a reduction of the rotating masses is attended with improved results.

The damping torque[1] is assumed proportional to the vector of the varying speed, and its amplitude $\mathbf{T}_d = b \times \omega_{vf}$ is

$$\mathbf{T}_o \sin \eta = \mathbf{T}_o \frac{2\pi f_e b}{\sqrt{(2\pi f_e b)^2 + (T_u - 4\pi^2 f_e^2 M k^2)^2}}$$

Since in the end the energy expended in damping comes from the engine or from the network driving a synchronous motor, the oscillation of output *of the prime mover or network* is proportional to a fluctuating torque of which the amplitude is $\sqrt{\mathbf{T}_d^2 + \mathbf{T}_{sf}^2}$, but the increase to balance damping concerns the prime mover and not the alternator output under consideration.[2]

§ 30. Maximum permissible values of cyclic speed-irregularity and phase displacement.—From the point of view of steadiness of the light, the cyclic speed irregularity ε in an engine driving an alternator should not be worse than $\frac{1}{150}$, but in practice smaller figures are in most cases obtained; the more usual figures which may be obtained without unduly heavy fly-wheels give limiting values of say $\frac{1}{180}$ or 0·55 of 1 per cent. for small machines, and $\frac{1}{250}$ for larger machines.

But the needs for parallel working are really the decisive considerations, and it is, therefore, the permissible fluctuation of output which is paramount rather than the actual variation of speed. Owing to their perfectly uniform speed of rotation, the water-wheel and water- or steam-turbine are the best prime movers for alternators which are to be run in parallel. But even the steam turbine may in exceptional cases have forced oscillations set up if the steam is admitted in recurring puffs by the action of the governor. In the engine, after every precaution has been taken to balance the reciprocating parts and to distribute the load properly between the several cylinders and within each cylinder, the want of uniformity in the speed which still remains must be brought within certain practical limits by adding sufficient fly-wheel effect in the machine as a whole. The heavy revolving mass of the magnet system may require to be supplemented by a fly-wheel proper, but any weight of fly-wheel greater than is really called for by genuine electrical reasons is not only an unnecessary expense, but is prejudicial to the proper action of the engine governor. It therefore becomes of great importance to determine the maximum displacement of the electrical phase from the position for perfectly uniform speed which is compatible with good parallel working.

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[1] For values of the ratio $\frac{b}{T_u}$ have been given the following figures, namely: (1) with laminated pole-pieces, 0·003–0·007; (2) with solid pole-pieces, 0·01–0·25; (3) with Leblanc dampers, 0·01–0·2 (O. Weisshaar, *Elektrotech. u. Maschinenbau*, Vol. 26, p. 555 ff.).

[2] For other articles dealing with the subject of the parallel working of alternators, the reader is referred to de Marchena, *Rev. Gén. d'Elect.*, Vol. 5, p. 405; A. Blondel, *Comptes Rendus*, Vol. 168, pp. 376 and 587, *Rev. Gén. d'El.* Vol. 13, p. 235; J. Frith, *Engineering*, Vol. 110, p. 657; A. W. Copley, *Elec. Journ.*, Vol. 17, p. 385; R. E. Doherty, *Gen. El. Rev.*, Vol. 23, p. 653; J. Fischer-Hinnen, *Electr.* Vol. 89, p. 185 (abstract).

What the designer has to secure is, in the last resort, that the final displacement, being the vectorial sum of the displacements due to all of the component harmonics of the variable driving torque combined with the effect of the synchronizing torque, *i.e.* δ_{ef}, should never exceed the maximum found permissible in practice. Obviously, with two similar alternators in parallel, if δ_{ef} ever exceeded 45°, the maximum theoretical synchronizing power on our approximate assumptions would have been passed, and the alternators must fall out of step. But even with values much short of this, parallel working may be attended with serious trouble. It may again be here mentioned that the possibility of one machine's variation of speed coinciding with that of another, but in the reverse sense, must be foreseen, so that the total divergence of phase β may be twice the divergence δ_e of either machine from an assumed vector of fixed position. As a practical limit, perhaps a maximum electrical displacement δ_{ef} of 7° may be adopted, although further evidence from theoretical analysis of actual cases (troublesome or otherwise) is much to be desired. The nature of the load requires too to be taken into account; rotary converters, for instance, form a most delicate load, and are peculiarly liable to rhythmic hunting or phase-swinging, which may upset the generators if their natural period approaches too closely to resonance with any speed-variation in the engines.

In the next place, the permissible final displacement in mechanical units must vary inversely as the number of pairs of poles, or $\theta_{mf} = \sigma\theta_0 = \frac{7^\circ}{p}$. In other words, if a point on the revolving portion of the alternator or on the fly-wheel were marked, and its position relatively to that which it would have with perfectly uniform rotation could be followed by the eye, its maximum angular displacement must not exceed $\frac{7^\circ}{p}$. With a standard speed for a large 1500 or 2000 kW engine-driven alternator of about 75 revs. per min. 40 poles are required for 25 cycles, or $p = 20$, and θ_{mf} must then be $\leq 0{\cdot}35$ degree. If the same output is to be given at a frequency of 60, the number of poles must be increased to 96, and for the same want of uniformity of engine speed and ratio of $\mathbf{T}_{so}$ to $\mathbf{T}_o$ or of f_o to f_e, the phase displacement would now be $7^\circ \times \frac{96}{40} = 16{\cdot}8$ electrical degrees, which might render parallel working difficult. Hence in the latter case, for the same phase displacement to be again maintained, a much heavier fly-wheel must be employed. A low frequency is, therefore, advantageous, and for a given frequency the advantage of a high speed is evident. Thus, as has already been stated (Chapter XXVIII, § 28), the greater the number of poles, the less must be θ_{mf}. But it does not necessarily follow that the initial mechanical displacement θ_o, as due solely to inequality of the driving torque, must be less with the greater number of poles, or the rotation due to the engine alone more uniform. The value of σ for any particular component of the variable driving torque of frequency f_e being when damping is neglected $\frac{1}{1-\left(\frac{f_o}{f_e}\right)^2}$, the most important factor is the ratio of the frequencies of the free undamped and forced oscillations; for if the conditions of resonance are approached, the increase of the displacement is so great that the value of the initial displacement θ_o becomes of but minor importance.

If the various harmonics of the variable torque and their percentages of the normal full-load torque are obtainable, the full process should be to deduce the displacement and oscillating output due to each, with allowance for damping, and adding them vectorially to limit the total fluctuating output, say, to 25 per cent. of the full load. It will be found that σ decreases very rapidly for the terms of the series containing 3Ω, 4Ω, etc., *i.e.* when in $f_e = \frac{w_c\Omega}{2\pi}$, w_c is 3, 4, or more. Thus even with multi-crank engines it is chiefly the components in Ω and 2Ω which require attention, and if these give satisfactory results the higher harmonics may be safely neglected. With single-crank engines the component with $w_c = 2$ must be carefully examined. But above all in every case the first term with $w_c = 1$, which is chiefly due to the obliquity of the connecting rod to the piston rod, to unbalanced weights, and to inequality between the impulses on the two sides of a double-acting piston, is the most important; in the first place, it is most likely to be resonant with the natural period, and also it may in multi-crank engines cause an initial displacement four times that due to the term in 2Ω.

If f_0 is lower than the lowest f_e by not less than 20 per cent. of f_e, $q = 0{\cdot}8^2 = 0{\cdot}64$, and then when damping is left out of account, $\sigma = \frac{1}{1-(0{\cdot}8)^2} = 2{\cdot}78$. While it is always desirable that f_e should thus exceed f_0, a similar value for σ (damping again ignored) is obtained on the other side of the resonant frequency when $q = 1{\cdot}36$ and f_0 is $1{\cdot}167 f_e$, *i.e.* exceeds f_e by $16\frac{2}{3}$ per cent. The synchronizing torque itself varies somewhat with the voltage and load, so that allowing some margin for the resulting difference in the value of f_0 and calculating for the two values of q below and above unity, two values of the fly-wheel effect are reached, which, when plotted in relation to revs. per min., define a " danger zone," as it has been called;[1] the fly-wheel effect must then be such that it does not fall within the danger zone. It is, however, not advisable, except for very cogent reasons to work with $f_0 > f_e$ for the term in Ω, since there is then a danger of resonance with the term in 2Ω when f_0 approaches $2f_e$. As a general rule, the natural periodic time of the alternator is in practice made greater than even the longest periodic time of any forced oscillation from inequality of the driving torque.

With two dissimilar prime movers, the possibility of resonance between the natural frequency of the combination and the frequency of irregular impulses in either of the two prime movers must be taken into consideration.

§ 31. **Further practical rules.**—The exact curve of the driving torque is seldom available for resolution of its variable portion into the various components by the usual methods of analysis. We then have to fall back upon examination of the periods for Ω and 2Ω, and in each case deduce the initial displacement from extreme values of $\mathbf{T}_0$, such as $0{\cdot}16$ and $0{\cdot}9$ of the normal torque in single-crank engines, or $0{\cdot}1$ and $0{\cdot}5$ of the normal torque in multi-crank engines ; or even in both cases to deduce it from the coefficient of speed variation, ε, by the empirical expression, namely, $\theta_0 = \frac{\varepsilon}{4} \times \frac{360°}{2\pi}$ mechanical degrees. The error is then at least on the safe side.

As a practical guide, without allowance for damping, if it is laid down that the natural frequency should not be more than 80 per cent. of any forced frequency, so that the final displacement is then $2{\cdot}78$ times the initial displacement, this condition must be combined with some further assumption as to the amount of the maximum permissible initial displacement, and it follows from our first rule that this should be $p\theta_0 = \pm 2\frac{1}{2}$ electrical degrees.

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[1] See Dr. E. Rosenberg, " Parallel Operation of Alternators," *Journ. I.E.E.*, Vol. 42, pp. 539–546, and Vol. 50, p. 540, where several curves of the danger zone have been plotted on certain defined assumptions. See also Prof. Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 233–238.

The 1920 Standardization Rules for Electrical Machinery of the British Electrical and Allied Manufacturers' Association recommend that the periodic angular displacement on either side of the position of uniform rotation should be limited to $2\frac{1}{2}$ electrical degrees, giving an interchange current of about 15 to 20 per cent. of full-load current; to secure this, the final ε must not exceed $n_c/12p$ (where p is the number of pole-pairs and n_c the number of engine impulses per rev.), as will be found by inserting $2\cdot4° = \delta$ in equation (316), which will retain the same form when the synchronizing torque is also taken into account; it is further recommended that f_0 should be at least 20 per cent. lower than the frequency of a possible forced oscillation. The lowest value of the latter will in most cases correspond to the crankshaft revs. per min., but in the special case of a 4-stroke cycle internal-combustion engine it will correspond to $\frac{1}{2}N$ since the cam-shaft speed in this case is half that of the engine. To secure the last condition, from equation (312)

$$f_0 \text{ per second} = \frac{0\cdot8 \times 2N}{\text{strokes per engine cycle} \times 60} = 0\cdot163\sqrt{\frac{f(kW_{eu})}{\text{foot-tons}}} \quad . \quad . \quad . \quad (323)$$

Taking (kW_{eu}) as 4·2 times the kVA rating of the machine, the required foot-tons of energy stored in the fly-wheel at the normal speed are[1]

$$\text{foot-tons} = \frac{1\cdot3 \times \text{no. of poles} \times (\text{strokes per engine cycle})^2 \times \text{kVA}}{N}$$

If f_0 is to be less than $0\cdot8 f_e$

$$\frac{12\cdot4}{N}\sqrt{\frac{f(kW_{eu})}{Wr^2}} < \frac{0\cdot8 N n_c}{60}$$

whence the fly-wheel effect in tons at 1 ft. radius must be

$$Wr^2 > \frac{f(kW_{eu})}{N^4 n_c^{\,2}} \times 865{,}000 \quad . \quad . \quad . \quad . \quad . \quad (324)$$

from which the great advantage of a quick-revolution engine in reducing the necessary size of fly-wheel is evident. The only part of the expression that the electrical designer can modify is kW_{eu}—by increasing z_a, but at the expense of the inherent regulation. The natural frequency is inversely proportional to the impedance, of which the inductance is the chief component; any increase in the inductance may be an advantage from the point of view of parallel running and ability to withstand short-circuits, but will worsen the regulation; an excitation lower than is normally employed will lower the natural frequency, and may perhaps surmount a difficulty, when with the full excitation an alternator cannot be run in parallel easily. There is, however, but little scope for any serious modification of the electrical constants, except by the addition of a damper winding to be described in § 38.

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[1] *Cp.* A. R. Everest, *Journ. I.E.E.*, Vol. 50, pp. 530, 531.

If, as a rough approximation, the identification of § 18 is accepted, the natural frequency becomes expressible in terms of the output and of I_{sc}/I_n, the ratio of the short-circuit to the full-load normal current under the same excitation, being that of full-load. Thus if η be the ratio of the terminal voltage at full-load to that on open circuit per phase, the apparent output in kVA is $\frac{N_{ph}\,\eta\,E_o\,I_n}{1000}$, and $(kW_{eu}) = \frac{N_{ph}\,E_o\,I_{sc}\sin\phi_a'}{1000}$. Thence $(kW_{eu}) = \text{kVA} \times \frac{I_{sc}}{I_n} \times \frac{\sin\phi_a'}{\eta}$, and since $\sin\phi_a'$ and η are not greatly different,

$$f_o = \frac{12{\cdot}42}{N}\sqrt{\frac{f \times (\text{kVA})}{Wr^2} \times \frac{I_{sc}}{I_n}}$$

$$\text{and } Wr^2 \text{ in tons-(ft)}^2 > \frac{f(\text{kVA})}{N^4 n_c^2} \times \frac{I_{sc}}{I_n} \times 865{,}000$$

where I_{sc}/I_n may be in practice from, say, 2½ to 3½.

But the identification is not sufficiently accurate to yield more than a rough guide to the moment of inertia that all the rotating parts must yield

§ 32. Sudden fluctuations of load.—If the load on two machines in parallel is suddenly reduced or increased, both rotors should move simultaneously and at the same rate towards the correct synchronous position for stable running at the reduced or increased load as fixed by the steam supplies given by their governors. But the rotating parts of the two machines may have different moments of inertia, and if the moment of inertia of one machine is greatly different from that of the other, their relative positions at each stage will be different, giving rise to a total divergence β_e, in their induced E.M.F.'s, to a synchronizing current between them, and to oscillations of both about their correct positions.

Or again, if the load of a synchronous motor running on a large network is suddenly altered, the synchronizing torque proceeds to retard or accelerate the rotor towards its position for stable running, but the rotor must necessarily execute a few swings about its normal position before settling down.

The free oscillations set up by fluctuation of the load are seldom likely to prove troublesome, provided that the initial disturbance does not itself throw a machine out of step. The amplitude of the oscillations will usually become smaller, although if the regulation of the machines is good, they may be accompanied by large transfers of energy backwards and forwards from one machine to another.

§ 33. Inequality of voltage when switching into parallel.—When we come to cases involving the possibility of the sudden closing of switches, as *e.g.* when machines are thrown into parallel far from coincidence of phase, a general caution applicable to the following sections must be entered. In such circumstances the full synchronous reactance cannot take effect instantaneously, so that any approximate calculation or explanation may require to be accompanied by the proviso that upon first closing switches the effective reactance may perhaps be only some 15 to 25 per cent. of the full synchronous reactance, and its value thence rises comparatively slowly as armature reaction develops. The mechanical stresses and shocks may, therefore, be more severe than would appear from a simple description.

Passing, then, to temporary conditions as arising on switching into parallel and first to voltage considerations, subject to the above-mentioned caution, let an alternator be run up on open circuit until its frequency is the same as that of the network to which it is to be connected, and let the phase of its open circuit E.M.F. E_0 be synchronized with that of the terminal voltage V_e of the bus bars, the throttle valve of the engine giving just enough steam to supply the losses from friction, windage, hysteresis, and eddy-currents. In the first place, let the network be fed by such powerful machines that the additional alternator when switched into parallel cannot sensibly affect their running, so that any current which flows through its armature produces a negligibly small effect upon the rest of the system; further, let E_0 differ in magnitude from V_e. If the ohmic resistance of the armature is negligible in comparison with its reactance, the voltage acting at the terminals of the machine is at the first moment simply the difference $E_0 - V_e$, and in phase therewith; to this corresponds a corrective current $\frac{E_0 - V_e}{x_a}$, of which the phase is at right angles in this case to either the induced or terminal E.M.F., and which is positive or negative according as E_0 or V_e is the larger. Such a corrective current accordingly either lags behind the armature E.M.F. by 90° and so partially demagnetizes the machine, or leads before the armature E.M.F. and assists in magnetizing it (Fig. 697*a*). In either case it so alters the magnetization that the vector sum of the induced E.M.F. and reactance voltage becomes equal to the terminal voltage of the network. But now, if the machine also has ohmic resistance in its armature, the cross current between the machine and the other generators, *i.e.* e/z_a lagging behind the cross E.M.F. e (or the vector joining E_ι and V_e) by some angle ϕ_a less than 90°, will have a component in phase with E_ι (Fig. 697*b*); it will, therefore, as it grows, cause the machine to act either as a generator (if $E_0 > V_e$) or as a motor (if $E_0 < V_e$), slightly retarding or accelerating its rotor; the driving torque from the steam supply is assumed to remain constant, so that the additional power is in the first case in reality obtained from the kinetic energy of the fly-wheel, and in the second case electrically from the network.

As the current, which is now synchronizing, grows up, it is accompanied *pari passu* by a shifting of the phase of E_0 or of the centre of the poles as the rotor is retarded or accelerated, and by a consequent shifting of the phase of the cross E.M.F. e which joins the extremities of E_i and V_e. The synchronizing power in watts per phase on the approximate assumption underlying a vector diagram may thus at the initial moment be taken as $\frac{E_0 - V_e}{z_a} E_0 \cos \phi_a$, and is a maximum; at any subsequent moment, according to the value which the angle between the terminal and induced E.M.F.'s has reached, it is as in § 4,

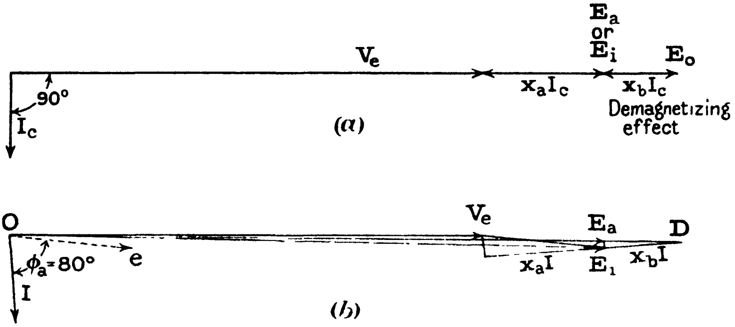

FIG. 697 —Alternator switched into parallel with $E_0 > V_e$

(*a*) Without armature resistance, (*b*) with armature resistance, before the final condition of stability is reached

$$\frac{E_i\{E_i \,.\, \cos\phi_a - V_e \cos(\phi_a + \gamma)\}}{z_a}$$

After a few oscillations a final steady state is reached in which (on our assumption that there is no surplus supply of steam available) any heat loss in the armature is derived from the network, and yet the terminal voltage of the alternator is equalized with that of the network. The final condition is that shown in Fig. 698, from which it will be seen that the cross-current I, which persists in this particular case, is at right angles to the induced E.M.F. of the alternator; it no longer, therefore, has any synchronizing power, but it may be resolved into a purely corrective component I_c which is at right angles to the terminal voltage, and a small negative component in phase with the terminal voltage and by its product therewith giving the watts expended over the armature resistance and supplied from the bus bars of the network. The final angle between E_i and V_e is then the value γ_1 at which $V_e \cos(\phi_a + \gamma_1) = E_i \,.\, \cos\phi_a$ as explained in § 4, and the armature ampere-turns are purely direct magnetizing, since ϕ_0 is 90°. In Fig. 698 E_0 was > V_e and, therefore, the current lags behind E_i, which is also equal to E_a, by 90°, and reduces the magnetization, the demagnetizing effect being indicated by FD, which is at right angles to I. If E_0 had been < V_e, E_i would lead V_e by the angle γ_1, and I would be 90° ahead of E_i.

From this point, if the throttle valve of the engine be opened so as to admit more steam, the driving torque is increased and the rotor is slightly accelerated. The angle γ between the induced and terminal E.M.F.'s gradually changes from negative to positive, so that we pass through stage (1) described in § 4. With further increase in the steam supply and positive value of γ, the alternator becomes a generator, and the successive values of stage (2) are passed through.

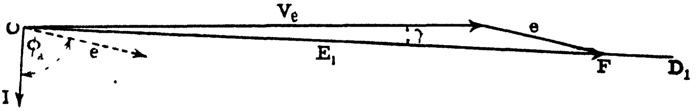

FIG 698.—Final condition of assumed alternator in parallel with $E_0 > V_e$, and supplying no power.

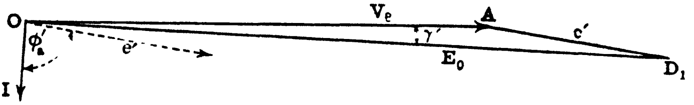

FIG 699—Preceding diagram on synchronous-impedance basis.

In Fig 698 the quantities assumed merely for the purposes of illustration are $E_i = 15$, $V_e = 12$, and for the alternator the same constants as in § 6. The final value of γ_1 is then given by the relation $12 \times \cos(80° + \gamma_1) = 15 \times 0{\cdot}1736$, whence $\gamma_1 = -2{\cdot}55°$, or an angle (negative) of lag.

On the synchronous-impedance hypothesis, x_a', z_a', and ϕ_a' replace x_a, z_a, and ϕ_a, and the angle γ' between E_0 and V_e replaces the angle γ between E_i and V_e Thus Fig 699 reproduces Fig. 698, and the line AD_1 is now $c' = z_a'I$, which precedes the current I by the angle ϕ_a' instead of by ϕ_a, and $OD_1 = E_0$

The drawing of the diagrams is thereby much simplified, although the true power of each machine is not obtained by the projection of the current vector upon E_0, but upon the induced E_i which no longer appears. The error is, however, but small, and the convenience of the method often warrants its adoption for approximate calculations, or to illustrate the actions which take place in alternators working in parallel. The results of § 4 can then be re-stated in terms of the open-circuit E.M F. and of the synchronous impedance, as

$$P = \frac{E_0}{z_a'}\{E_0 \,.\, \cos\phi_a' - V_e \,.\, \cos(\phi_a' + \gamma')\}$$

and

$$P' = \frac{V_e}{z_a'}\{E_0 \,.\, \cos(\phi_a' - \gamma') - V_e \,.\, \cos\phi_a'\}$$

while for particular relations γ' assumes new values which may be marked as γ_1', γ_2', etc.

But the case must also be considered of a machine A synchronized and connected to another machine B which is comparable in size and which is already feeding the network. The effect of inequality of voltage is then similar, but in addition machine B is oppositely affected If the machines are of equal size and similar in design, and further, have negligibly small ohmic resistance, then on no-load the corrective current simply rises to a value $I_c = \frac{E_{01} - E_{02}}{2\,x_a'}$ such that when acting upon the series circuit of pure inductance it produces a terminal voltage equal to the arithmetical mean of the two no-load voltages; being correctly synchronized, the phases of the E.M.F.'s coincide and the effect of the wattless current is purely magnetic. But since the armatures have ohmic resistance, when the alternator A giving E_{01} volts on open-circuit is synchronized with the terminal voltage of the second machine V_e, then, as before, the difference $E_{01} - V_e$ causes a synchronizing current through machine A which at the initial moment may be taken as $\frac{E_{01} - V_e}{z_a}$ lagging ϕ_a° behind the terminal voltage. It is, therefore, a generator current, forming an additional load on machine A if E_{01} is $> V_e$, slightly retarding it when the steam supply is assumed to remain constant. But the same synchronizing current also flows through the armature of machine B, and in relation to this machine is a motor current; or, more correctly speaking, it enters as a component into the total armature current of machine B, and from its direction has the effect of reducing that current. It thus allows the rotor of machine B to be accelerated, and the two machines oscillate on either side of their final steady position. After a few oscillations equilibrium is again established when the terminal voltages are equalized, and also the power developed by each machine is in accordance with its respective steam supply.

If at the moment of switching into parallel, V_e is greater than E_{01}, the reverse holds good; the synchronizing current is at first a motor current through machine A, accelerating it, while machine B is retarded by more work being thrown on to it.

Since the reactance of ordinary machines is many times greater than their resistance, the final position of the vector of the cross current is in both cases very nearly in quadrature with the E.M.F. of either machine, so that it is almost wattless. The component current which at the moment of switching into parallel begins by transferring energy from one machine to the other, after the phase angles of the two machine pole-centres have settled down into their correct relative positions, ends by having a nearly pure magnetic effect. The general result is that the final corrective current which flows through the armatures in series, on the assumption that the steam supplies of both machines are fixed and that their governors do not come into action, affects to some slight extent the watts lost over the resistance of the armature of machine B, and in consequence its output into the external network is indirectly affected.

§ 34. Effect of want of synchronism of phase when switching into parallel.—If γ' be the initial divergence of phase between the open-circuit E.M.F. E_0 of the incoming machine A and the terminal voltage V_e from a number of powerful alternators which may be credited with a vector of terminal pressure revolving with perfectly uniform velocity, the mechanical power which machine A will absorb and convert into electrical energy, or, *vice versa*, is given by the equation for P on p. 543. This is positive when γ' is positive and machine A leads, or negative when γ' is negative and machine A lags. But now if the steam supply is limited to the amount required to make up the constant losses, the driving torque remains constant; hence, if γ' is positive, it is evident that as a synchronizing current grows up it causes an additional electrical resisting torque which can only be balanced by a retardation of the rotor. The rotor and fly-wheel then part with some of their stored kinetic energy, which appears as electrical energy. Or, if γ' is negative, the rotor has work done on it by the network, and is accelerated so that the divergence of phase is reduced.

If the network is fed by a single similar machine B of equal size, the synchronizing current reacts on machine B in exactly the contrary sense. Thus the temporary changes in the speeds of the rotating parts supply a certain amount of positive or negative watts, and, after some free oscillations, the machines by their mutual control pull into step and a state of equilibrium is reached. If machine A is not supplied with any more steam, and E_0 originally $= V_e$, its rotor will have fallen into synchronism with V_e, and it will be carrying no load or current, the synchronizing current having itself disappeared.

Starting with such an expression as (303), δ_e during the oscillations is not constant, but wanes and waxes as the alternators draw nearer to synchronism and then draw apart again; the frequency is approximately constant at the value f_0, but the amplitude decreases from a maximum by an exponential of time. Since f_0 is very low as compared with the machine frequency f, approximately if at any stage δ_0 is the maximum amplitude of the oscillation

$$\Delta P_1 + \Delta P_2 = N_{ph}\, E_t^2 \frac{x_a}{z_a^2} \sin 2\,(\delta_0 \sin 2\pi f_0 t)$$

The average value of this during a half cycle of oscillation, being the time during which it is drawing the two machines together or separating them farther apart, may be represented by

$$N_{ph} E_i^2 \frac{x_a}{z_a^2} \times \frac{1 - \cos 2\delta_0}{2\delta_0}$$

and the time of the half cycle being $1/(2f_0)$, the energy transfer is

$$N_{ph} E_i^2 \frac{x_a}{z_a^2} \times \frac{1 - \cos 2\delta_0}{4f_0 \, \delta_0}$$

which reaches its maximum when $\delta_0 = \frac{\pi}{2}$.

§ 35. Inequality of frequency when switching in.—The action when the machines have not quite the same frequency at the moment of parallel connexion, which was already considered in § 1, is in some respects similar to the last case. We have in effect a progressive succession of cases of want of synchronism of phase, and as before, but to an even greater degree, the case cannot be truly represented by a simple clock diagram of virtual values with fixed phase relations, since the instantaneous values of the quantities are not altering after a sine law, and their vectors have not a strictly constant velocity.

The action depends, too, upon the exact instantaneous phase of the two E.M.F.'s at the moment of switching in. Even if these initially coincide, and so are in direct opposition with regard to the series circuit between the armatures, a resultant difference of potential at once begins to arise, as explained in § 1. A cross current grows up which is, however, no longer determined almost solely by the resistance of the phase lamps, as in § 1, but by the resistance and inductance of the two armatures in series. The phase of the cross current is thereby shifted more nearly into agreement with the E.M.F. of the machine which is at the moment leading and into opposition to the E.M.F. of the lagging machine. The synchronizing current corresponds to the changes in the kinetic energy of the rotors of the machines, the lagging machine being accelerated and the leading machine retarded, while if the inequality of frequency is considerable, the mechanical governors of the prime movers may further be called into action.

If one alternator has an average frequency $(1 - s)f$, say, $= (1 - 0{\cdot}01)50 = 49{\cdot}5$, and the other similar alternator an average frequency $(1 + s)f$, say, $1{\cdot}01 \times 50 = 50{\cdot}5$, so that they differ in frequency by $2sf$, the poles of the two alternators are slipping past each other with the frequency $2sf$. *E.g.* assuming the process to start when one phase E.M.F. of each machine is zero and poles of the same sign are in line, after $1/4s = 25$ cycles $= 0{\cdot}5$ second, a N. pole of one alternator has fallen back so far, and a S. pole of the other

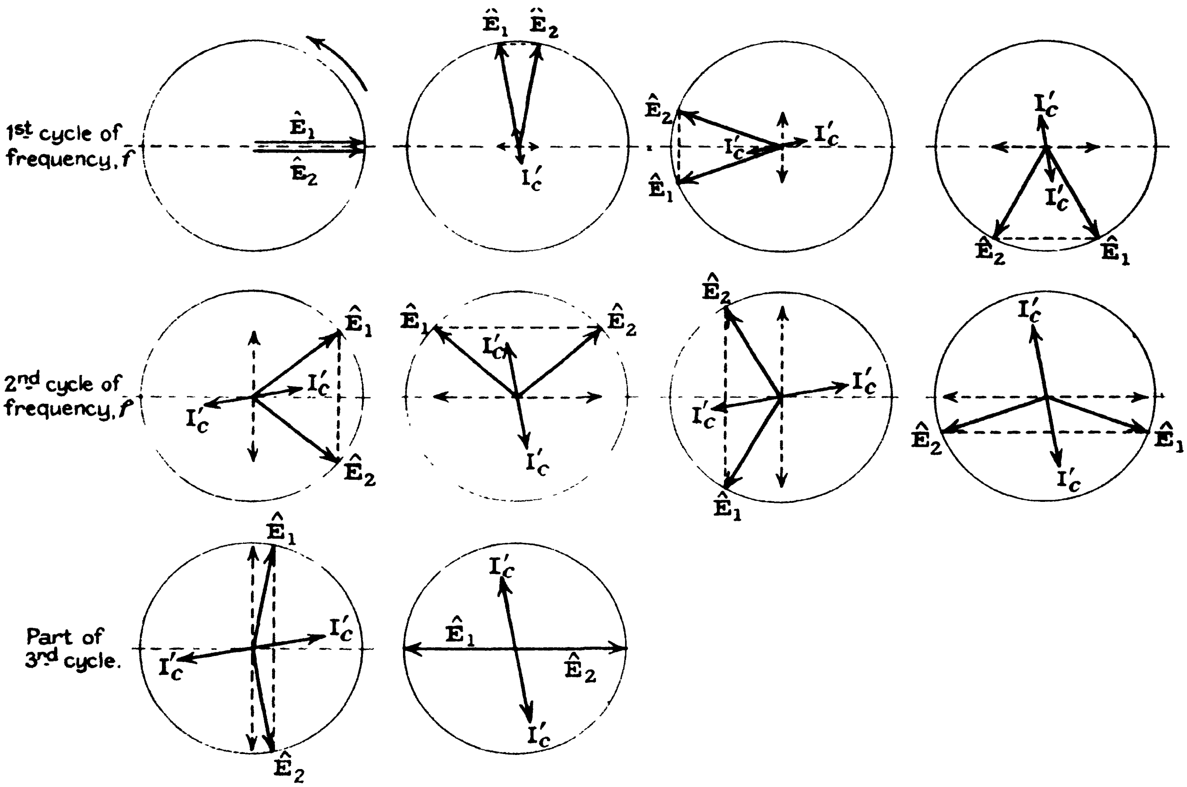

FIG. 700.—Two alternators slipping past each other; one-half of a cycle of frequency, $2sf$; $s=1/9$; $f=10$.

alternator has gone ahead so far, from the position for equal frequency that the two are in line, and after $1/2s = 50$ cycles $= 1$ second, two N. poles are again in line.

To take an exaggerated case, let the slip, s, be $1/9 = 0{\cdot}111$ and f be 10; then for every 1/4 of a cycle of the mean frequency, f, the faster machine, E_1, will diverge an additional 10° (electrical) ahead of the position corresponding to the mean frequency. Two and a quarter cycles are shown at intervals of a quarter period in Fig 700, at the end of which the alternators have passed from in-phase to in-opposition, one machine having gone ahead 90° and the other having fallen behind by 90°, so that their total divergence is 180° (electrical). The dotted vector E_1E_2 is shown transferred to the centre, and behind it lags the interchange current I_c' by an assumed angle $\phi_a = 80°$. I_c' has grown from zero to a maximum, and if the diagram were continued for another two and a quarter cycles, making in all a cycle of frequency, $2sf$, it would again sink to zero, and the E M.F.'s would coincide on the left horizontal line. Poles of the same sign being then again in line, everything repeats, so that there is no occasion to consider more than the cycle of frequency, $2sf$, or one " beat " as in Fig 677 With similar machines, the vector of the terminal voltage revolves at the constant rate corresponding to the mean frequency intermediate between the two machine frequencies, but varies in amplitude, its instantaneous value becoming zero not only twice in every cycle of frequency, f, but also in the middle of every cycle of frequency, $2sf$, when the machine E M.F.'s are in direct opposition.[1]

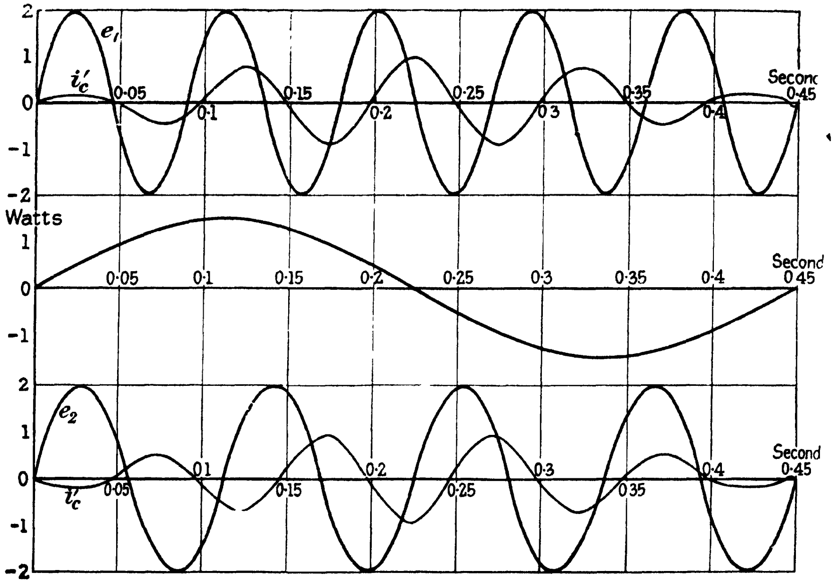

FIG 701 —$s = \frac{1}{9}$; $f = 10$.

Top part = faster machine
Bottom part = slower machine

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[1] In the case of a single self-synchronizing machine coupled to a large network of *constant voltage* and frequency f when coming up to speed from slip s, the cycle to be considered is that of frequency, sf. *Cp.* Dr. E. Rosenberg, " Self-synchronising Machines," *Journ. I.E.E.*, Vol. 51, p. 70.

The instantaneous E.M.F.'s of the two alternators from the assumed starting-point being

$$e_1 = \mathbf{E} \sin (1 + s)\, 2\pi ft$$
$$e_2 = \mathbf{E} \sin (1 - s)\, 2\pi ft$$

the electrical divergence between the two E.M.F.'s is $2\delta_e = 4\pi sft$. The vector $\overline{\mathbf{E}_1\mathbf{E}_2}$ is then $2\mathbf{E} \sin \delta_e = 2\mathbf{E} \sin 2\pi sft$. But this varying vector, as will be seen from Fig. 700, when transferred to the centre, revolves round it with frequency f, and the angle which it makes with the horizontal being $2\pi ft$, its instantaneous projection on the vertical time-axis is proportional to $\sin 2\pi ft$. The total cross-circuit E.M.F. is thus

$$2e_c = 2\mathbf{E} \sin 2\pi sft \times \sin 2\pi ft$$

When s is small, the rate of change of $\sin 2\pi sft$ is so slow as compared with that of $\sin 2\pi ft$, that within any one cycle of frequency f, $\sin 2\pi sft$ may be regarded as constant ; the reactance of each machine, whether for frequency $(1 + s)f$ or $(1 - s)f$, may thus be identified with $x_a = 2\pi f\ /_a$ for the mean frequency. The virtual value of the interchange current is $I_c' = \frac{2\sqrt{2}E_t}{2z_a} \sin 2\pi sft$, and its instantaneous value (Fig 701)

$$i_c' = \frac{\mathbf{E}}{z_a} \sin 2\pi sft \cos (2\pi ft - \phi_a)$$

for the current $\mathbf{I}_c'$ lags by the angle ϕ_a behind $\mathbf{e}_c$, and this precedes the horizontal axis by $(90° + 2\pi ft)$, so that the angle which $\mathbf{I}_c'$ makes with the horizontal is $90° + 2\pi ft - \phi_a$, and its projection on the vertical time-axis is proportional to $\sin (90° + 2\pi ft - \phi_a) = \cos (2\pi ft - \phi_a)$

The instantaneous powers per phase of the two machines separately are

$$e_1 i_c' = \frac{\mathbf{E}^2}{z_a} \sin 2\pi sft \cos (2\pi ft - \phi_a) \sin (1 + s)2\pi ft$$

$$- e_2 i_c' = - \frac{\mathbf{E}^2}{z_a} \sin 2\pi sft \cos (2\pi ft - \phi_a) \sin (1 - s)2\pi ft$$

and their difference, $e_1 i_c' - (- e_2 i_c')$ reduces to[1]

$$\frac{\mathbf{E}^2}{2z_a} \sin 4\pi sft \left[\sin (4\pi ft - \phi_a) + \sin \phi_a \right]$$

The term of double frequency, $\sin (4\pi ft - \phi_a)$, has to do with the periodic storage and release of energy in the inductive circuit, and the power transfer (middle curve of Fig 701), being half the net difference of powers, is

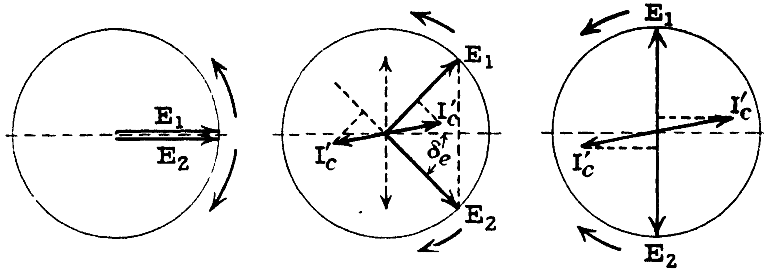
$$\frac{(\sqrt{2}E_t)^2}{4z_a} \sin 4\pi sft \sin \phi_a$$

and in all phases, since $\sin \phi_a = x_a/z_a$

$$= N_{ph}\, E_t^2\, \frac{x_a}{2z_a^2} \sin 4\pi sft$$

---

[1] Beginning with the machines in phase at the moment of their maximum, instead of zero, E.M.F., the same expression is reached, but with $- \sin \phi_a$ instead of $+ \sin \phi_a$. *Cp.* C. P. Steinmetz, " Power Control and Stability of Electric Generating Stations," *Trans. A.I.E.E.*, Vol. 39, Part II, p. 1236, upon which the following treatment is in principle based.



Moment in 1st half

$\delta_e = 0$ $\delta_e = 90°$

$I_c' = 0$ $I_c' = \text{max}$

Angle between E_1 and I_c'

from $(90° - \phi_a)$ through $(90° - \phi_a - \delta_e)$ to ϕ_a

(lead) (lag)

Machine 2 receives energy from 1

⟶ ⟵

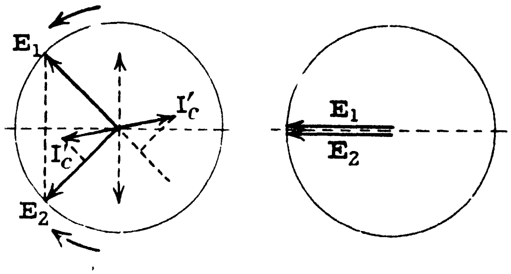
pulling together

Speed of slower machine 2

$\omega(1 - s - s_1)$ rising through ωs to $\omega(1 - s + s_1)$

Speed of faster machine 1

$\omega(1 - s + s_1)$ falling through ωs to $\omega(1 - s - s_1)$



Moment in 2nd half

$\delta_e = 180°$

$I_c' = 0$

Angle between E_1 and I_c'

$90° - \phi_a - \delta_e$ to $-(\phi_a + 90°)$

Machine 1 receives energy from 2

⟵ ⟶

pulling apart

FIG. 702.—One cycle of virtual values, of frequency $2sf$.

We thus pass to virtual values and to consideration of the interchange current I_c', varying comparatively slowly, and of the synchronizing watts throughout a cycle of frequency, $2sf$. The case is then represented by regarding the vector of terminal voltage as stationary in space, although varying in virtual value, while the two vectors E_{i1} and E_{i2} slowly rotate in opposite directions (Fig. 702). When $t = 1/8\,sft$ = a quarter of a cycle, the power transfer $N_{ph} E_i^2 \frac{x_a}{2z_a^2}$ is a maximum : its average value is $2/\pi$ of this, and this average value maintained over half a cycle of frequency $2sf$, *i.e.* for a time $1/4sf$, transfers in energy $\frac{1}{2\pi sf} N_{ph} E_i^2 \frac{x_a}{2z_a^2}$ joules.

The effect of the energy transfer is that the slip from the mean frequency, instead of remaining constant at the value s, which has above been given as its *average* value, is made to vary between values $s \pm s_1$. During the first half of a cycle of frequency $2sf$, when the machines are passing from in-phase to in-opposition, it will be seen from Fig. 702 that the projection of I_c' on the E.M.F. E_2 of the slower machine gives negative or motor watts ; consequently the slower machine during that time receives from the faster machine energy to the last-mentioned amount in joules ; this appears as kinetic energy, the rotor being speeded up from $\omega(1 - s - s_1)$ to $\omega(1 - s + s_1)$, where ω is the angular velocity at the mean frequency f. The effect on the faster machine is the reverse, so that the two machines are brought nearer to each other in speed, *i.e.* they are pulled towards synchronism. During the next half-cycle, when the machines are passing from in-opposition to in-phase, it is the faster machine which receives energy from the slower one, so that they pull apart again. Supposing that the pulling apart and drawing together are exactly equal, the amplitude of the speed oscillation which is superimposed on the average speed of either machine can now be calculated. If M is the mass of its rotating parts, and k the radius of gyration, the gain of stored energy by the slower machine during the first half of a cycle as it is being speeded up is

$$\tfrac{1}{2}\,\omega^2 Mk^2\,[(1 - s + s_1)^2 - (1 - s - s_1)^2]$$

which for small values of s and s_1 is nearly equal to $2Mk^2\omega^2 s_1$ or in ft.-lbs. $\frac{W}{g} r^2 \left(\frac{2\pi N}{60}\right)^2 \times 2240 \times 2s_1$ where W is the weight in tons This must be equal to $N_{ph} E_i^2 \frac{x_a}{2z_a^2} \times \frac{1}{2\pi sf} \times \frac{550}{746}$ ft.-lb., whence

$$s_1 = \frac{N_{ph} E_i^2 \frac{x_a}{2z_a^2} \times \frac{1}{2\pi sf} \times \frac{550}{746}}{2\frac{W}{g} r^2 \left(\frac{2\pi N}{60}\right)^2 \times 2240}$$

Unless, therefore, the energy transferred during the half-cycle is greater than the energy required to speed up the rotor of the slower machine to synchronism, *i.e.* unless

$$N_{ph} E_i^2 \frac{x_a}{2z_a^2} \times \frac{1}{2\pi sf} \times \frac{550}{746} > 2s_1 \frac{W}{g} r^2 \left(\frac{2\pi N}{60}\right)^2 \times 2240$$

the machines continue to slip past each other, drawing together during one-half of the cycle of frequency $2sf$, and drawing apart by the same amount during the next half-cycle. But as soon as the superimposed speed oscillation ωs_1 is equal to the average ωs, or $s_1 = s$, they pull into synchronism. By making $s_1 = s$ in the above equation, it is thus found that the maximum slip from which they can pull into synchronism is

$$s_{max} = \frac{E_i}{2}\sqrt{\frac{N_{ph} \frac{x_a}{2z_a^2} \times \frac{550}{746}}{\pi f \frac{W}{g} r^2 \left(\frac{2\pi N}{60}\right)^2 \times 2240}} \quad . \quad . \quad . \quad . \qquad (325)$$

But in the practical application of all such estimates, two difficulties arise; in the first place, as contrasted with Fig. 677, where the current is negligible, E_i does not remain constant, but varies periodically with the frequency of the current fluctuation owing to the variation in amplitude and phase of the armature reaction from the interchange current; and, secondly, the magnetizing or demagnetizing effect of the armature reaction does not take effect instantaneously, but lags behind the armature reaction by some amount difficult to predict.[1]

§ 36. **Influence of the governor.**—In addition to the rhythmical variation in the driving torque of steam engines, there may also be forced oscillations set up by the governors under sudden variations of load. Since the period of such governor oscillations is fairly long, there is the more likelihood of resonance with the natural period of the alternator, and the only remedy is then to damp the action of the governor by means of a dash-pot. An unduly heavy fly-wheel with long natural period is especially to be avoided in this connection, as well as from the fact that it prolongs the time required for synchronizing when switching into parallel.[2] The action of the mechanical governor introduces so many complexities, since there is a certain lag between the change of steam pressure or steam supply and the change of speed to which it is due, that space prevents a further consideration of the subject.

§ 37. **Influence of the shape of the E.M.F. curve.**—So far, the E M F waves of the alternators have been assumed to have the same sinusoidal shape, but when, as may be the case, they are very unlike in form, the resulting cross E.M.F acting round the circuit of the armatures may cause such a powerful cross current and such violent oscillations as to render parallel working impossible It is evident that a slight divergence of phase between two very peaked waves of E M F. will produce a much larger resultant cross E M F than the same divergence between smooth sine waves Further, the oscillograph shows that on light loads there may in such cases of different wave-forms be alternations of the current more rapid than the true frequency, and consequently rapid changes from generator to motor action. The fundamental sine curves may be correctly opposed to each other in relation to the cross circuit of the armatures, but the harmonics may be in phase and act in series round it. If the frequency of the reversals of power is such that they are reinforced by any approach to resonance with the natural period, hunting may result, even though there is no periodic fluctuation of driving torque from the prime mover.[3]

§ 38. **Damper winding.**—For the purpose of rendering polyphase machines more stable when run in parallel, and more independent of any fluctuation of the prime mover's speed, M. Leblanc introduced the device of an additional *damper* or "*amortisseur*" winding. This consists in principle of a number of uninsulated copper bars embedded in holes or half-closed slots close to the faces of the pole-pieces parallel to the armature slots and joined up to a copper or bronze ring on each side of the poles, so as to form a grid (Fig. 703); there may also too be a similar copper bar on either side of each pole-piece.

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[1] For further continuation of the subject with allowance for the above secondary effects, see C. P. Steinmetz, *loc. cit.*, pp. 1241–1244.

[2] For further discussion on the influence of governor and fly-wheel, see Van der Stegen, *Soc. Belge Electr. Bull.*, 1902, Vol 19, p. 14; C. F. Guilbert, *Bull. Soc. Intern. Elect.*, 1902, Vol. 2, p. 281; P Boucherot, *Trans. Intern. Electr. Congress St. Louis*, 1904, Vol. 1, pp. 697–705, and *Bull. Soc. Intern. Electr.*, 1905, Vol. 5, p. 509, H. H. Barnes, *Trans. Amer. I.E.E.*, Vol. 21, p. 343, "Notes on Fly-wheels"; de Marchena ,*Bull. Soc. Intern. Electr.*, 1902, Vol. 2, p. 754; C. E. Lucke, *Trans. Amer. I.E.E.*, Vol. 26, Part I, p. 1.

[3] See A. F. T. Atchison, *Journ. I.E E.*, Vol. 33, p. 1119.

The projecting ends of the bars are either brazed or electrically welded to the end-rings, and these may either be cast solid or in segments, united by connecting pieces bolted and sweated on. The bars must be tight in the slots owing to the racking nature of the alternating forces to which they are subjected. At high speeds the stress in the end-rings requires calculation. With a whole number of stator slots per pole, the pitch of the half-closed damper slots in a pole-piece is preferably such that the numbers of stator and damper slots in about the effective width of the pole-shoe differ by one; the effect of the damper slots on the shape of the E.M.F. wave is visible, and if their pitch is an exact multiple or submultiple of the stator slot-pitch, they will magnify the possible tooth-ripples. Especially disadvantageous is a stator tooth-pitch divided equally between tooth and slot and a damper slot-pitch equal to the width of either stator tooth or stator slot, which gives rise to high-frequency currents that only heat the damper winding without assisting the mechanical damping.

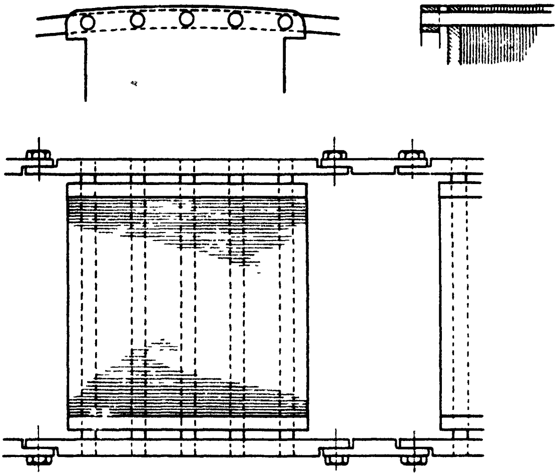

FIG. 703.—Diagram of damper winding.

Fig. 704 shows the damper winding of a 1500 kVA 750 r.p.m. synchronous condenser; the copper rods are brazed to brass end-segments bolted and sweated together between poles. Fig. 705 shows the similar damper on an 1125 H.P. 214 r.p.m. motor.

The tufting of the lines as they sweep over open stator slots with a small air-gap may also set up eddy-currents in the damper bars at no-load—an effect which is obviated if the damper bars are thinly insulated and skewed axially through a stator slot-pitch.[1]

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[1] See Prof. Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, p. 101, and for other sources of loss on load, p. 102.

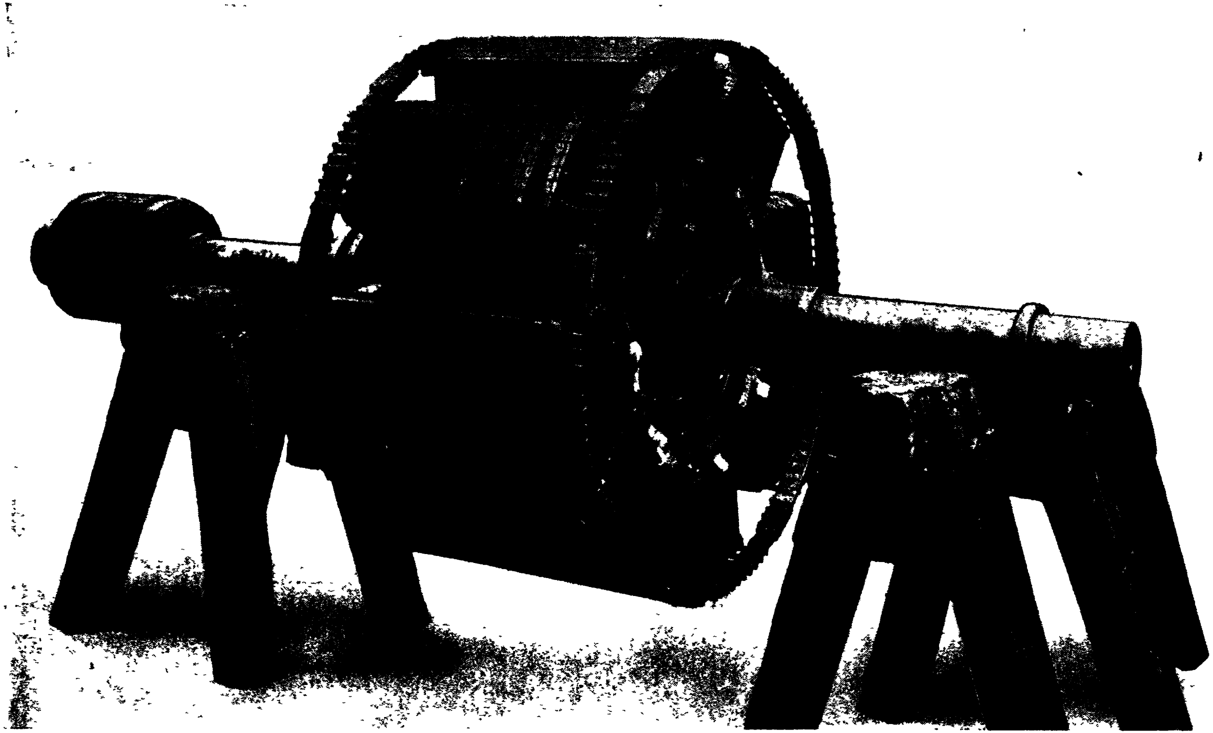


FIG. 704 —Damper winding on 1500 kVA synchronous condenser.

(The Metropolitan-Vickers Electrical Co., Ltd.)

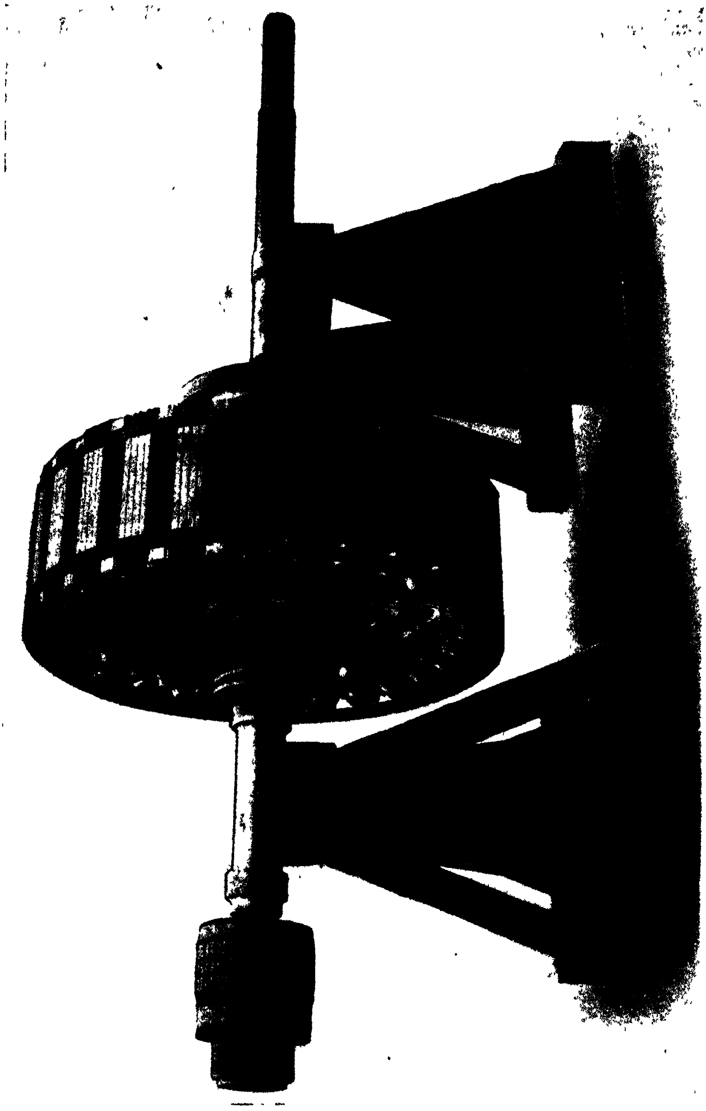
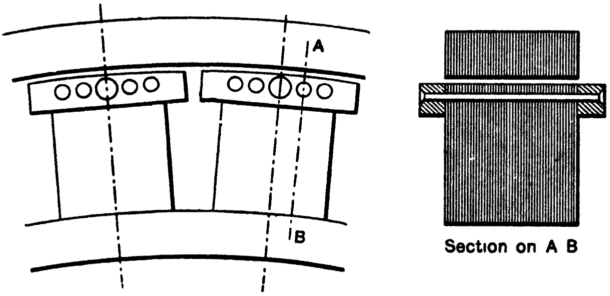


FIG. 705.—Damper winding on 1125 H.P. motor.
(The Metropolitan-Vickers Electrical Co., Ltd.)

On the non-salient-pole rotors of turbo-alternators, whether polyphase or single-phase, the wedges closing the slots that hold the magnet coils, if of copper or bronze, are best joined up by a tongued copper sheet or bronze ring, so as to form an effective damper system without contact with the end-bells of the rotor.

Although it is not essential that the rings short-circuiting the damper bars should be continuous, provided that there is little interference in the paths round which the secondary currents flow, the gaps occurring midway between the poles being only small, yet if not continuous, they cannot be equally effective at all power-factors. The bars of copper may also in the interpolar gaps be replaced by brass grids. Fig. 706 shows a form of damper winding in which each pole is self-contained; the pole-faces are encircled by brass castings, into which a few round copper bars passing through the laminations of the pole are brazed. The casting at the same time serves to hold in place the winding of the field-coils (*cp.* also Fig. 578).



Section on A B

FIG. 706—Damper winding.

As explained in Chapter XXVII, § 9, the progressive rise and fall of the current in the armature windings of a polyphase generator sets up a fundamental joint wave of M.M.F. (*A* Fig. 707), which rotates at the constant velocity corresponding to the average machine speed and which in reality combines with the M.M.F. of the field *AT* at any moment. When only one generator is at work at a constant velocity, the field-magnet rotates at the same speed, and the two M.M.F.'s rotate together in a fixed position relatively to one another depending on the power-factor of the load, *i.e.* are relatively stationary, so that the bars of the damper winding are unaffected, and the same is true if several machines run in parallel with velocities which are perfectly constant.

But if an oscillation is set up in any machine, upon the synchronous speed of the poles (the field-magnet being assumed to be the rotor) there is superposed a small swing of their centre-lines of electrical amplitude δ_e relatively to the stator. Thus, *e.g.* in a non-salient-pole turbo-alternator the trapezoidal wave of field M.M.F. virtually moves backwards and forwards across the sinusoidal wave of armature M.M.F. through a small angle. The rotating field-magnet regarded as a secondary then lags or leads at regular intervals relatively to the armature regarded as a primary, and the differences in the algebraic summation of the two waves so far as they both act on the pole-faces alter the value and spacial distribution of the flux thereat. The secondary currents set up thereby in the bars of the damper winding have the effect described in § 21 as due to currents crossing the pole-faces, so that their function is to make b positive and damp out the oscillation. The eddy-currents which would flow in a solid pole-piece are canalized or confined to paths at right angles to the flux-direction, and are thereby rendered fully effective.

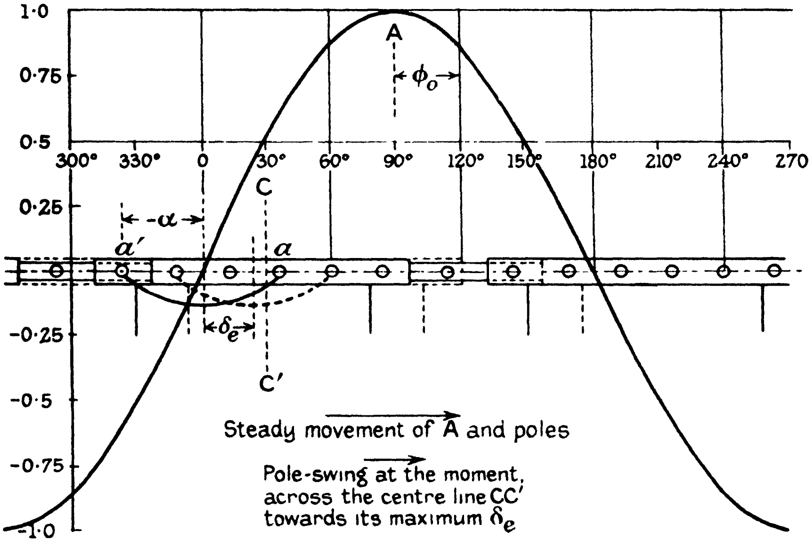

1·0
0·75
0·5
0·25
-0·25
-0·5
-0·75
-1·0
300°
330°
0
30°
60°
90°
120°
150°
180°
210°
240°
270°
A
ϕ_o
C
C′
$-\alpha$
a′
a
δ_e
Steady movement of A and poles
Pole-swing at the moment, across the centre line CC′ towards its maximum δ_e

FIG. 707.—Damper action on non-salient-pole rotor

If the swinging of the magnet does not alter the magnetic effect of the armature M.M.F.—a case which would be practically approached if the smooth-surface cylindrical rotor were uniformly slotted—for the M.M.F.'s may be substituted component air-gap fluxes, and the armature component field over a pole-pitch would be symmetrical in each of its two halves. The damper bars move in this component field, and starting from the moment when they are in the position of steady motion but are moving with the maximum velocity of oscillation across this mean position, a bar a (Fig. 707) will thence move with diminishing velocity into a field of, say, increasing density, but if so, another bar a' at the same distance on the other side of the zero-line of the component field with which the first bar may be paired, will be moving into a field of decreasing density; the pair forming a loop is thus virtually moving with simple harmonic motion through flux-densities B_x and $-B_x'$ dependent on the points x and x' or angles α and $-\alpha$ at which the bars are situated when in their mid-position for steady motion. In Fig. 707 for the particular angle of lag $\phi_0 = 30°$, the full-line poles indicate the steady position, and the dotted lines the position of the poles (shown as if salient for ease of following) and loop $a\ a'$ at the end of a swing of electrical amplitude $\delta_e = 24°$; the span of the loop is $2\alpha = 36°$, its two sides being at 18° and $-$ 18°. In the supposed case, B_x would numerically be equal to $-B_x'$, and if the symmetrical curve of flux was actually sinusoidal, $B_x = B_{max} \sin \alpha$. The instantaneous E.M.F. round the loop at radius R is then $2B_x\, L_f\, \omega_v\, R \times 10^{-8}$ volts.

The average value of the angular velocity of oscillation is equal to the amplitude of the mechanical displacement divided by the time of one-quarter of a cycle or $1/4f_n$, where f_n is the frequency of the oscillation, free or forced. The average E.M.F. of one bar or one side of the loop is, therefore,

$$B_x\, L_f \frac{\delta_e}{p} \times 4f_n\, R \times 10^{-8}$$

and assuming a simple harmonic motion, its virtual E.M.F. is

$$E = \frac{\pi}{2\sqrt{2}} B_x\, L_f \frac{\delta_e}{p} \times 4f_n\, R \times 10^{-8}$$

If $X_2 =$ the reactance of the bar at frequency f_n, *i.e.* $2\pi f_n\ '$, its impedance is $\sqrt{R_2^2 + X_2^2}$, where R_2 must take into account a share of the end-ring resistance, of which the reactance is practically negligible; its virtual current $E/\sqrt{R_2^2 + X_2^2}$ lags behind the E M.F. by the angle $\phi_2 = \tan^{-1} \frac{X_2}{R_2}$, or $\cos \phi_2 = R_2/Z_2$. Hence in Fig. 708 the current has been shown as changing its direction after the direction of the component field has changed.

Any secondary current set up must either by Lenz's law oppose the originating cause by producing a torque tending mechanically to damp the movement or reduce the originating cause by a magnetic damping of the variation of the embraced flux. As the loop swings to one side or the other of its mean position, either more positive or more negative lines from the supposed armature field enter it, and the magnetic effect of the loop ampere-turns is to reduce the initial variation of flux. If the reactance were very high relatively to the resistance, the angle of lag of the current would approach 90° (electrically), and at the end of the swing when the difference in the embraced flux of the armature field is at its greatest, the current and the secondary ampere-turns would be nearly at their maxima. If the relatively high ratio of the reactance to the resistance is due to the reactance having a very high absolute value, the current would be so small that its torque would yield little mechanical damping and its ampere-turns little magnetic damping. The reactance of the embedded damper bars, although appreciable, is not in fact great, owing to the comparatively low frequency of the oscillation of the order of 2 cycles per sec., but there remains the further possibility that the resistance may be very low. If so, the magnetic damping is very great, but the mechanical damping still remains inappreciable. If the loop were perfectly conducting, *i.e.* of zero resistance, the magnetic damping would be perfect, whatever the value of the reactance, so long as it was finite; the slightest tendency for the flux to vary would be instantly checked by opposing ampere-turns, so that in such an ideal case the secondary E.M.F. and the current would each end by remaining zero, and there would be no mechanical damping of the oscillation. A thick copper plate over the pole-face would approach such an ideal case.

For mechanical damping, therefore, there must be resistance in the loop, which calls for a consumption of mechanical energy in the movement. In the equally ideal case of negligible reactance, the E.M.F. and current would be in step, and the mechanical damping torque would be greatest for a given resistance, being greatest in the mid-position and thence diminishing, but there would be little or no magnetic damping. When both reactance and resistance are present, as must necessarily be the case, the resultant flux-change just rises to such a value as to give the secondary current which by its displaced ampere-turns reduces the initial flux-change to its resultant amount. The maximum torque due to the ampere-conductors in the resulting flux-density then occurs at some intermediate point. The resultant change of flux through the loop lags behind that which would be due solely to movement in the armature field if acting alone. The amount of mechanical damping is thus proportional to the amplitude of the resulting flux-change, which is reduced by increased conductivity, and to its phase-difference from the change that would be due to the armature M.M.F. alone. Hence, as Prof. B. Hopkinson has pointed out,[1] it is possible to increase the conductivity too far, when the diminished amplitude outbalances the increased phase difference; in other words, it is possible to put too much copper into the damper, and the extent to which it is embedded in the iron admits of some variation in the design so as to secure the right relation between its inductance and its resistance to exert maximum effect for the frequency of the oscillations which it is especially to damp out.

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[1] *Proc. Royal Society*, Vol. 72, p. 235; and *Electr. Eng.*, Vol. 32, p. 471, *Electr.* Vol. 51, pp. 656, 704, 886.

§ 39. Damper winding in the salient-pole machine.—The case of the salient-pole machine is somewhat different, since, as the poles swing, the effect of the armature M.M.F. wave on the varying magnetic circuit presented to it alters ; hence, instead of remaining stationary relatively to a rotor that moved with constant velocity, the distribution of the component armature flux-curve alters periodically. Thus the particular shape of Fig. 708, assumed for some given angle of lag of current, is only true for one particular moment, and the moment for which this diagram has been drawn is that when the centre-lines *CC′* of the poles are actually moving across the position of steady motion.

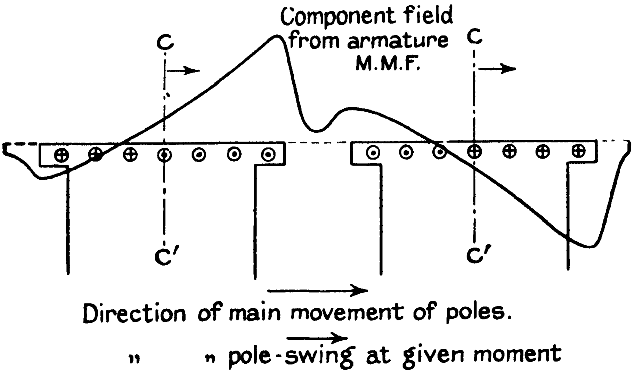
Component field from armature M.M.F.
C
C
C′
C′
Direction of main movement of poles.
" " pole-swing at given moment

FIG 708.—Damper action on salient-pole rotor.

Resolving the amplitude A of the joint fundamental of the armature reaction F_{AR} into two components, one above the centre-line of the pole and one above the centre-line of the interpolar gap (Fig. 709), the amplitude of the back effect becomes $A \sin(\phi_0 + \delta)$, and that of the cross effect $A \cos(\phi_0 + \delta)$, where δ is the electrical displacement from the position of steady motion. The time-value of the latter is $\delta_e \sin 2\pi f_n t$, where δ_e is the amplitude of the displacement and t is time reckoned from the moment when the centre-lines of the poles cross the position of steady motion in the direction of rotation. In Fig. 709 it is assumed in the dotted lines that $t = 1/4f_n$, so that the displacement has its maximum value δ_e.

Now the curve of *a* changing to *a′* always stands symmetrically over the pole, and the curve of *b* changing to *b′* always stands symmetrically over the interpolar gap. All that affects the damper loops is, therefore, in either case the pulsation of the embraced flux due to the changing amplitudes. Reckoning the spacial average of the back M.M.F. as a scalar value, its change is

$$1{\cdot}257\,A\,\{\sin\phi_0 - \sin(\phi_0 + \delta_e \sin 2\pi f_n t)\}\,\frac{2Y}{\pi y}\sin\frac{\pi}{2}\frac{y}{Y}$$

at any moment, and since δ_e is small, this may be identified with

$$1{\cdot}257\,A\cos\phi_0\sin(\delta_e\sin 2\pi f_n t)\,\frac{2Y}{\pi y}\sin\frac{\pi}{2}\frac{y}{Y}$$

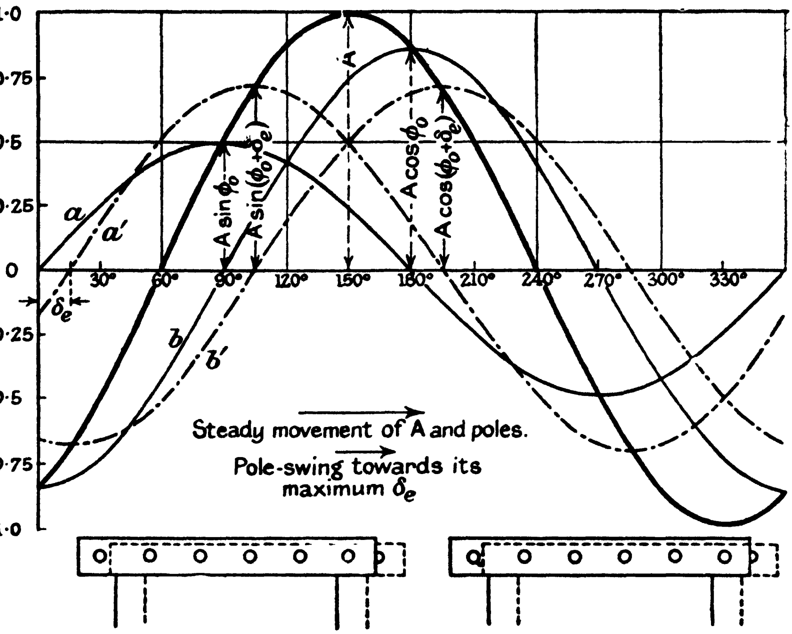

FIG. 709 —Resolution of joint armature M.M.F. wave into components above and at right angles to pole-centres, for two positions.

Dividing by l_g, the density of the back flux B_b is obtained, and the flux linked with any damper loop with sides at equal distances x from the centre of the pole is proportional to its total width. Its instantaneous E.M.F. is then $\propto 2xB_bL_f \times 2\pi f_n \cos(\delta_e \sin 2\pi f_n t)$.

Similarly, there is a changing amplitude of flux through each damper loop with sides at equal distances from the interpolar line of symmetry, and in both cases the resulting flux-change is limited in amount by the secondary ampere-turns and made to lag in point of time. The virtual current in each bar is then the square root of the sum of the squares of the two component virtual currents which are spacially in quadrature.[1] The point of normal air-gap density still oscillates on either side of its mean position, but the oscillation is reduced in extent and displaced temporally.

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[1] For a more detailed analysis, see especially M. Liwschitz, " Torque and Design of Damping Windings for Polyphase Synchronous Machines in Parallel Operation," *Archiv für Elektrotechnik*, Vol. 10, p. 96 ; and J. Bethenod, *Soc. Franc. Elect Bull.*, Vol. 3, p. 29.

§ 40. Approximate calculation of damper winding.—At the moment when the poles are in the position of steady motion and the E.M.F. induced in each bar is at its maximum, the situation is analogous to that of the squirrel-cage armature of an induction motor when the slip frequency sf is equal to f_n of the damper, for the armature field is at that moment moving past the damper bars with the maximum velocity ω_0. The resemblance is not, however, exact, in so far as there may be no damper bar at the point of maximum density of the component field; each loop has its own virtual E.M.F. differing from that of others according to its position in the pole, and the process is not maintained as a continuous rotation. Each bar is not, therefore, subjected in turn to the maximum field, as in the squirrel-cage armature of the motor. Yet at the given moment assumed above, the analogy holds, and since the torque of the squirrel-cage armature can be calculated, it assists in an estimate of the damping torque in the alternator.

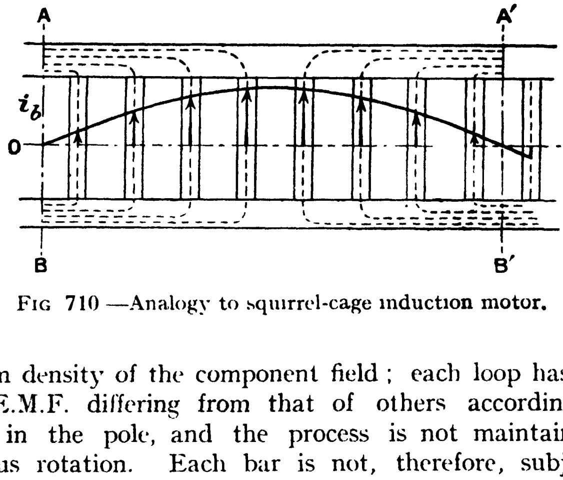

FIG 710 —Analogy to squirrel-cage induction motor.

Passing then to the squirrel-cage analogy, let I_b = the virtual current in one damper bar, of which there are g_2 in a pole-pitch and $S_2 = 2pg_2$ on the whole machine, and let R_b = its resistance in ohms when hot, with allowance for the joints to the end-rings. The watts in it are, therefore, $R_bI_b^2$. A given section of one end-ring when opposite the dividing line where the current changes direction in the bars from back to front or *vice versa* (Fig. 710), collects the current of $g_2/2$ bars; the average current in each bar being $\sqrt{2}I_b \times 2/\pi$, this amounts to $g_2\sqrt{2}I_b/\pi$, which is the maximum current in the end-ring section. The same also holds for all other sections as the dividing line moves along, so that the virtual current everywhere in the ring is $I_{er} = g_2I_b/\pi$.

Let R_{er} be the resistance of a complete end-ring, when hot, also with allowance for bolted joints, if any. Then the resistance of two half pole-pitches, one on either side of the pole, is $\frac{R_{er}}{2p} \times \frac{1}{2} \times 2 = R_{er}/2p$, and the watts therein

$$\frac{R_{er}}{2p} \times \frac{g_2^2}{\pi^2} \times I_b^2$$

Dividing by $g_2/2$, the end-ring watts attributable to one bar are

$$\frac{R_{er}}{p} \times \frac{g_2}{\pi^2} I_b^2$$

Thence from the total watts may be found the resistance of a bar and its share of the end-rings as

$$R_2 = R_b + \frac{R_{er}}{p} \times \frac{g_2}{\pi^2}$$

Let the stator winding now be treated as the primary of an induction motor, to which is supplied 3-phase electrical energy at a rate somewhat in excess of $3V_eI \cos \phi_e$ or the normal full-load kilowatts; part of the excess being balanced against the stator ohmic watts and the losses in the iron core of the stator which would occur if it actually formed part of an induction motor, the remainder of the power is transmitted across the air-gap into the rotor, and let this amount be equal to the full kW output of the generator ($3V_eI \cos \phi_e$) *plus* the rotor ohmic watts. This rotor electrical input appearing as mechanical power and rotor copper loss, the slip which, as is well known, is

$$s = \text{rotor electrical loss/rotor input}$$

becomes $$= \frac{2pg_2(R_2I_b^2)}{1000 \times \text{full-load kW} + 2pg_2(R_2I_b^2)}$$

The ampere-turns of the damper must be an exact image or replica of the ampere-turns of the stator, so far as these are due to the load component current which is a factor in the mechanical power transmitted, *i.e.* in the full kilowatt load.

The ratio $$\frac{\text{rotor current per damper bar}}{\text{equivalent stator current per phase}} = \frac{I_b}{I_2'}$$

$$= \frac{\text{No. of primary phases} \times \text{primary conductors per phase} \times \text{distribution factor}}{\text{No. of secondary phases}}$$

$$= \frac{Z \times 0{\cdot}955}{S_2},$$

and I_2' on the basis supposed $= J \cos \phi_e$. Hence approximately the virtual current per damper bar must be

$$I_b = \frac{JZ \cos \phi_e}{S_2}$$

$$\text{and } s = \frac{R_2\,(JZ \cos \phi_e)^2}{S_2 \times 3V_e I \cos \phi_e + R_2\,(JZ \cos \phi_e)^2} \quad . \quad . \quad . \quad (326)$$

where $S_2 = 2pg_2 =$ the total number of damper slots or bars.

Taking some data for R_2 and S_2 according to the design proposed, the above provides a description of the damper, since if $100s$ is the percentage slip at which it gives full-load torque, the damper would then be described as a $100s$ per cent. damper.[1]

Now the full-load torque in lb-ft. is

$$7{\cdot}05\,\frac{3V_e I \cos \phi_e}{N}$$

and this has been obtained by the above damper when the relative angular velocity between the rotor and the revolving stator field is $s \times 2\pi N/60$ radians per second. The damping torque being proportional to the relative angular velocity, its value for $d\theta/dt = 1$ radian per second is

$$b = 7{\cdot}05\,\frac{3V_e I \cos \phi_e \times 60}{2\pi N^2 s} = 67{\cdot}4\,\frac{1000 \times \text{kW}}{N^2 s} \text{ lb.-ft.} \quad . \quad (327)$$

§ 41. The effectiveness of damper winding.—The natural frequency of the machine is not greatly modified by the presence of the damping unless b is quite large, and the function of the damper in the case of the free oscillations is mainly to limit their amplitude.

Forced oscillations, if of long period as due to the hunting of the engine governor, are likewise reduced by damper winding. But if the forced oscillations are of short period, as due to unequal turning moment, the usefulness of the damper is great when the fly-wheel, although heavy enough from the point of view of cyclic irregularity of a single machine, gives a condition of resonance when two or more machines are connected in parallel. From Fig. 696 it will be seen that to reduce appreciably the component of the initial variable torque which is at right angles to the damping torque, the latter must approach equality to the initial variable torque.[2] But when there is resonance, and actual equality of the damping and initial variable torques holds, the displacement is so greatly reduced, that damper winding may render parallel working possible, *e.g.* with slow-speed alternators driven by gas-engines,[1] even when there is actual resonance between f_0 and a forced f_e. But the 1920 Standardization Rules for Electrical Machinery of the British Electrical and Allied Manufacturers' Association state that reliance should not in such cases be placed upon damper winding, if the co-efficient of fluctuation of energy (§ 28) exceeds 10 per cent.

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[1] See Prof. Miles Walker, *Specification and Design of Dynamo-electric Machinery*, p. 354; and E Rosenberg, *Journ. I.E.E.*, Vol. 42, p. 548.

[2] *Cp.* Dr. E. Rosenberg, *Journ. I.E.E.*, Vol. 42, pp. 535 and 547.

The proportion that the section of copper in the damper bars bears to the cross-section of the stator conductors varies greatly and may range from 25 to 70 per cent.

§ **42. Damper winding on single-phase alternators.**—In the single-phase alternator the action of the damper winding is somewhat different. The secondary currents set up in it partially neutralize the armature ampere-turns, and so reduce the armature reaction or increase the output for the same drop of volts. The effect from the armature ampere-turns as acting on the main magnetic circuit does not rotate, but, although stationary, it pulsates in strength; it may, therefore, as already explained in Chapter XXVII, § 34, be replaced in imagination by two rotary fields, each of constant strength equal to half the maximum of the varying field. The one component field when the speed is perfectly steady rotates synchronously with the magnet or the damper, and has, therefore, no effect on it, but when free oscillations are set up the currents set up in the damper by this component have a damping effect and render the running stable. The other component rotates at the same speed of synchronism, but in the opposite direction, so that its velocity relatively to the damper is 2ω, and currents are set up in the damper of twice the machine frequency. These currents are nearly opposed to the ampere-turns causing the field to which they are themselves due, so that in the ideal case the inductance of the armature should be halved. In practice so large a reduction is not fully realized owing to the leakage of lines in the relatively large air-gap between the armature coils and the damper, but there is a marked reduction in the value of ζ as compared with the same machine without the damper winding. Since the damper bars demand a considerable amount of copper (in many cases more than half of that of the exciting coils) and add to the expense of manufacture, it is sometimes the case that the same results could be more cheaply obtained by increasing the field strength and by adding part only of the extra weight of copper to the magnet-coils.[2] But the great reduction in the flux pulsation which is secured by dampers is shown by the reduction in the temperature of the same single-phase turbo-alternator that is obtained when they are employed.[1]

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[1] *Cp.* Prof. Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 226–7, and p. 237.

[2] *Cp* Fischer Hinnen, *Elec World and Eng.*, Vol. 38, p. 1058; and *Elec. Eng.*, Vol. 20, pp. 597 and 654.

In the single-phase case the stator ampere-conductors, which above formed a datum for the calculation of the damper, must now be taken as one-half only of the total ampere-conductors of the armature.

If it is not required to reduce the virtual inductance of the armature, the damper winding may be embedded more deeply in the pole-faces, so that only its function of damping oscillations of long period for which it is practically non-inductive remains.

Both in polyphase and single-phase machines, reducing the resistance of the exciting circuit by adding more weight of copper to it would in some cases be as effective in damping out free oscillations and rendering the working stable as an actual damper winding.

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[1] *Cp.* W. L. Waters, *Trans. Amer. I.E.E.*, Vol. 27, Part II, p. 1069.

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